Effort Levels in Contests with Two Asymmetric Players*

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I. Introduction

A contest is a situation in which players compete with one another by expending effort to win a prize. Examples abound. Firms compete by spending R&D expenditures to win a patent which will guarantee a flow of high profits. Firms compete to obtain a monopoly or a procurement contract from a government. People in different locations compete to win the designation of the location of a government institution, a government-owned corporation, or a new highway. Political candidates compete by their campaign activities to win an election. Candidates compete for a job or to win promotion to a higher rank.

Such contests have been studied by many economists. Loury [17], Lee and Wilde [16], Dasgupta and Stiglitz [5], Gilbert and Newbery [10], Harris and Vickers [11], Reinganum [21], and Delbono and Denicolo [6] have studied R&D competition. Tullock [25; 26], Krueger [14], Posner [20], Rogerson [23], Appelbaum and Katz [1], Hillman and Riley [12], Hirshleifer [13], Ellingsen [8], Nitzan [18], and Baik [2; 3] have studied rent-seeking contests. Lazear and Rosen [15], O'Keeffe, Viscusi, and Zeckhauser [19], and Rosen [24] have examined performance incentives associated with reward schemes. Riley [22] has analyzed the war of attrition and auctions. Dixit [7] has considered strategic commitments in contests with different applications. These are only a part of the literature on the theory of contests.

Despite the vast literature, however, the effects of asymmetries between players have not been clarified, except by Harris and Vickers [11] who analyze, in a patent "race" model, the strategic consequences of asymmetries. The purpose of this paper is to examine the effects of asymmetries between two players on individual and total effort levels in a model with a logit-form probability-of-winning function. Effort levels expended by the players deserve attention. Effort in a rent-seeking contest is interpreted as social costs and thus total effort level is a measure of economic efficiency; effort in an R&D contest is interpreted as R&D expenditures and thus effort levels determine the expected date of invention.

We focus on asymmetries between the players resulting from their different valuations of the prize, their different abilities to convert effort into probability of winning, or both. Such asymmetries are common in contests. For example, people in different locations may receive different

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economic impacts from winning the designation of the location of a new government-owned corporation. Their abilities to influence the political decision may differ. In the case where a government monopoly franchise contract is reconsidered every period, the previous contract holder may value the contract more highly than its rivals because it has invested (and sunk) resources into the particular industry and has experience in that industry. The previous contract holder may be more powerful and more efficient than its rivals in the contest for this period's contract, partly because it has been able to establish a relationship with government officials [23, 392] and partly because its accumulated knowledge and experience in that industry is respected by the authority. In the R&D context, a firm which also participates in other related industries, may put a higher value on a patent and have more ability to win the patent, compared with its rival firms which do not participate in a related industry. Finally, an incumbent monopolist may value the patent for a new substitute product more highly than potential entrants [10, 516; 11, 195]. The reason is that if the incumbent wins the patent, then it earns monopoly profits; if one of the potential entrants wins the patent and enters into the market, then the entrant earns duopoly profits. As for abilities to win the patent, the incumbent may well have more ability than potential entrants.

In section II, we set up the basic model and derive players' reaction functions. We show that the reaction functions are nonmonotonic.

Section III assumes that the players choose their effort levels simultaneously and employs a Nash equilibrium as the solution concept. Let the strong (weak) player be the player who has more (less) ability. Let the Nash winner (Nash loser) be the player who has a probability of winning greater (less) than 1/2 at the Nash equilibrium. Section III shows that the weak player can be the Nash winner. The weak player tries harder than the strong player and becomes the Nash winner if his relative valuation of the prize is high enough to overcome his relative weakness in ability.

Section IV examines how individual and total effort levels at the Nash equilibrium respond when valuation and ability asymmetries between the players change. Let the even contest be a contest in which both players have the same probability of winning at the Nash equilibrium. We find the following. As a player gets hungrier for the prize, (i) he always exerts more effort; (ii) his opponent exerts more effort until the even contest is reached but after the even contest she exerts less effort; and (iii) total effort level becomes larger until the even contest is reached but after the even contest total effort level may become larger or smaller. Starting from the even contest, as a player becomes stronger relative to his opponent, both players expend less effort. This implies that individual and total effort levels are maximized in the even contest.

