Explicit formulas for e-positivity of chromatic quasisymmetric functions

Seung Jin Lee

Abstract

In 1993, Stanley and Stembridge conjectured that a chromatic symmetric function of any $(3+1)$-free poset is e-positive. Guay-Paquet reduced the conjecture to $(3+1)$- and $(2+2)$-free posets which are also called natural unit interval orders. Shareshian and Wachs defined chromatic quasisymmetric functions, generalizing chromatic symmetric functions, and conjectured that a chromatic quasisymmetric function of any natural unit interval order is e-positive and e-unimodal. For a given natural interval order, there is a corresponding partition λ and we denote the chromatic quasisymmetric function by X_λ.

In this talk, I introduce local linear relations for chromatic quasisymmetric functions, and how they can be generalized, which we call Rectangular Lemma. For example, when λ is contained in a rectangle such that the width of the rectangle is greater than or equal to the height of the rectangle, one can expand X_λ in terms of X_μ's where μ is a rectangle, and the lemma states the positive explicit formula for the coefficients of the expansion. Moreover, the similar formula holds for e-positivity of chromatic quasisymmetric functions when λ is contained in a rectangle. If time permits, we discuss a relation with q-hit numbers, counting certain rook placements. This is a joint work with Sue Kyoung Soh.

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Mathematical tools for analyzing Deep Neural Networks : Bifurcation analysis and Lipschitz constant

Byungjoon Lee

Abstract

In spite of the great success of deep learning, its training process has not been completely understood. The objective of this talk is to introduce two useful tools for mathematical analysis in training of Deep Neural Networks (DNNs), one for bifurcation analysis and the other for Lipschit constant. Based on these tools, strategies to avoid gradient explosion phenomena in training Recurrent Neural Networks (RNNs) and to determine the optimal learning rate in traininge DNNs are presented and discussed in this talk.

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Fast extra gradient method for smooth structured nonconvex-nonconcave minimax problems

Donghwan Kim

Abstract

Modern minimax problems, such as generative adversarial network and adversarial training, are often under a nonconvex-nonconcave setting. So, developing an efficient method for such setting is of interest. Recently, two variants of the extragradient (EG) method are studied in that direction. First, a two-time-scale variant of the EG, named EG+, was proposed under a smooth structured nonconvex-nonconcave setting, with a slow $O(1/k)$ rate on the squared gradient norm, where k denotes the number of iterations. Second, another variant of EG with an anchoring technique, named extra anchored gradient (EAG), was studied under a smooth convex-concave setting, yielding a fast $O(1/k^2)$ rate on the squared gradient norm. Built upon EG+ and EAG, this work proposes a two-time-scale EG with anchoring, named fast extragradient (FEG), that has a fast $O(1/k^2)$ rate on the squared gradient norm for smooth structured nonconvex-nonconcave problems. This work further develops its backtracking line-search version, and studies its stochastic version. This is a joint work with Sucheol Lee.

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Some initiatives on Hankel matrices, orthogonal polynomials, and Riordan group theory

Minho Song

Abstract

A Hankel matrix is a square matrix in which each ascending skew-diagonal from left to right is constant. In this talk, we first introduce necessary and sufficient conditions for having a nice LDU decomposition of a Hankel matrix and give a combinatorial interpretation. From this observation, we examine which orthogonal polynomials have constants that coincide with the moments up to sign. Second, we introduce a map that sends a general k-Bell matrix to a (k+1)-Bell matrix. Using this map, we give a conjecture of the explicit form of the map for the k-Bell matrix generated by the Catalan generating function. Finally, we introduce relations among the RNA, Motzkin, Catalan, and Pascal matrices and give them simple combinatorial interpretations. Introducing the B-sequence, we interpret more general situations in terms of separating peaks of paths.

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Positivity phenomenon in classical orthogonal polynomials

Donghyun Kim

Abstract

We introduce positivity phenomenon happening in coefficients and moments of classical orthogonal polynomials. We will start by giving some background on orthogonal polynomials and connection to the asymmetric simple exclusion process. Then various results toward this positivity phenomenon will be discussed.

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Consensus Analysis for Multi-agent Systems

Arumugam Parivallal

Abstract

Multi-agent systems are special type of complex dynamical systems that incorporates multiple self-organizing interacting agents. In general, an agent may practically describe a robot, a sensor, an autonomous train, an aircraft and so on. In particular, multi-agent systems can accomplish complex assignments in a cooperative manner which are too complicated to carry out by isolated systems. To be precise, the complex assignment will be partitioned into numerous small assignments and then allocated to particular agents. Finally, the complex assignment will be fulfilled via the interaction among agents.

Generally, the conventional centralized control method is a widely applied technique to deal with isolated control systems. However, due to computation capability, the conventional control methods fail for interconnected systems. In order to deal with these problems an efficient control architecture known as the distributed control technique is developed. In the distributed control technique, separate controllers have been developed to deal with each subsystem of an interconnected system. The subsystems can exchange their information by using the communication among these local controllers. Owing to this fact, the distributed controllers are considered to be more efficient than conventional controllers. Cooperative control is one of the most important techniques built based on the architecture of distributed control. Cooperative control of multi-agent systems has gained attractive research consideration among the scientific community due to its wide potential applications in systems and control theory. Consensus is one of the most fundamental and important features in cooperative control for multi-agent systems. The objective of consensus in multi-agent systems is to develop an agreement protocol such that the agents achieve a common value subject to a certain quantity of interest by negotiating with their neighbors.

In this talk, we briefly explain the types of consensus problems for multi-agent systems and control techniques used to ensure the consensus.

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Dynamics of non-smooth ADMM

Bomi Shin

Abstract

In this talk we analyze topological perspective with dynamics on the alternating direction method of multipliers (ADMM). We remind the ordinary differential inclusion derived from the non-smooth ADMM. We give properties of the omega-limit set for the multi-valued flow derived from non-smooth ADMM and prove for certain parameters that the omega-limit sets contains an equilibrium. Also, we consider some conditions for the existence of global attractor of the differential inclusion.

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Introduction to dynamical system of Lucas polynomial over the ring of 2-adic integers

Myunghyun Jung

Abstract

Like the relation between Fibonacci numbers and Fibonacci polynomials, Lucas polynomials are a polynomial sequence which can be considered as a generalization of the Lucas numbers.

$$L_n(x) = \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{n}{n-k} \binom{n-k}{k} x^{n-2k}$$

We show the dynamical systems of Lucas polynomials over the ring of 2-adic integers by investigating the minimal decomposition which consists of minimal subsets and the attracting basin.

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