Finding needles in haystacks: combinatorial perturbation theory

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Many problems in combinatorial matrix theory can be phrased as:

Does there exist a matrix M with property P having structure S?

For example: does there exist a matrix M with the property that its eigenvalues are $1, 2, \ldots, n$ having the tridiagonal structure

$$\begin{bmatrix} a_1 & b_1 & & & & \\ b_1 & a_2 & b_2 & & & \\ & \ddots & \ddots & \ddots & & \\ & & b_{n-2} & a_{n-1} & b_{n-1} \\ & & & b_{n-1} & a_n \end{bmatrix}, \text{ where } b_1, \ldots, b_{n-1} \text{ are each nonzero?}$$

These problems tend to be difficult, much like finding a needle in a haystack. In this talk we will describe a newly developed method, based on the implicit function theorem, that gives conditions under which the existence of a needle in a given haystack guarantees the existence of a needle in each nearby haystackes.

The technique will be illustrated through some examples related minimum rank, inverse eigenvalues problems and orthogonal matrices.