2019 Joint Conference of AORC-IRCTMT-IDEaS

International Conference on Matrix Theory and Applications:

Combinatorics, Optimization & Data Analysis

Jeju National University

Jeju, Korea

May 23-27, 2019

Organized by

AORC (Applied Algebra and Optimization Research Center, SKKU, Korea)

IRCTMT (International Research Center for Tensor and Matrix Theory, Shanghai Univ, China)

IDEaS (Institute for Data Engineering and Science, Georgia Tech, USA)

Hosted by

AORC (Applied Algebra and Optimization Research Center, SKKU, Korea)

Abstracts

Abstracts

Plenary Talks (Ara Hall-1, May 24)

- **Richard A. Brualdi, University of Wisconsin - Madison, USA**

Title : **Stirling Permutations**

Abstract : Stirling permutations were introduced by Gessel and Stanley [1] in 2005 in their investigation of certain polynomial associated with the classical Stirling numbers of the first and second kinds. Since then, there has been considerable work done, primarily concerned with certain associated statistics, e.g. [2] and the references contained therein. A *Stirling permutation* of order n and length $2n$ is a permutation of the multiset $\{1,1,2,2,\ldots,n,n\}$. For example,

$$\pi = (1,2,2,4,5,6,6,5,4,3,3,1)$$

is a Stirling permutation of order 6 and length 12. It follows by induction that the number of Stirling permutations of length n is $(2n-1)!! = (2n-1)(2n-3)(2n-5)\cdots 1$. A Stirling permutation π of order n determines a unique pair (π_1,π_2) of permutations of $\{1,2,\ldots,n\}$ determined by the first and second occurrences of the integers $1,2,\ldots,n$ in π. For the above example we have:

$$\pi = (1,2,2,4,5,6,6,5,4,3,3,1) \to (\pi_1,\pi_2) \text{ where } \pi_1 = (1,2,4,5,6,3) \text{ and } \pi_2 = (2,6,5,4,3,1).$$

A special case is when $\pi_1 = (1,2,\ldots,n)$. We shall discuss various properties of Stirling permutations and their associated Stirling permutation pairs [3].

References

[1] I. Gessel and R.P. Stanley, *Stirling polynomials*, J. Combinatorial Theory Ser. A **72** (2005), 95-117.

[2] S.-M. Mah, Y.N. Yeh, *Stirling permutations, cycle structure of permutations and perfect matchings*, Electr.J. Combin. **24(4)** (2015), #P4.27.

[3] R.A. Brualdi, Stirling pairs of permutations, in preparation.

Time : 09:00 – 09:50

- **Michael K. Ng, Hong Kong Baptist University, China**

Title : **Robust Tensor Completion and Its Applications**

Abstract : In this talk, we report the results of robust tensor completion using tubal singular value decomposition, and its applications. Several applications and theoretical results are discussed. Numerical examples are also presented for demonstration.

Time : 14:00 – 14:50

Invited Talks (Ara Hall-1, May 24)

- **Qing-Wen Wang, Shanghai University, China**

  Title : **The new developments of the Sylvester-like matrix equations with applications**

  Abstract : In this talk, we introduce some new developments of Sylvester-like matrix equations including the solvability conditions, the expressions of the general solutions to some systems of matrix equations and give a real application. (Joint work with Zhuo-Heng He and Yang Zhang)

  References

  [1] Q.W. Wang, Z.H. He, Y. Zhang *Constrained two-sided coupled Sylvester-type quaternion matrix equations*, Automatica **101** (2019), 207–213.

  Time : 10:20 – 10:50

- **Yang Zhang, University of Manitoba, Canada**

  Title : **Consistency of quaternion matrix equations** $AX^{\star} - XB = C$ **and** $X - AX^{\star}B = C$

  Abstract : For a given ordered units triple $\{q_1, q_2, q_3\}$, we discuss the solutions to the quaternion matrix equations $AX^{\star} - XB = C$ and $X - AX^{\star}B = C, X^{\star} \in \{X, X^{\eta}, X^{*}, X^{\eta *}\}$, where X^{*} is the conjugate transpose of X, $X^{\eta} = -\eta X \eta$ and $X^{\eta *} = -\eta X^{*} \eta$, $\eta \in \{q_1, q_2, q_3\}$. Some new real representations of quaternion matrices are used, which enable one to convert η-conjugate (transpose) matrix equations into some real matrix equations. By using this idea, the conditions for the existence and uniqueness of solutions to the above quaternion matrix equations are derived. Also, the methods to construct the solutions from some related real matrix equations are presented. (Joint work with Xin Liu and Qing-Wen Wang)

  Time : 10:50 – 11:20

- **Zhuo-Heng He, Shanghai University, China**

  Title : **Some quaternion matrix equations involving** ϕ**-skew-Hermicity**

  Abstract : In this talk, we consider some quaternion matrix equations involving ϕ-skew-Hermicity. We derive some practical necessary and sufficient conditions for the existence of a ϕ-skew-Hermitian solution to a system of quaternion matrix equations involving ϕ-skew-Hermicity. Apart from proving an expression for the general ϕ-skew-Hermitian solution to this system, we derive the $\beta(\phi)$-signature bounds of the ϕ-skew-Hermitian solution in terms of the coefficient matrices. Moreover, we obtain necessary and sufficient conditions for the system to have $\beta(\phi)$-positive definite, $\beta(\phi)$-positive semidefinite, $\beta(\phi)$-negative definite and $\beta(\phi)$-negative semidefinite solutions.