Section V considers a case of endogenous timing. We model a game in which the players first announce simultaneously and independently when they will expend their effort and then, based on this timing, they choose their effort levels. Defining the subgame-perfect winner as the player who has a probability of winning greater than 1/2 in the subgame-perfect equilibrium, we show that the Nash winner is also the subgame-perfect winner. We also show that in a lopsided contest endogenous timing leads the players to expend less effort, compared with the simultaneous-move Nash equilibrium.[1] In the even contest, however, endogenous timing does not make any difference with respect to effort levels, compared with the simultaneous-move Nash equilibrium.

Section VI provides conclusions.

1. A lopsided contest is defined as a contest in which the probability of winning for one of the players is greater than 1/2 at the Nash equilibrium. In a lopsided contest, then, there exist the Nash winner and the Nash loser.

II. The Basic Model

Consider a contest in which two risk-neutral players, 1 and 2, compete with each other to win a prize. Let x_1 and x_2 represent the two players' irreversible effort levels in units commensurate with the prize and let p represent the probability that player 1 wins. We assume that the probability-of-winning function for player 1 is

$$p = \sigma h(x_1)/(\sigma h(x_1) + h(x_2)), \tag{1}$$

where $\sigma > 0$.[2] The parameter σ represents player 1's *relative* ability to player 2. A value of the ability parameter greater than unity implies that player 1 has more ability than player 2. In this case, if both players exert the same level of effort, player 1's probability of winning is greater than a half. A value of σ less than unity implies the opposite and $\sigma = 1$ implies that both players have equal ability. We assume that $h(0) \geq 0$ and $h(x_i)$ is increasing in x_i. For a mathematical reason, if $h(0) = 0$, we define $p = 0$. We then obtain $\partial p/\partial x_1 > 0$ for $x_2 > 0$ and $\partial p/\partial x_2 < 0$ for $x_1 > 0$. Each player's probability of winning is increasing in his own effort and decreasing in his opponent's effort. We also assume that

$$h''(x_1)(\sigma h(x_1) + h(x_2)) < 2\sigma(h'(x_1))^2 \tag{2}$$

and

$$h''(x_2)(\sigma h(x_1) + h(x_2)) < 2(h'(x_2))^2, \tag{3}$$

where h' and h'' denote the first and second partial derivatives of the function h. From function (1) and inequality (2), we have $\partial^2 p/\partial x_1^2 < 0$ for $x_2 > 0$ and from function (1) and inequality (3), we obtain $\partial^2 p/\partial x_2^2 > 0$ for $x_1 > 0$. This means that the marginal effect of each player's effort on his own probability of winning decreases as his effort increases.

Evaluation of the prize is different between the two players. Player 1 values the prize at αv and player 2 values the prize at v, where $\alpha > 0$. A value of the valuation parameter α greater than unity implies that player 1 is hungrier for the prize than player 2, while a value of the parameter less than unity implies that player 2 is hungrier than player 1.

Let π_i represent player i's expected payoff. We have then

$$\pi_1 = \alpha\sigma v h(x_1)/(\sigma h(x_1) + h(x_2)) - x_1 \tag{4}$$

and

$$\pi_2 = v h(x_2)/(\sigma h(x_1) + h(x_2)) - x_2. \tag{5}$$

We assume that all of this is common knowledge. The first-order conditions for maximizing π_1 and π_2 are

$$\partial \pi_1/\partial x_1 = B_1(x_1, x_2; \alpha, v, \sigma) - 1 = 0 \tag{6}$$

and

2. Logit-form probability-of-winning functions have been extensively used in the contest literature. Examples include Loury [17], Dasgupta and Stiglitz [5], Tullock [26], Rogerson [23], Rosen [24], Appelbaum and Katz [1], Dixit [7], Hillman and Riley [12], Reinganum [21], Ellingsen [8], Nitzan [18], Baik and Shogren [4], and Baik [3].

$$\partial \pi_2/\partial x_2 = B_2(x_1, x_2; v, \sigma) - 1 = 0, \tag{7}$$

where

$$B_1(x_1, x_2; \alpha, v, \sigma) = \alpha\sigma v h'(x_1) h(x_2)/(\sigma h(x_1) + h(x_2))^2$$

and

$$B_2(x_1, x_2; v, \sigma) = \sigma v h(x_1) h'(x_2)/(\sigma h(x_1) + h(x_2))^2.$$

Note that player 2's marginal gross payoff B_2 is independent of the parameter α. Using inequalities (2) and (3), we obtain $\partial B_1/\partial x_1 < 0$ and $\partial B_2/\partial x_2 < 0$.[3] This implies that π_i is strictly concave in x_i and thus the second-order condition for maximizing π_i is satisfied.

