

# 제6회 정수론 페스티벌

Number Theory Festival 2017

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- Place : 성균관대학교 자연과학캠퍼스  
(Sungkyunkwan Univ. National Science Campus)

- Date : 2017. 2. 20 (Mon) - 2017. 2. 22 (Wed)

- Organizer :

- \* Chang Heon Kim (SungKyunKwan Univ.)
- \* Daeyeol Jeon (Kongju National Univ.)

Sponsored by



한국연구재단  
National Research Foundation of Korea



AORC  
응용대수 및 최적화 연구센터

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## Schedules

	2017. 2. 20. (Mon)	2017. 2. 21. (Tue)	2017. 2. 22. (Wed)
9:30 - 10:10		김병두 (V.U.W)	이현석 (연세대)
10:30 - 11:10		김찬호 (KIAS)	임수봉 (성균관대)
11:30 - 12:10		최도훈 (한국항공대)	Free Discussion
		Lunch	
14:00 - 14:40	최준화 (POSTECH)	박철 (KIAS)	
15:00 - 15:40	김광섭 (KIAS)	양재현 (인하대)	
16:00 - 16:40	박지훈 (POSTECH)	권용재 (인하대)	
17:00 - 17:40		주장원 (서울대)	
18:00 -		Banquet	

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## Abstracts (가나다 순)

권용재\* (인하대학교), 양재현 (인하대학교)

**Title:** The rank of an elliptic curve

**Abstract:** In this talk, we survey the recent works about the rank of an elliptic curve and the Selmer groups of an elliptic curve that were done by Manjul Bhargava and his colleagues. In the end of this talk, we make a comment on the new directions to the study of the rank of an elliptic curve.

김광섭 (KIAS)

**Title:** The minimal number of Frobenius elements of  $G_{K,S}$  whose conjugacy classes generate the whole group  $G_{K,S}$

**Abstract:** Assume that  $K$  is a number field and  $S$  is a finite set of primes of  $K$ . Let  $G_S(K)$  be the Galois group  $\text{Gal}(K_S/K)$ , where  $K_S$  is the maximal extension of  $K$ , which is unramified outside the primes in  $S$ . Suppose that the rank of  $G^{ab_{K,S}}$  is  $r$ . In this talk, we use topological methods to show that  $G_{K,S}$

김병두 (Victoria Univ. of Wellington)

**Title:** The Mordell-Weil ranks over cyclotomic extensions of abelian varieties defined over number fields whose primes above  $p$  are totally ramified.

**Abstract:** Continuing Kim's earlier work ("Ranks of the Rational Points of Abelian Varieties over Ramified Fields, and Iwasawa Theory for Primes with Non-Ordinary Reduction," preprint) on the Mordell-Weil ranks over cyclotomic extensions of elliptic curves over number fields whose primes above  $p$  are totally ramified, we study the Mordell-Weil ranks of abelian varieties under similar conditions.

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Our model is Barry Mazur’s “Rational Points of Abelian Varieties with Values in Towers of Number Fields” (*Inventiones math.* 18, 183–266 (1972)), and Bernadette Perrin-Riou’s “Theorie d’Iwasawa  $p$ -adique locale et globale” (*Invent. math.* 99, 247–292 (1990)). There are two main difficulties: good non-ordinary reduction, which is generally harder than good ordinary reduction, and the ramification of the field over which the abelian varieties are defined.

We follow Kim’s earlier work, which in turn adopted and further developed Perrin-Riou’s ideas. However, what we have to overcome is not trivial at all. In particular, whereas elliptic curves have only one kind of good non-ordinary reduction—supersingular reduction, abelian varieties can have a mix of ordinary and non-ordinary reduction types, and since abelian varieties generally have higher dimensions, they have multiple logarithm generators, and therefore, the local points we generate (mimicking universal norms) can have different norm relations. Finally, our technique involves constructing a power series which functions as a logarithm much as Shinichi Kobayashi (Iwasawa theory for elliptic curves at supersingular primes. *Inventiones Mathematicae* 152 (2003), no.1, 1–36) does, but finding the right constant becomes a non-trivial matter if we work at the level of generality in our work.

Overcoming all these issues, we generalize Kim’s work (which means we also generalize Mazur’s work, and Perrin-Riou’s work in our context), and in particular, obtain bounds for the Mordell-Weil ranks of abelian varieties defined over the above-mentioned number fields. In particular, we obtain the Mordell-Weil ranks over cyclotomic extensions of the Jacobian varieties of hyperelliptic curves of large  $p$ -power exponents.

김찬호 (KIAS)

**Title:** an anticyclotomic Mazur-Tate conjecture for modular forms

**Abstract:** We discuss what the Mazur-Tate conjecture and its anticyclotomic variant are and how to approach to the the anticyclotomic one under certain assumptions.