  Time : 11:30 – 12:00

- **Hyun-Min Kim, Pusan National University, Korea**

  Title : **On the extreme solutions of a class of nonlinear matrix equation**

  Abstract : The nonlinear matrix equation $X + A^{*}\bar{X}^{-1}A = Q$, where both the coefficient matrices and the unknown are complex matrices and Q is Hermitian positive definite, is considered. Some new and useful properties of the maximal and minimal positive definite solutions of the matrix equation is obtained. We show that an iteration method with a quadratic convergence rate can be effectively applied to compute the extreme solutions. This iteration method is based on cyclic reduction. Numerical examples and comparisons are provided to show the efficiency of the proposed iteration method. (Joint work with Jie Meng and Hosoo Lee)

References

[1] W. N. Anderson Jr., T. D. Morley, and G. E. Trapp, *Positive solutions to* $X = A - BX^{-1}B^*$, Linear Algebra Appl. **134** (1990), 53–62.

[2] J. Meng and H.-M. Kim. *The positive definite solution of the nonlinear matrix equation* $X^p = A + M(B + X^{-1})^{-1}M^*$, J. Comput. Appl. Math. **322** (2017), 139–147.

[3] B. Zhou, G. Cai, and J. Lam, *Positive definite solutions of the nonlinear matrix equation* $X + A^H \bar{X} A = I$, Appl. Math. Comput. **219** (2013), 7377–7391.

Time : 12:00 – 12:30

- **Rajendra Bhatia, Ashoka University, India**

Title : **The Bures - Wasserstein distance between positive definite matrices**

Abstract : The metric $d(A,B) = [tr(A+B) - 2tr(AB)^{1/2}]^{1/2}$ on positive definite matrices is of great interest in diverse areas like statistics, optimal transport, quantum information and differential geometry. A unified treatment from the perspective of matrix analysis is presented. (Joint work with T. Jain and Y. Lim.)

References

[1] R. Bhatia, T. Jain and Y. Lim *On the Bures Wasserstein distance between positive definite matrices*, Expositiones Math. (2019).

Time : 15:00 – 15:30

- **Yongdo Lim, Sungkyunkwan University, Korea**

Title : **Off-diagonal constant Hermitian matrices**

Abstract : It is not always easy to see whether a certain Hermitian matrix is positive semidefinite. In this talk we consider a real $m \times m$ positive semidefinite matrix with $a_1, \ldots, a_m$ on the main diagonal and x in all off-diagonal positions and the extremal problem of finding the minimum and maximum of x. We discuss a new mean of positive reals obtained from the extremal problem and similar problems on Hermitian matrices over Euclidean Hurwitz algebras; complex numbers, quaternions, and octonions(Cayley numbers).

Time : 15:30 – 16:00

- **Dragana Cvetković Ilić, University of Niš, Serbia**

Title : **Completion problems of operator matrices to Fredholm, Weyl and Kato operators**

Abstract : We will address some recent results on certain problems of completions of operator matrices. In particular, we will consider the problems of completions of operator matrices to Fredholm, Weyl and Kato operators. Also we will show interesting applications of these results in solving some problems related to the invertibility and Fredholmness of linear combinations of operators from $\mathcal{B}(\mathcal{H})$, in particular those of idempotents and projectors.

References

[1] V. Pavlović, D.S. Cvetković-Ilić, *Completions of upper-triangular operator matrices to Kato nonsingularity*, Journal of Math. Anal. Appl. **433(2)** (2016) 1230-1242.

[2] D.S. Cvetković-Ilić, *Completions of upper-triangular matrices to left-Fredholm operators with non-positive index*, Journal of Operator Theory, **76:1** (2016) 101-114.

[3] D.S. Cvetković-Ilić, *Fredholm consistence of upper-triangular operator matrices*, Journal of Spectral Theory, **7(4)** (2018) 1023-1038.

Time : 16:30 – 17:00

- **Takeaki Yamazaki, Toyo University, Japan**

Title : **Power monotonicity for a path of operator means**

Abstract : A function f is called *power monotone increasing* (PMI) if and only if $f(x)^r \leq f(x^r)$ holds for all $r \geq 1$ and $x > 0$. Also we can define *power monotone decreasing* (PMD) by the similar way. The power monotonicity is closely related to the Ando-Hiai inequality, a very famous operator inequality. In this talk, we shall prove the power monotonicity of the function

$$F_{p,q}(x) := \left(\frac{p}{p+q} \cdot \frac{x^{p+q} - 1}{x^p - 1} \right)^{\frac{1}{q}}.$$

$F_{p,q}$ is a representing function of so called the *extension of the power difference mean*. It includes several representing functions of operator means. (Joint work with Jun-Ichi Fujii)

Time : 17:00 – 17:30

Invited Talks (Ara Hall-2, May 24)

- **Bryan Shader, University of Wyoming, USA**

Title : **Diabolical Diameter Dilemmas**

Abstract : Much of combinatorial matrix theory is devoted to exploring how the combinatorial structure (e.g., the position of the zeros and nonzero entries) of a matrix influence the spectral properties of the matrix. In turn spectral properties are often connected to parameters of importance in applications (e.g., Vapnik–Chervonenkis dimension measures the communication complexity of a problem and is the smallest rank of a matrix with a given corresponding sign-pattern). Several authors have established very interesting results along these lines when the nonzero entries of a symmetric matrix A correspond to a tree. One example is Fielder's result that the path on n vertices is the unique graph on n vertices for which there is no matrix with that graph having multiple eigenvalues. Another example is Godsil's result that a matrix A whose graph is a tree has at least $d(T) + 1$ eigenvalues, where $d(T)$ is the diameter of T. In this talk, we will further study the relationship between the diameter of T and the number of distinct eigenvalues of A. In particular, we will show that for each positive integer d, there exists a constant $c(d)$ and an polynomial $p(x)$ of degree $c(d)$ such that for each tree T of diameter d there is a symmetric matrix A whose graph corresponds to T with $p(A) = O$. In other words, there exists a "mother" polynomial whose factors include a minimal polynomial for at least one matrix whose graph is any tree of diameter d. Additionally, we will discuss a characterization

of the graphs G for which no matrix with graph G has a small number multiple eigenvalues. Both these results naturally lead to some seemingly difficult, yet interesting open problems that we will mention.