Let $x_1 = r_1(x_2)$ denote player 1's reaction function. Since it is derived from condition (6), we have

$$\alpha\sigma v h'(x_1) h(x_2) = (\sigma h(x_1) + h(x_2))^2 \tag{8}$$

along player 1's reaction function. Similarly, we have

$$\sigma v h(x_1) h'(x_2) = (\sigma h(x_1) + h(x_2))^2 \tag{9}$$

along player 2's reaction function, $x_2 = r_2(x_1)$, which is derived from condition (7). Let curve MN in Figure 1 represent the locus of points which satisfy $h(x_2) = \sigma h(x_1)$.[4] We describe the shapes of the reaction functions in Lemma 1.[5]

LEMMA 1. *As x_2 increases from zero, player 1's reaction function lies below curve MN and increases in x_2, lies on the curve and reaches the maximum, and then lies above the curve and decreases in x_2. As x_1 increases from zero, player 2's reaction function lies above curve MN and increases in x_1, lies on the curve and reaches the maximum, and then lies below the curve and decreases in x_1.*

III. Simultaneous Moves and Nash Equilibrium

This section assumes that the players choose their effort levels simultaneously and we employ a Nash equilibrium as the solution concept. Since a Nash equilibrium occurs at an intersection of the reaction functions, it satisfies equations (8) and (9) simultaneously.[6] Let (x_1^*, x_2^*) denote the interior Nash equilibrium. Then we have

3. If $h(0) = 0$, then we have $B_i = 0$ and $\partial B_i/\partial x_i = 0$ given $x_j = 0$, for $i \neq j$. Since this case does not affect our analysis, we ignore it for concise exposition.

4. Figure 1 represents the case where $\sigma > 1$ and $h(0) \neq 0$. If $h(0) = 0$, then curve MN emanates from the origin and both reaction functions start from an arbitrarily small positive number.

5. To shorten the paper, we omit the proofs of Lemmas 1 and 2. They are available from the author upon request.

6. Let $\mathbf{J}(x_1, x_2)$ be the Jacobian of equations (8) and (9). Then

$$\mathbf{J}(x_1, x_2) = \begin{bmatrix} \partial^2\pi_1/\partial x_1^2 & \partial^2\pi_1/\partial x_2 \partial x_1 \\ \partial^2\pi_2/\partial x_1 \partial x_2 & \partial^2\pi_2/\partial x_2^2 \end{bmatrix}.$$

If $\mathbf{J}(x_1, x_2)$ is negative quasidefinite for all strategy profiles, then the game has a unique Nash equilibrium [9, 86].

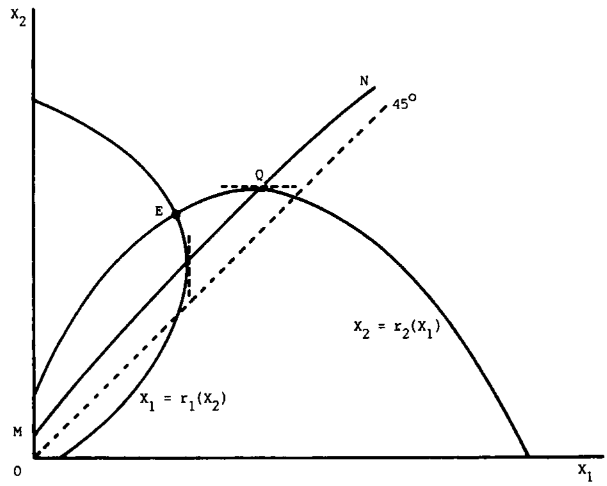

Figure 1. Reaction Functions When Player 2 Is the Weak Player but Is the Nash Winner