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## Abstracts (가나다 순)

### 박지훈 (POSTECH)

**Title:** Inverse values of the modular  $j$ -function and homotopy Lie theory

**Abstract:** The goal of this talk is to give a simple arithmetic application of the enhanced homotopy (Lie) theory for algebra varieties developed by myself and my self and my collaborators (Dokyung Kim, Yesule Kim, and Jae-Suk Park). Namely, we compute the inverse values of the modular  $j$ -function by using deformation theory for period matrices of elliptic curves based on homotopy Lie theory. The key other ingredient in our approach is J. Carlson and P. Griffiths' explicit computation regarding infinitesimal variations of Hodge structures. This is a joint work with Kwang Hyun Kim and Yesule Kim.

### 박철 (KIAS)

**Title:** A first step towards mod  $p$  Langlands program for  $GL_n(\mathbb{Q}_q)$

**Abstract:** Let  $F/\mathbb{Q}$  be a CM field in which  $p$  splits completely and  $\bar{r} : \text{Gal}(\bar{\mathbb{Q}}/F) \rightarrow GL_n(\bar{\mathbb{F}}_p)$  a continuous automorphic Galois representation. We assume that  $\bar{r}|_{\text{Gal}(\bar{\mathbb{Q}}_p/F_w)}$  is an ordinary representation at a place  $w$  above  $p$ . In this talk, we discuss a problem about local-global compatibility in the mod  $p$  Langlands program for  $GL_n(\mathbb{Q}_p)$ . It is expected that if  $\bar{r}|_{\text{Gal}(\bar{\mathbb{Q}}_p/F_w)}$  is tamely ramified, then it is determined by the set of modular Serre weights and the Hecke action on its constituents. However, this is not true if  $\bar{r}|_{\text{Gal}(\bar{\mathbb{Q}}_p/F_w)}$  is wildly ramified, and the question of determining  $\bar{r}|_{\text{Gal}(\bar{\mathbb{Q}}_p/F_w)}$  from a space of mod  $p$  automorphic forms lies deeper than the weight part of Serre's conjecture. We define a local invariant associated to  $\bar{r}|_{\text{Gal}(\bar{\mathbb{Q}}_p/F_w)}$  in terms of Fontaine-Laffaille theory, and discuss a way to prove that the local invariant associated to  $\bar{r}|_{\text{Gal}(\bar{\mathbb{Q}}_p/F_w)}$  can be obtained in terms of a refined Hecke action on a space of mod  $p$  algebraic automorphic forms on a compact unitary group.

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## 양재현 (인하대학교)

**Title:** Stable Jacobi forms and the universal Jacobian locus

**Abstract:** In this talk, I will explain the notion of the stability of Jacobi forms, present some properties of stable Jacobi forms and discuss the relations between stable Jacobi forms and the universal Jacobian locus. I also give a brief review on the work on stable modular forms that had been done by E. Freitag forty years ago, and the recent work of G. Codogni and N. I. Shepherd-Barron. Finally I present some interesting open problems.

## 이현석 (연세대학교)

**Title:** MZV (Multiple Zeta Values)

**Abstract:** L. Euler (1707-1783) investigated the values of the numbers

$$\zeta(s) = \sum_{n \geq 1} \frac{1}{n^s}$$

for  $s$  a rational integer, and B. Riemann (1826-1866) extended this function to complex values of  $s$ . For  $s$  a positive even integer,  $\zeta(s)/\pi^s$  is a rational number. Our knowledge on the values of  $\zeta(s)$  for  $s$  a positive odd integer is extremely limited. Recent progress involves the wider set of numbers

$$\zeta(s_1, \dots, s_k) = \sum_{n_1 > \dots > n_k \geq 1} \frac{1}{n_1^{s_1} \dots n_k^{s_k}}$$

for positive integers with  $s_1, \dots, s_k$  positive integers with  $s_1 \geq 2$ .

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## Abstracts (가나다 순)

### 임수봉 (성균관대학교)

**Title:** Limits of traces of singular moduli

**Abstract:** Zagier proved that traces  $\text{Tr}_d(j_1)$  of singular moduli for  $d < 0$  are coefficients of a weakly holomorphic modular form  $g_1$  of weight  $3/2$ . Duke, Imamoglu, and Tóth defined a modular trace  $\text{Tr}_d(j_1)$  for  $d > 0$  by using the cycle integral of  $j_1$ , and showed that its generating function is a mock modular form whose shadow is  $g_1$ . In this talk, we introduce connections between  $\text{Tr}_d(j_1)$  for  $d > 0$  and  $\text{Tr}_d(j_1)$  for  $d < 0$  by considering a certain asymptotic behavior of twisted sums of  $\text{Tr}_d(j_1)$  over  $d > 0$  and  $d < 0$ , respectively.