Time : 10:20 – 10:50

- **Mao-Ting Chien, University of Manitoba, Canada**

Title : **A combinatorial method for computing Riemann matrices of quartic curves**

Abstract : Let T be an $n \times n$ complex matrix. The real ternary form $F_T(x, y, z)$ associated to T is defined by

$$F_T(x, y, z) = \det(zI_n + x\Re(T) + y\Im(T)),$$

where $\Re(T) = (T + T^*)/2$, $\Im(T) = (T - T^*)/(2i)$. The ternary form $F_T(x, y, z)$ completely determines the numerical range of T. Assume the complex projective curve $\Gamma = \{[x, y, z] \in \mathbb{CP}^2 : F_T(x, y, z) = 0\}$ has no singular points, Γ is a Riemann surface with genus g. The homology group $H_1(\Gamma) \cong \mathbb{Z}^{2g}$ has a canonical basis $\{a_1, \ldots, a_g, b_1, \ldots, b_g\}$. Let $\{\omega_1, \ldots, \omega_g\}$ be a basis for the holomorphic differential 1-form on Γ. The Riemann period matrix Ω of Γ is a $g \times 2g$ complex matrix defined as $\Omega = [A\,|B]$, where $A = (a_{ij})_{i,j=1}^g$ and $B = (a_{ij})_{i,j=1}^g$ are given by $a_{ij} = \oint_{a_j} \omega_i$, $b_{ij} = \oint_{b_j} \omega_i$. It is known the matrix A is invertible. The Riemann matrix of Γ is defined by $R = A^{-1}B$, which plays a key role by Helton and Vinnikov to prove the Lax conjecture. In this talk, we present a combinatorial approach to compute the Riemann period matrix for the Riemann surface of hyperbolic quartic curves $F_T(x, y, z) = 0$ when $n = 4$. (Joint work with Hiroshi Nakazato)

[1] M. T. Chien, H. Nakazato, *Computation of Riemann matrices for the hyperbolic curves of determinantal polynomials*, Ann. Funct. Anal. **8**(2017), 152-167.

[2] J. W. Helton, V. Vinnikov, *Linear matrix inequality representations of sets*, Comm. Pure Appl. Math. **60**(2007), 654-674.

[3] P. D. Lax, *Differential equations, difference equations and matrix theory*, Comm. Pure Appl. Math. **6**(1958), 175-194.

[4] C. L. Tretkoff, M. D. Tretkoff, *Combinatorial group theory, Riemann surface and differential equations*, Contemp. Math. **33**(1984), 467-517.

Time : 10:50 – 11:20

- **Xiao-Dong Zhang, Shanghai Jiao Tong University, China**

Title : **Equitable partition theorem of tensors and tpectrum of generalized power hypergraphs**

Abstract : In this talk, we introduce an equitable partition theorem of tensors, which gives the relations between H-eigenvalues of a tensor and its quotient equitable tensor and extends the equitable partitions of graphs to hypergraphs. Furthermore, with the aid of it, some properties and H-eigenvalues of the generalized power hypergraphs are obtained, which extends some known results, including some results of Yuan, Qi and Shao. (Joint work with Ya-lei Jin and Jie Zhang)

Time : 11:30 – 12:00

- **Shangzhi Li, Beihang University, China**

Title : **Subgroup structure of Classical groups**

Abstract : My main research work is on the subgroup structure of classical groups over arbitrary fiels, including infinte fields. All the overgroups are classified, for many important series of subgroups of classical groups over division rings, including arbitrary fields. And thus many series of maximal subgroups are classified. My main method is based on an elementary but rather technical matrix approach, following Professor Hua Luogeng.

Time : 12:00 – 12:30

- **Jein-Shan Chen, National Taiwan Normal University, Taiwan**

  Title : **On unitary elements defined on Lorentz cone and their applications**

  Abstract : In this talk, we illustrate a new concept regarding unitary elements defined on Lorentz cone, and establish some basic properties under the so-called unitary transformation associated with Lorentz cone. As an application of unitary transformation, we achieve a weaker version of the triangle inequality and several (weak) majorizations defined on Lorentz cone.

  Time : 15:00 – 15:30

- **Donghwan Kim, KAIST, Korea**

  Title : **Accelerated First-order Methods for Large-scale Optimization**

  Abstract : Many modern applications, such as machine learning, require solving large-dimensional optimization problems. First-order methods, such as a gradient method, are widely used to solve such problems, since their computational cost per iteration mildly depends on the problem dimension. However, they suffer from slow convergence rates, compared to second-order methods such as Newton's method. Therefore, accelerating first-order methods has received a great interest, and this led to the development and extension of a conjugate gradient method, a heavy-ball method, and Nesterov's fast gradient method, which we will review in this talk. This talk will then present recently proposed accelerated first-order methods, named optimized gradient methods. (Joint work with Jeffrey A. Fessler, Department of EECS, University of Michigan, USA)

  Time : 15:30 – 16:00

- **Woong Kook, Seoul National University, Korea**

  Title : **Harmonic cycles and network decentralization**

  Abstract : Harmonic cycles form the kernel of combinatorial Laplacians of simplicial networks. In this talk, we will review combinatorial Hodge theory highlighting energy-minimizing property of harmonic cycles as a basis for addressing network decentralization, which is one of the key objectives of current network science. Our notion of network decentralization aims to increase uniform distribution of communication capacity, and we will describe how this goal may be interpreted in terms of effective conductance. If time permits, we will present a real world application, called "gift forests".

