Online list colouring of graphs

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Abstract

Assume G is a graph and $f : V(G) \to \{1, 2, \ldots\}$ is a mapping which assigns to each vertex a positive integer. The *f-painting game* on G is played by two players: Lister and Painter. Initially, each vertex v of G has $f(v)$ tokens. In each round, Lister chooses a subset X of the remaining vertices of G and removes one token from each of the chosen vertices. Painter chooses a subset I of X which is an independent set of G, and delete vertices in I from G. The game ends if either all vertices are deleted, or there are remaining vertices with no token left. If all vertices are deleted, then Painter wins the game. Otherwise, Lister wins the game. We say a graph G is f-paintable if Painter has a winning strategy for the f-painting game on G. We say G is k-paintable if G is f-paintable for the constant function $f \equiv k$. The painting game is also called the on-line list colouring game. If X_i is the set of vertices chosen by Lister on the ith round, then we say i is a permissible colour for vertices in X_i. If I_i is the independent subset of X_i chosen by Painter, then we say vertices in I_i are coloured with colour i. For each vertex v, $f(v)$ is the number of permissible colours for v. Painter's goal is to colour all the vertices with permissible colours, and Lister's goal is construct a list assignment so that Painter cannot colour all vertices. The *painter number* or the *online choice number* of G, denoted by $\chi_P(G)$, is the minimum k for which G is k-paintable. It follows from the definition that if G is f-paintable, then G is f-choosable. The converse is not true. So for any graph $\chi_P(G) \geq \mathrm{ch}(G)$, where $\mathrm{ch}(G)$ is the choice number of G. For many problems concerning the choice number of graphs, the corresponding problems for the paint number are interesting and usually more challenging. For example, many upper bounds for the choice number of families of graphs remain to be upper bound for their paint number. However, some of the proofs are quite different, and shed new lights on the original problem for choice number. In this talks, I shall survey results and open problems concerning the paint number of graphs.