$$\alpha\sigma v h'(x_1^*)h(x_2^*) = (\sigma h(x_1^*) + h(x_2^*))^2 \tag{10}$$

and

$$\sigma v h(x_1^*)h'(x_2^*) = (\sigma h(x_1^*) + h(x_2^*))^2. \tag{11}$$

From (10) and (11), we also have

$$\alpha h'(x_1^*)h(x_2^*) = h(x_1^*)h'(x_2^*). \tag{12}$$

Lemma 2 describes the location of the Nash equilibrium.

LEMMA 2. *The Nash equilibrium* (x_1^*, x_2^*) *is located below curve MN if* $\alpha\sigma h'(x_1^*) > h'(x_2^*)$ *holds. It is located above curve MN if* $\alpha\sigma h'(x_1^*) < h'(x_2^*)$ *holds. Finally, it is located on curve MN if* $\alpha\sigma h'(x_1^*) = h'(x_2^*)$ *holds.*

Note that all the conditional statements in Lemma 2 are stated in terms of the parameters α, v, and σ since $x_1^* = x_1^*(\alpha, v, \sigma)$ and $x_2^* = x_2^*(\alpha, v, \sigma)$. Suppose that the function h is an affine function: $h(x_i) = a + bx_i$ where a is a nonnegative constant and b is a positive constant. Then, we can rewrite Lemma 2 as follows: The Nash equilibrium is located below curve MN if $\alpha\sigma > 1$; it is located above curve MN if $\alpha\sigma < 1$; it is located on curve MN if $\alpha\sigma = 1$. We have defined the Nash winner as the player who has a probability of winning greater than 1/2 at the Nash equilibrium. Therefore, if the Nash equilibrium is located below curve MN, player 1 is the Nash winner. If the Nash equilibrium is located above curve MN, player 2 is the Nash winner. Lemma 2 shows that the Nash winner of the contest is determined by "composite" strength of the players—the winner is determined by their valuations of the prize as well as their abilities. An important implication is that the weak player can be the Nash winner. As Figure 1 illustrates, the weak player tries harder than the strong player and becomes the Nash winner if his relative valuation of the prize is high enough to overcome his relative weakness in ability. This confirms that motivation is an important element of success.

IV. Degree of Asymmetries and Effort Levels

This section examines how individual and total effort levels at the Nash equilibrium respond when the valuation parameter α or the ability parameter σ changes. We begin by showing how an increase in α affects the reaction functions.

LEMMA 3. *When the valuation parameter α increases, player 1's reaction function shifts to the right while player 2's reaction function remains unchanged.*

Proof. Player 1's reaction function is derived from condition (6). We obtain $\partial B_1/\partial\alpha > 0$ and $\partial B_1/\partial x_1 < 0$. This implies that given player 2's effort level, when α increases, player 1's effort level must increase in order to satisfy condition (6).

Player 2's reaction function is derived from condition (7) which is independent of α. ∎

Given opponent's effort level, when a player gets hungrier for the prize, his marginal gross payoff increases at any level of his effort and thus his best response increases.

Proposition 1 summarizes how individual and total effort levels at the Nash equilibrium are affected as α increases.[7]

PROPOSITION 1. *As the valuation parameter α increases from an arbitrarily small positive number, both players expend more effort and thus total effort level becomes larger. This is true until the even contest is reached. As α increases beyond the even contest, player 1 expends more effort while player 2 expends less. Whether total effort level becomes larger or smaller depends on the derivative of player 2's reaction function.*

The proof of Proposition 1 is immediate from Lemma 3 and is omitted. Proposition 1 shows the following. As a player gets hungrier for the prize, (i) he always exerts more effort; (ii) his opponent exerts more effort until the even contest is reached but after the even contest she exerts less effort; and (iii) total effort level becomes larger until the even contest is reached but after the even contest total effort level may become larger or smaller. In the case where the function h is an affine function, we can obtain more definite result about total effort level: For $\sigma \le 2$, total effort level is always increasing in the valuation parameter α; for $\sigma > 2$, total effort level is increasing in α until α reaches $1/(\sigma - 2)$ and then is decreasing in α. Total effort level is not maximized in the even contest which occurs when $\alpha = 1/\sigma$.