  Time : 16:30 – 17:00

- **Delin Chu, National University of Singapore, Singapore**

  Title : **An SVD-Based Algorithm for Orthogonal Low Rank Approximation of Tensors**

  Abstract : In this work we study the orthogonal low rank approximation problem of tensors in the general setting in the sense that more than one matrix factor is required to be mutually orthonormal, which includes the completely orthogonal low rank approximation and semi-orthogonal low rank approximation as two special cases. We develop an SVD-based algorithm. Our SVD-based algorithm updates two vectors simultaneously and maintains the required orthogonality conditions by means of the polar decomposition. The convergence behavior of our algorithm is analyzed for both objective function and iterates themselves and is illustrated by numerical experiments.

Time : 17:00 – 17:30

Plenary Talks (Ara Hall-1, May 25)

- **Haesun Park, Georgia Institute of Technology, USA**

  Title : **Joint Nonnegative Matrix Factorization for Unsupervised and Semi-Supervised Clustering**

  Abstract : Constrained Low Rank Approximation (CLRA) is a powerful framework for a variety of important tasks in Big Data analytics such as topic discovery in text data and community detection in social network data. In this talk, a hybrid method called Joint Nonnegative Matrix Factorization (JointNMF) is introduced for latent information discovery from data sets that contain both text content and connection structure information. The method jointly optimizes an integrated objective function, which is a combination the Nonnegative Matrix Factorization (NMF) objective function for handling text content/attribute information and the Symmetric NMF (SymNMF) objective function for handling relation/connection information. An effective algorithm for the joint NMF objective function is proposed utilizing the block coordinate descent (BCD) method.

  The proposed hybrid method simultaneously discovers content associations and related latent connections without any need for post-processing or additional clustering. Another capability is prediction of unknown connection information which is illustrated using some real world problems such as citation recommendations of papers and leader detection in organizations. It is also shown that JointNMF can be applied when the text content is associated with hypergraph edges, and hypergraph structure can be introduced to incorporate partially known relationships among the data items for semi-supervised clustering.

  Time : 09:00 – 09:50

- **Kim-Chuan Toh, National University of Singapore, Singapore**

  Title : **Exploiting Second Order Sparsity in Big Data Optimization**

  Abstract : In this talk, we shall demonstrate how second order sparsity (SOS) in important optimization problems such as sparse optimization models in machine learning, semidefinite programming, and many others can be exploited to design highly efficient algorithms. The SOS property appears naturally when one applies a semismooth Newton (SSN) method to solve the subproblems in an augmented Lagrangian method (ALM) designed for certain classes of structured convex optimization problems. With in-depth analysis of the underlying generalized Jacobians and sophisticated numerical implementation, one can solve the subproblems at surprisingly low costs. For lasso problems with sparse solutions, the cost of solving a single ALM subproblem by our second order method is comparable or even lower than that in a single iteration of many first order methods. Consequently, with the fast convergence of the SSN based ALM, we are able to solve many challenging large scale convex optimization problems in big data applications efficiently and robustly. For the purpose of illustration, we present a highly efficient software called SuiteLasso for solving various well-known Lasso-type problems. (Joint work with Xudong Li and Defeng Sun)

References

[1] X.D. Li, D.F. Sun and K.C. Toh, *On efficiently solving the subproblems of a level-set method for fused lasso problems*, SIAM J. Optimization **28** (2018), 1842–1866.

[2] X.D. Li, D.F. Sun and K.C. Toh, *A highly efficient semismooth Netwon augmented Lagrangian method for solving Lasso problems*, SIAM J. Optimization **28** (2018), 433–458.

Time : 10:20 – 11:10

- **Lajos Molnár, University of Szeged, and Budapest University of Technology and Economics, Hungary**

  Title : **Symmetries of structures of positive (semi)definite matrices**

  Abstract : In this talk we survey the results of our recent research on the ’symmetries’ of different structures on the following subsets of positive semi-definite matrices: the positive definite cone, the matrix unit interval (effect algebra), the set of density matrices. In those results we give exact descriptions of certain algebraic endomorphisms/automorphisms (such as Jordan triple endomorphisms, sequential endomorphisms), isometries corresponding to different distances, and transformations preserving divergences (among others, different sorts of relative entropies).

  Time : 14:00 – 14:50

Invited Talks (Ara Hall-1, May 25)

- **Jimmie Lawson, Louisiana State University, USA**

  Title : **Extending Matrix and Operator Means to Barycentric Maps**

  Abstract : Let $\{G_n\}, n \geq 2$ be a family of matrix or operator means on the cone Ω of positive elements, where G_n is a mean in n-variables. If the family is symmetric (invariant under coordinatewise permutation) and further is iterative and contractive (notions that will be defined in the talk), then it extends uniquely to a contractive (in the Wasserstein metric) barycentric map on the space of integrable probability measures equipped with the Wasserstein metric. We show connections of this theory with the Bochner integral, with extending the operator geometric mean to a barycentric map, and with doing probability theory for random variables with values in metric spaces equipped with a barycentric map. We discuss some recent results about barycentric maps for the cone of positive elements in an arbitrary unital C^*-algebra. (Joint work with Yongdo Lim)

  Time : 11:30 – 12:00

- **Fumio Hiai, Tohoku University, Japan**

  Title : **Matrix limit theorems of reciprocal Lie-Trotter type**

  Abstract : We discuss limit theorems for $\Phi(A^p)^{1/p}$ and $(A^p \sigma B)^{1/p}$ as $p \to \infty$ for positive matrices A, B, where Φ is a positive linear map between matrix algebras (in particular, $\Phi(A) = KAK^*$) and σ is an operator mean (in particular, the weighted geometric mean), which are considered as certain reciprocal Lie-Trotter formulas and also a generalization of Kato’s limit to the supremum $A \vee B$ with respect to the spectral order.