### 주장원 (서울대학교)

**Title:** A generalization of Gauss' triangular theorem

**Abstract:** A quadratic polynomial  $\Phi_{a,b,c}(x, y, z) = x(ax+1) + y(by+1) + z(cz+1)$  is called universal if the diophantine equation  $\Phi_{a,b,c}(x, y, z) = n$  has an integer solution  $x, y, z$  for any nonnegative integer  $n$ . In this talk, we show that if  $(a, b, c) = (2, 2, 6), (2, 3, 5)$  or  $(2, 3, 7)$ , then  $\Phi_{a,b,c}(x, y, z)$  is universal. These were conjectured by Zhi-Wei Sun. This is a joint work with Byeong-Kweon Oh.

### 최도훈 (한국항공대학교)

**Title:** 자기 동형 표현의 유리성 체에 관한 산술

**Abstract:** 주어진 자기 동형 표현에 관해 동형 사상을 주는 갈루아 표현들의 고정되는 복소수 상에 가장 작은 체를 자기 동형 표현의 유리성 체라 한다. 자기 동형 표현과 혹은 특수한 자기 동형 표현의 모임들에 대한 유리성 체에 관한 산술에 대해 발표한다.

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## 최준화 (POSTECH)

**Title:** Quadratic twists of Gross curves

**Abstract:** Let  $K = \mathbb{Q}(\sqrt{-q})$ , where  $q$  is any prime  $q \equiv 7 \pmod{8}$ , and write  $\mathcal{O}$  for the ring of integers of  $K$ . Let  $H = K(j(\mathcal{O}))$  denote the Hilbert class field of  $K$ , where  $j$  is the classical  $j$ -function. There is a unique elliptic curve defined over  $\mathbb{Q}(j(\mathcal{O}))$ , which is called the Gross curve, whose  $j$ -invariant is equal to  $j(\mathcal{O})$ , whose ring of  $H$ -endomorphisms is equal to  $\mathcal{O}$ , whose minimal discriminant ideal in  $H$  is equal to  $(q^3)$ , and which is isogenous to all of its conjugates under the action of the Galois group of  $H$  over  $K$ . In this talk, we will discuss the Iwasawa theory of quadratic twists of these Gross curves, especially the non-vanishing theorem on Iwasawa  $\mu$ -invariants.

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## How To Come



### 지하철

지하철 1호선 성균관대역 하차 후 도보로 5~10분 거리  
(4호선을 이용하시는 경우, 금정역에서 1호선 수원행 열차로 환승가능)

### 시외버스

#### 7790

성대입구 (학교후문, 율전동사무소앞) → 사당역 → 성균관대 (율전동사무소맞은편)

#### 7800

성대입구 (학교후문, 율전동사무소앞) → 사당역 → 성균관대 (율전동사무소맞은편)

#### 3003

율전성당 → 예술의전당 → 서초역 → 강남역 → 뱅뱅사거리 → 양재역 → 율전성당

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## How To Come

### 셔틀버스

(동계방학기간에는 사당행 셔틀버스만 운행합니다.)

### 정류장

사당역 5번출구 홈플러스 앞(사당역 4번출구 남태령 방면 300m 직진)

사당역 출발 시간
08:10   08:30   09:30   10:30   11:30   12:30   13:30   14:30   15:30   16:30   17:30
자과캠 출발 시간
09:40   10:50   11:50   12:50   13:50   14:50   15:50   16:50   18:00   19:00

### 요금

현금 : 1200원

회수권 : 1150원 (10매 구매시)

### 일반버스

#### 62-1

서수원버스터미널 → 삼성아파트 → 성균관대학🚌 → 성균관대역🚌 → 율전성당 → 신안아파트 → 풍림아파트 → 경기일보 → 동수원세무소 → 장안문 → 팔달문 → 동수원병원 → 아주대입구 → 매탄초등학교 → 한빛마을 → 중동사거리 → 장안문 → 동남보건대 → 율전성당 → 성균관대학🚌

#### 82-2

영통롯데아파트 → 매탄공원 → 중부소방서 → 수원역 → 서수원버스터미널 → 삼성아파트 → 성균관대학🚌 → 성균관대역🚌 → 율전성당 → 밤밭저수지 → 매탄초등학교 → 매탄공원 → .....(이하 순환)

#### 39

서수원버스터미널 → 수원역 → 팔달문 → 상공회의소 → 화서역 → 성대정문 → 성대입구🚌 → 성균관대역(회차지)🚌

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## How To Come

### 숙박지원장소(ibis 수원)에서 오시는 경우

#### 주소

경기도 수원시 팔달구 권광로 132, (수원시청역 2번출구 근처)

#### 지하철

수원시청역(분당선) → 수원역(1호선) → 성균관대역(1호선) → 성균관대학 📍

#### 강의실 :

성균관대학교 자연과학캠퍼스 31316 (제1과학관 3층)

