TORIC RICHARDSON VARIETIES AND BRUHAT INTERVAL POLYTOPES

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The standard action of a complex torus $T = (\mathbb{C}^*)^n$ on the complex vector space $\mathbb{C}^n$ induces an action of T on the full flag variety $\mathcal{F}\ell(\mathbb{C}^n)$. The set of fixed points can be identified with the set $\mathfrak{S}_n$ of permutations on $\{1, 2, \ldots, n\}$. For given permutations v and w in $\mathfrak{S}_n$ with $v \leq w$, we define the Richardson variety X^v_w, which is a T-invariant subvariety of the full flag variety $\mathcal{F}\ell(\mathbb{C}^n)$ and the fixed point set is identified with permutations $\{z \in \mathfrak{S}_n \mid v \leq w \leq w\}$. The moment map image of the Richardson variety X^v_w is a Bruhat interval polytope, which is the convex hull of points $(z(1), \ldots, z(n)) \in \mathbb{Z}^n$ for permutations $v^{-1} \leq z \leq w^{-1}$. We first observe the relation between Richardson varieties and Bruhat interval polytopes. Then, we study sufficient condition on v and w such that the Richardson variety X^v_w to be a smooth toric variety. In particular, we show that smooth toric Richardson varieties are Bott manifolds. This is joint work with Mikiya Masuda and Seonjeong Park.

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