  Time : 12:00 – 12:30

- **Chi-Kwong Li, College of William and Mary, USA**

  Title : **Mappings between matrix spaces**

  Abstract : We consider mappings between matrix spaces with different dimensions that leave invariant some special functions, sets, or relations. A brief survey on the topic will be given. Some recent recent results and open problems will be mentioned.

  Time : 15:00 – 15:30

- **Yiu-Tung Poon, Iowa State University, USA**

  Title : **Submultipicativity of the numerical radius of commuting matrices of order two**

  Abstract : Denote by $w(T)$ the numerical radius of a matrix T. An elementary proof is given to the fact that $w(AB) \leq w(A)w(B)$ for a pair of commuting matrices of order two. Characterization is given to commuting matrices A and B, which attain the equality $w(AB) = w(A)w(B)$. (Joint work with Chi-Kwong Li)

  Time : 15:30 – 16:00

- **Seung-Hyeok Kye, Seoul National University, Korea**

  Title : **Entangled edge states of corank one with positive partial transposes**

  Abstract : We construct a parameterized family of $n \otimes n$ PPT states of corank one for each $n \geq 3$. With a suitable choice of parameters, we show that they are $n \otimes n$ PPT entangled edge states of corank one for $3 \leq n \leq 800$. They violate the range criterion for separability in the most extreme way. Note that corank one is the smallest possible corank for such states. The corank of the partial transpose is given by $2n - 3$, which is also the smallest possible corank for the partial transposes of PPT entangled edge states of corank one. They provide the first explicit examples of such states for $n \geq 4$. This is coworked with Young-Hoon Kiem and Jinwon Choi, and posted at arXiv 1903.10745.

  Time : 16:30 – 17:00

- **Gustavo Corach, Instituto Argentino de Matemática, Argentina**

  Title : **On a bilateral Schur complement in the sense of Ando**

  Abstract : Given a subspace S of a (finite dimensional) Hilbert space H, T. Ando defined in 1979 the class of what he called S-complementable operators; for these operators A, a generalized Schur complement S(A) can be defined, and for matrices this construction is equivalent to the classical Schur complement. Ando's approach has been applied by Mitra-Puri and Carlson-Haynworth to operators from one space H with a fixed subspace S to another space K with a subspace T; with a kind of S,T-complementability property a bilateral Schur complement S,T(A) can be defined. Antezana et al (2006) extended this to infinite dimensional Hilbert spaces, bounded linear operators and closed subspaces. Here, we continue their research and find new properties and new characterizations of S,T-complementability and bilateral Schur complements. We also apply these results to the general Wiener-Hopf operators of Devinatz-Shinbrot (1969), namely PAQ where P (resp. Q) is the orthogonal projection onto T (resp. S). (Joint work with Maria Laura Arias and Alejandra Maestripieri)

  Time : 17:00 – 17:30

Invited Talks (Ara Hall-2, May 25)

- **Sangwoon Yun, Sungkyunkwan University, Korea**

  Title : **Probabilistic K-means with Sparsity**

  Abstract : The aim of clustering is to partition a set of data points into groups of similar data points, called clusters. Clustering is one of the most fundamental problems in unsupervised learning and has many applications in various disciplines. Due to its importance, there are tremendous algorithms and works in the literature and they may be classified as hard and soft clustering. In hard clustering such as k-means, every data point belongs to exactly one cluster. On the contrary, soft clustering allows more than one clusters since they can overlap in real data sets. Fuzzy c-means is a typical case of it. In this work, we combine the benefit of two methods, clarity of hard clustering and uncertainty of soft clustering. For example, let us consider assigning a number between 0 and 1 to indicate the clusters 0 and 1. When the affiliation of a data point is clearly noticeable, it is desirable to assign the discrete values 0 and 1. Only when its affiliation is not clear, it is proper to assign some neutral values such as 0.5. If such affiliation is expressed by a matrix and the discrete values 0 and 1 are neglected, the matrix is sparse. To promote exclusive assignments on the majority of data points which have clear associations, we add the ℓ_1 norm in the model. Owing to sparsity in solutions, the ℓ_1 norm constraint is widely used in compressed sensing, image processing, statistics and machine learning. With the ℓ_2 norm in prevalent clustering models, we intend to enhance the prediction and interpretability of the models. Actually, coupling penalty terms are popular and successful in many branches of applied mathematics. In addition, data sets are contaminated by outliers which should not belong to any cluster. Outliers are caused by high variability in data, measurement errors, faulty data, erroneous procedures, for example. Since they can cause serious problems in statistical analyses, outlier detection has an extensive literature. However, most existing algorithms address either overlap or outlier detection and do not tackle the problem in a unified way. Recently, there are several efforts to handle clustering and outlier detection simultaneously. Along with this direction of research, we also propose a model which incorporate overlapping clustering and outlier detection. We propose an alternating minimization method for solving the proposed model and analyze its convergence property. Numerical results on synthetic data are presented. (Joint work with Yoon Mo Jung and Joyce Jiyoung Whang)

Time : 11:30 – 12:00

- **Defeng Sun, The Hong Kong Polytechnic University, Hong Kong**

  Title : **Spectral Operators of Matrices**

  Abstract : The class of matrix optimization problems (MOPs) has been recognized in recent years to be a powerful tool to model many important applications involving structured low rank matrices within and beyond the optimization community. The Lowner operator, which generates a matrix valued function via applying a single-variable function to each of the singular values of a matrix, has played an important role for a long time in solving matrix optimization problems. However, the classical theory developed for the Lowner operator has become inadequate in these recent applications. The main objective of this talk is to introduce a class of new matrix valued functions, coined as spectral operators of matrices. Several fundamental properties of spectral operators, including the well-definedness, continuity, directional differentiability, Frechet-differentiability and the ssemismoothness will be presented. (Joint work with Chao Ding, Jie Sun and Kim-Chuan Toh)

  Time : 12:00 – 12:30

- **Louis Shapiro, Howard University, USA**

  Title : **Palindromes and Pseudo-involutions in the Riordan group**

  Abstract : We present some new methods for constructing pseudo-involutions. Often these lead to elements in the k-Bell subgroup and often there are interesting combinatorial results. This takes place in the Riordan group of infinite lower triangular matrices.