Next, we perform comparative statics with respect to the ability parameter σ. Let σ_1 be the initial value of σ and let σ_2 be its new value. Assume that σ_2 is greater than σ_1 and that the difference between the two values, $\sigma_2 - \sigma_1$, is small. In Figures 2a and 2b, curve M_1N_1 represents the locus of points which satisfy $h(x_2) = \sigma_1 h(x_1)$ and curve M_2N_2 satisfies $h(x_2) = \sigma_2 h(x_1)$. The following two lemmas show how a small increase in σ affects the reaction functions.

LEMMA 4. *When the ability parameter σ increases from σ_1 to σ_2, above curve M_2N_2 player 1's (player 2's) reaction function shifts to the right (upward) but below curve M_1N_1 player 1's (player 2's) reaction function shifts to the left (downward).*

Proof. Consider first player 1's reaction function which is derived from condition (6). Using $\partial B_1/\partial\sigma = \alpha v h'(x_1)h(x_2)(h(x_2) - \sigma h(x_1))/(\sigma h(x_1) + h(x_2))^3$, we obtain the following: When σ

7. Figure 1 is useful in following Proposition 1. Note that the even contest occurs when player 1's reaction function passes through point Q and that curve MN is independent of the valuation parameter α.

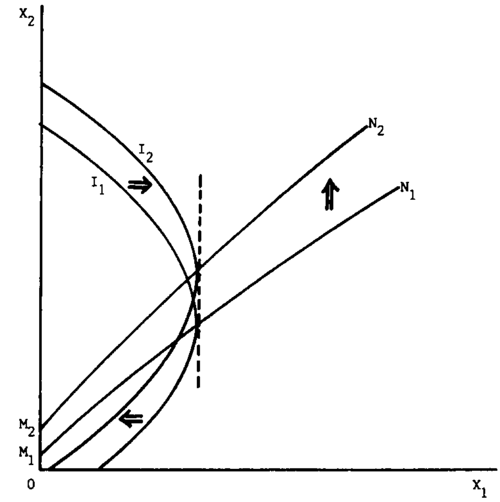

Figure 2a. A Shift of Player 1's Reaction Function When the Ability Parameter Increases

increases from σ_1 to σ_2, $\partial B_1/\partial\sigma > 0$ holds at the points above curve M_2N_2 but $\partial B_1/\partial\sigma < 0$ holds at the points below curve M_1N_1. This follows from the fact that $h(x_2) > \sigma_2 h(x_1)$ holds at the points above curve M_2N_2 but $h(x_2) < \sigma_1 h(x_1)$ holds at the points below curve M_1N_1. Using inequality (2), we also obtain that $\partial B_1/\partial x_1 < 0$. Thus $\partial B_1/\partial x_1 < 0$ and $\partial B_1/\partial\sigma > 0$ hold above curve M_2N_2 but $\partial B_1/\partial x_1 < 0$ and $\partial B_1/\partial\sigma < 0$ hold below curve M_1N_1. Therefore, given player 2's effort level, when σ increases from σ_1 to σ_2, above curve M_2N_2 player 1's effort level must increase but below curve M_1N_1 it must decrease, in order to satisfy condition (6).

The proof of the second part is similar to the above and is omitted. ∎

LEMMA 5. *When the ability parameter σ increases from σ_1 to σ_2, the maximum value of each reaction function remains constant.*

Proof. It follows from condition (6) and Lemma 1 that at the maximum point of player 1's reaction function, $B_1 = 1$ and $h(x_2) = \sigma h(x_1)$ hold. These two equations are reduced to $\alpha v h'(x_1) = 4h(x_1)$. This implies that the value of x_1 satisfying the two equations does not depend on σ.