  Time : 15:00 – 15:30

- **I-Chiau Huang, Academia Sinica, Taiwan**

  Title : **Matrices from Numerical Semigroups**

  Abstract : A numerical semigroup is a monoid generated by coprime positive integers, For instance, the numerical semigroup $\langle 3,5\rangle$ consists of $0,3,5,6$ and all integers greater than 7. In spite of the simple definition, numerical semigroups have rich structure [1]. The study of numerical semigroups attracts people working on combinatorics, commutative algebra, algebraic geometry, number theory, *etc*. Let $\kappa[\![\mathbf{u}]\!]$ be the power series ring of one variable over a field κ. Given a numerical semigroup S, the numerical semigroup ring $\kappa[\![\mathbf{u}^S]\!]$ is a subring of $\kappa[\![\mathbf{u}]\!]$ consisting of all power series $\sum a_i\mathbf{u}^{s_i}$, where $a_i \in \kappa$ and $s_i \in S$. One may study singularities of numerical semigroup rings, such as Gorenstein and complete intersection. We remark that numerical semigroup rings are always Cohen-Macaulay.

  Following Grothendieck's philosophy on algebraic geometry, we study numerical semigroups from a relative viewpoint [2]. More precisely, we study numerical semigroup rings as algebras over a numerical semigroup ring. These algebras are called numerical semigroup algebras. We found a sufficient condition for numerical semigroup algebras to be complete intersection [3]. The proof involves certain matrices, whose entries are rational numbers. The positivity of the determinants of the matrices plays an important role in the proof. In the talk, we will discuss questions regarding these matrices.

  References

  [1] José Carlos Rosales and Pedro A. García-Sánchez. *Numerical semigroups*, Springer, New York, 2009.

  [2] I-Chiau Huang and Mee-Kyoung Kim. *Numerical semigroup algebras*, Preprint, 2016.

  [3] I-Chiau Huang and Raheleh Jafari, *Factorizations in numerical semigroup algebras*, J. Pure Appl. Algebra **203** (2019), no. 5, 2258–2272.

  Time : 15:30 – 16:00

- **Huihui Zhu, Hefei University of Technology, China**

  Title : **Weighted pseudo core inverses in rings**

  Abstract : Let R be a unital $*$-ring and let $a, e, f \in R$ with e, f invertible Hermitian elements. In this paper, we define two types of outer generalized inverses, called pseudo e-core inverses and pseudo f-dual core inverses. An element $a \in R$ is pseudo e-core invertible if there exist an element $x \in R$ and some positive integer n such that $xax = x, xR = a^nR$ and $Rx = R(a^n)^*e$. Dually, a is pseudo f-dual core invertible if there exist an element $x \in R$ and some positive integer m such that $xax = x, Rx = Ra^m$ and $fxR = (a^m)^*R$. Moreover, we investigate both of them for their characterizations and properties. Also, the relations between the pseudo e-core inverse (resp. the pseudo f-dual core inverse) and the inverse along an element are given. (Joint work with Qing-Wen Wang)

  Time : 16:30 – 17:00

- **Libin Li, Yangzhou University, China**

  Title : **From representation theory of fusion ring to matrix equations**

  Abstract : The non-integer matrix (NIM) representations over near group fusion rings are closely related to the NIM solution of matrix equations and the module categories over near group categories. In this talk we shall introduce a general theory of irreducible NIM representations over near group fusion rings. We give the minimum upper the bound of rank of NIM representation over a near group fusion ring, and the general classification methods of irreducible NIM representations over near group fusion rings. We give explicitly the classifications of irreducible NIM representations over some near group fusion rings. (Joint work with Chengtao Yuan)

Time : 17:00 – 17:30

Plenary Talk (Ara Hall-1, May 26)

- **Xuding Zhu, Zhejiang Normal University, China**

Title : **List colouring and Alon-Tarsi number of planar graphs**

Abstract : It is known that every planar graph is 5-choosable, however, there are planar graphs that are not 4-choosable. One natural question is how far can a planar graph be away from being 4-choosable? This question is not well-posed, unless there is some kind of measurement that refines the choosability of graphs. We shall discuss a few such measurements. One measure is to define a graph G to be $(4+\epsilon)$-choosable if for any subgraph H of G, there is a subset X of $V(H)$ with $|X| \leq \epsilon|V(H)|$ such that for any list assignment L of G with $|L(x)| = 5$ for $x \in X$ and $|L(x)| = 4$ for $x \notin X$, H is L-colourable. With this definition, a meaningful question is what is the smallest number ϵ such that every planar graph is $(4+\epsilon)$-choosable. It follows from a result of Mirzkahani that $\epsilon \geq 1/63$. Recently, in a joint paper with Grytczuk, we proved that $\epsilon < 1/2$. Another measurement is to consider defective choosability. A d-defective colouring of a graph G such that every vertex v has at most d-neighbours coloured the same colour as v. We say a graph G is d-defective k-choosable if for any k-list assignment L of G, there exists a d-defective L-colouring of G. It was proved by Cushing and Kierstead that every planar graph is 1-defective 4-choosable. We strengthen this result by showing that every planar graph G contains a matching M such that $G - M$ is online 4-choosable. This implies that every planar graph is 1-defective 4-paintable. These results are proved by considering the polynomial associated to G and by applying combinatorial nullstellensatz. We shall prove that every planar graph G has Alon-Tarsi number at most 5, and that G has a matching M such that $G - M$ has Alon-Tarsi number at most 4. (Joint work with Jaroslaw Grytczuk and Huajin Lu)

References

[1] W. Cushing and H. A. Kierstead, *Planar graphs are 1-relaxed, 4-choosable*, European Journal of Combinatorics, **31**(5)(2010), 1385 —1397.

[2] H. Lu and X. Zhu, *The Alon-Tarsi number of planar graphs without adjacent short cycles*, manuscript, 2019.

[3] J.Grytczuk and X.Zhu, *The Alon-Taris number of a planar graph minus a matching*, https://arxiv.org/abs/1811.12012.

[4] X. Zhu, *The Alon-Tarsi number of planar graphs*, Journal of Combinatorial Theory series B **134** (2019), 354–358.

Time : 09:00 – 09:50

Invited Talks (Ara Hall-1, May 26)

- **Jang Soo Kim, Sungkyunkwan University, Korea**

Title : **Generalized Schur function determinants using Bazin-Sylvester identity**

Abstract : In the literature there are several determinant formulas for Schur functions: the Jacobi–Trudi formula, the dual Jacobi–Trudi formula, the Giambelli formula, the Lascoux–Pragacz formula, and the Hamel–Goulden formula, where the Hamel–Goulden formula implies the others. In this talk we use the Bazin–Sylvester identity to derive a determinant formula for Macdonald's ninth variation of Schur functions. As

consequences we obtain a generalization of the Hamel–Goulden formula and a Lascoux–Pragacz-type determinant formula for factorial Schur functions conjectured by Morales, Pak and Panova. (Joint work with Meesue Yoo)

Time : 10:20 – 10:50

- **Gabriel Larotonda, Universidad de Buenos Aires, Argentina**

  Title : **Geodesics of the unitary group for weakly unitarily invariant norms**

  Abstract : It is well-known that one-parameter groups are minimizing geodesics for unitarily invariant norms in the unitary group [3, 6, 1] of complex matrices. In this talk we will discuss the extension of these result to weakly unitarily invariant norms (such as the C-numerical radius), which in general do not preserve spectral order [4]. In the process we hope to clarify some technical aspects of previous known proofs given for strongly unitarily invariant norms [2]. This research is part of the work on the metric geometry of Lie groups carried out in [5].

References

[1] E. Andruchow and G. Larotonda, *Hopf-Rinow theorem in the Sato Grassmannian*, J. Funct. Anal. **255** (2008), no. 7, 1692–1712.

[2] J. Antezana, G. Larotonda, and A. Varela, *Optimal paths for symmetric actions in the unitary group*, Comm. Math. Phys. **328** (2014), no. 2, 481–497.

[3] C. J. Atkin, *The Finsler geometry of groups of isometries of Hilbert space*, J. Austral. Math. Soc. Ser. A **42** (1987) no. 2, 196–222.

[4] R. Bhatia and J. A. R. Holbrook, *A softer, stronger Lidskii theorem*, Proc. Indian Acad. Sci. Math. Sci. **99** (1989), no. 1, 75–83. Journal of Combinatorial Theory series B **134** (2019), 354–358.

[5] G. Larotonda *The metric geometry of infinite dimensional Lie groups and their homogeneous spaces*, arxiv preprint (2019), arXiv:1805.02631.

[6] H. Porta and L. Recht, *Minimality of geodesics in Grassmann manifolds*, Proc. Amer. Math. Soc. **100** (1987) no. 3, 464–466.

Time : 10:50 – 11:20

- **Seog-Jin Kim, Konkuk University, Korea**

  Title : **Coloring squares of graphs with mad constraints**

  Abstract : The square G^2 of a graph G is the graph defined by $V(G) = V(G^2)$ and $uv \in E(G^2)$ if and only if the distance between u and v is at most two. We denote by $\chi(G^2)$ the chromatic number of G^2, which is the least integer k such that a k-coloring of G^2 exists. In this paper, we prove that the square of every graph G with $Mad(G) < 4$ and $\Delta(G) \geq 8$ is $(3\Delta(G) + 1$-choosable and even correspondence- colorable. Furthermore, we show a family of 2-degenerate graphs G with $Mad(G) < 4$, arbitrarily large maximum degree, and $\chi(G^2) \geq \frac{5\Delta(G)}{2}$, improving a result of Kim and Park (2016).(Joint work with Hervé Hocquard and Théo Pierron, Uniersity of Bordeaux, France)

  Time : 11:30 – 12:00

- **Marcell Gaál, University of Szeged, Hungary**

  Title : **A class of convex combination preservers on density operators**

  Abstract : Density operators are the mathematical representatives of the quantum states in the framework of quantum information theory. Entropy is a numerical quantity which is of fundamental importance in this area. In fact, there are several concepts of entropy among which the most famous one is due to von Neumann. In this talk, we completely characterise those transformations on the set of density operators, acting on a finite dimensional Hilbert space, which preserve certain entropylike quantities of *any* convex combination of operators. We also consider maps that preserve the von Neumann entropy or Schatten p-norm of *every* convex combination on the τ-density space of a C^*-algebra carrying a faithful trace τ. A part of the talk is based on a joint work with Gergő Nagy.