Similarly, we obtain $vh'(x_2) = 4h(x_2)$ at the maximum point of player 2's reaction function. Therefore, the maximum of player 2's reaction function is independent of σ. ∎

Using Lemmas 1, 4, and 5, we draw Figures 2a and 2b. The figures illustrate shifts of the reaction functions when σ increases from σ_1 to σ_2. In Figure 2a, reaction functions I_1 and I_2 represent player 1's reaction functions when $\sigma = \sigma_1$ and $\sigma = \sigma_2$, respectively. In Figure 2b, reaction functions J_1 and J_2 represent player 2's reaction functions when $\sigma = \sigma_1$ and $\sigma = \sigma_2$, respectively. The shift of each reaction function is not unidirectional. In Figure 2a, player 1's best response to player 2's "low" effort level decreases while his best response to player 2's "high" effort level increases. In Figure 2b, player 2's best response to player 1's "low" effort level increases while his best response to player 1's "high" effort level decreases. An increase in σ means that player 1's relative ability to player 2 increases. It also means that player 2's relative ability to player 1 decreases. Hence, the above can be rephrased as follows: Given low effort level of the opponent, a player responds to an increase (decrease) in his relative ability by decreasing (in-

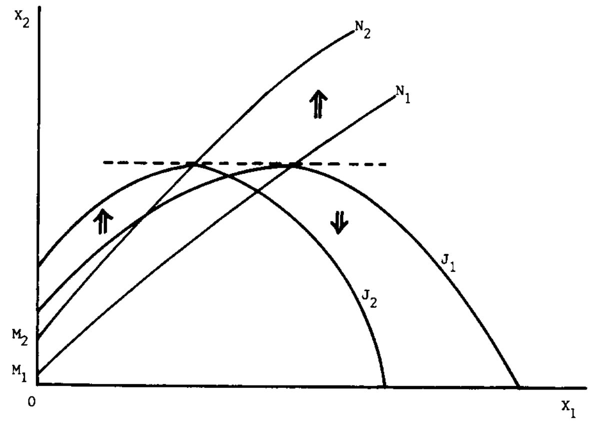

Figure 2b. A Shift of Player 2's Reaction Function When the Ability Parameter Increases

creasing) his effort level; given high effort level of the opponent, a player responds to an increase (decrease) in his relative ability by increasing (decreasing) his effort level.

Figures 3a and 3b are useful in proving Proposition 2. In Figures 3a and 3b, the even contest occurs if player 1 has reaction function I and player 2 has reaction function J. This is because the reaction functions I and J are maximized at point E, which implies that the players have the same probability of winning at the Nash equilibrium (see Lemma 1). Figure 3a shows that player 1's reaction function shifts to I' and player 2's reaction function shifts to J' when σ increases from the even contest. Figure 3b shows that player 1's reaction function shifts to I'' and player 2's reaction function shifts to J'' when σ decreases from the even contest. Points E' and E'' are the resulting Nash equilibria.

Proposition 2 describes how individual and total effort levels at the Nash equilibrium respond when σ changes.

PROPOSITION 2. *Starting from the even contest, as the ability parameter σ increases (decreases), both players exert less effort and thus total effort level becomes smaller.*

Proof. Expression (12) is satisfied at the Nash equilibrium. Partially differentiating expression (12) with respect to σ, we obtain:

$$(\alpha h''(x_1^*)h(x_2^*) - h'(x_1^*)h'(x_2^*))(\partial x_1^*/\partial\sigma)$$
$$= (h(x_1^*)h''(x_2^*) - \alpha h'(x_1^*)h'(x_2^*))(\partial x_2^*/\partial\sigma). \tag{13}$$

We first consider the case where σ increases, starting from the even contest. From Figure 3a, it is easy to see that $\partial x_1^*/\partial\sigma < 0$. Using expression (13) and assuming that $h''(x_i) \leq 0$, we also obtain $\partial x_2^*/\partial\sigma < 0$.