  Time : 12:00 – 12:30

Invited Talks (Ara Hall-2, May 26)

- **Ren-Cang Li, University of Texas at Arlington, USA**

  Title : **Convergence of Lanczos Methods for Trust-Region Subproblem**

  The problem of minimizing a quadratic function over an ellipsoid constraint has long been known as the Trust-Region Subproblem (TRS) in the famous trust-region method in optimization. Besides playing a critical role in the trust-region method, TRS by itself also arises in other real-world applications. For a large scale TRS, an efficient Krylov subspace method was proposed in [Gould, Lucidi, Roma and Toint, *SIAM J. Optim.*, 9:504–525 (1999)] as a natural extension of the classical Lanczos method for the linear system and eigenvalue problems. But only until recently, the convergence analysis of the Lanczos method for TRS has been completed. In this talk, I review some recent developments that are concerned with convergence behaviors, restarting techniques as well as possible extensions and applications. (Joint work with Lei-hong Zhang and Chungen Shen)

  References

  [1] L.-H. Zhang and C. Shen, *A nested Lanczos method for the trust-region subproblem.*, SIAM J. Sci. Comput. **40**(4) (2018), A2005–A2032.

  [2] L.-H. Zhang, C. Shen, and R.-C. Li., *On the generalized Lanczos trust-region method*, SIAM J. Optim. **27**(3) (2017), 2110–2142.

  [3] L.-H. Zhang, C. Shen, W. H. Yang, and J. J. Júdice, *A Lanczos method for large-scale extreme Lorentz eigenvalue problems*, SIAM J. Matrix Anal. Appl. **39**(2) (2018) 611–631.

  Time : 10:20 – 10:50

- **Pei Yuan Wu, National Chiao Tung University, Taiwan**

  Title : **Equality of Numerical Ranges of Matrix Powers**

  Abstract : For an $n \times n$ matrix A, we determine when the numerical ranges $W(A^k)$, $k \leq 1$, are all equal to each other. More precisely, we show that this is the case if and only if A is unitarily similar to a direct sum of matrices B and C, where B is idempotent and C is such that $W(C^k)$ is contained in $W(B)$ for all k. We then

consider, for each $n \leq 1$, the smallest integer k_n for which every $n \times n$ matrix A with $W(A) = W(A^k)$ for all k up to k_n implies that A has an idempotent direct summand. It seems to be the case that $k_n = p_n$, the largest prime less than or equal to $n + 1$. We are able to show that (1) k_n is larger than or equal to p_n for all n, (2) $k_1 = 2, k_2 = k_3 = 3$, and (3) if A is an $n \times n$ normal matrix, then $W(A) = W(A^k)$ for k up to p_n implies that A has an (orthogonal) projection direct summand. (Joint work with Hwa-Long Gau and Kuo-Zhong Wang)

References

[1] H.-L. Gau, K.-Z. Wang and P. Y. Wu, *Constant norms and numerical radii of matrix powers*, Oper. Matrices, to appear.

[2] H.-L. Gau, K.-Z. Wang and P. Y. Wu, *Equality of numerical ranges of matrix powers*, preprint.

Time : 10:50 – 11:20

- **Raymond Nung-Sing Sze, The Hong Kong Polytechnic University, Hong Kong**

Title : **Star-shapedness of (p, q)-Matricial Range**

Abstract : Let $\mathbf{A} = (A_1, \ldots, A_m)$ be an m-tuple of bounded linear operators acting on a Hilbert space H. Their joint (p, q)-matricial range $\Lambda_{p,q}(\mathbf{A})$ is the collection of $(B_1, \ldots, B_m) \in M_q^m$, where $I_p \otimes B_j$ is a compression of A_j on a pq-dimensional subspace. This definition covers various kinds of generalized numerical ranges for different values of p, q, m. In this paper, it is shown that $\Lambda_{p,q}(\mathbf{A})$ is star-shaped if the dimension of H is sufficiently large. If $\dim H$ is infinite, we extend the definition of $\Lambda_{p,q}(\mathbf{A})$ to $\Lambda_{\infty,q}(\mathbf{A})$ consisting of $(B_1, \ldots, B_m) \in M_q^m$ such that $I_\infty \otimes B_j$ is a compression of A_j on a closed subspace of H, and consider the joint essential (p, q)-matricial range

$$\Lambda_{p,q}^{ess}(\mathbf{A}) = \bigcap \{cl(\Lambda_{p,q}(A_1 + F_1, \ldots, A_m + F_m)) : F_1, \ldots, F_m \text{ are compact operators}\}.$$

The above set is shown to be non-empty, compact and convex. Similar results for real joint (p, q)-matricial range and joint congruence matricial ranges will be discussed too. (Joint work with P.S. Lau (Nevada), C.K. Li (William & Mary) and Y.T. Poon (Iowa State))

References

[1] Pan-Shun Lau, Chi-Kwong Li, Yiu-Tung Poon, and Nung-Sing Sze, *Convexity and star-shapedness of matricial range*, Journal of Functional Analysis **275** (2018), 2497-2515.

Time : 11:30 – 12:00

- **Jorge Antezana, National University of La Plata, Argentina**

Title : **Polar Decompositions and Aluthge Transforms**

Abstract : In this talk I will talk about several polar decompositions on the Lie group of invertible matrices. These polar decompositions have the form $M = f(P)UP$, where U is a unitary matrix, P is a positive definite matrix, and f is a self-map on the convex cone of positive definite matrices. I will comment some basic properties of the corresponding unitary factor and absolute value, and I will compare them with those of the classical polar decomposition. Finally, I will discuss the special case $f(X) = X^t$, and its connection with Aluthge transforms. (Joint work with Yongdo Lim)

Time : 12:00 – 12:30