Next, it follows immediately from Figure 3b that as σ decreases from the even contest, player 2's effort level at the Nash equilibrium decreases: $\partial x_2^*/\partial\sigma > 0$. Using expression (13) and assuming that $h''(x_i) \leq 0$, we also obtain $\partial x_1^*/\partial\sigma > 0$. ∎

An increase (decrease) in σ means that player 1 (player 2) becomes stronger relative to player 2 (player 1). Therefore, Proposition 2 establishes the following. Starting from the even contest, as one player becomes stronger relative to his opponent, the player expends less effort. Furthermore, his opponent also expends less effort. In the case where the function h is an affine

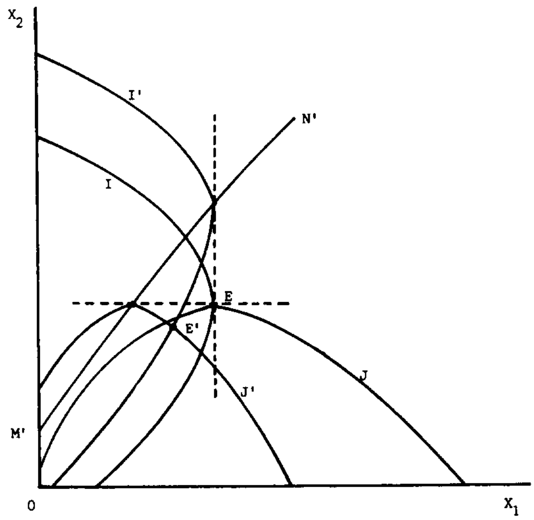

Figure 3a. Shifts of the Reaction Functions When the Ability Parameter Increases from the Even Contest

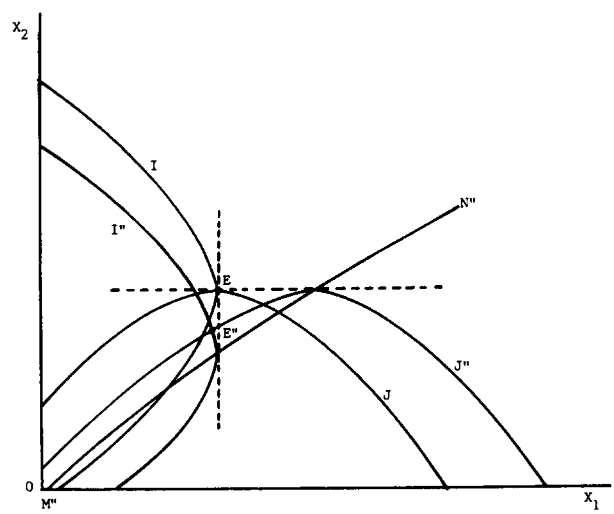

Figure 3b. Shifts of the Reaction Functions When the Ability Parameter Decreases from the Even Contest

function, the even contest occurs when $\alpha\sigma = 1$ (see Lemma 2). Therefore, each player's effort level is increasing in σ until σ reaches $1/\alpha$, and then is decreasing in σ. The following corollary follows immediately from Proposition 2.

COROLLARY. *When we perform comparative statics with respect to the ability parameter σ, individual and total effort levels are maximized in the even contest.*

This result contrasts with a result in Proposition 1. When we perform comparative statics with respect to the valuation parameter α, only player 2's effort level is maximized in the even contest.

V. Endogenous Timing

Sections III and IV have assumed that the players choose their effort levels simultaneously. But this simultaneous-move assumption may be too restrictive. In many contests, we observe that players have opportunities to communicate with each other before they expend their effort. We also observe that players announce their plans strategically or nonstrategically before they expend their effort. In these circumstances, we can expect players to determine the order of their moves endogenously. This section considers a case of endogenous timing. Formally, we extend the basic model described in section II to the following game. There are two periods, the first and the second, in which the players expend their effort. The players first announce simultaneously and independently when they will expend their effort. Then, knowing who will move when, the players choose their effort levels in the period which they chose in the announcement stage.

Employing a subgame-perfect equilibrium as the solution concept, we find that in a lopsided contest the Nash loser expends his effort before the Nash winner does.[8] Let the subgame-perfect winner (subgame-perfect loser) be the player who has a probability of winning greater (less) than 1/2 in the subgame-perfect equilibrium. We also find Propositions 3 and 4.

PROPOSITION 3. *A player is the subgame-perfect winner (subgame-perfect loser) if and only if he is the Nash winner (Nash loser).*

Proposition 3 establishes that the simultaneous-move and endogenous-timing frameworks yield the same result with respect to the winner (loser) of the contest. However, the two frameworks yield different results with respect to individual and total effort levels. Proposition 4 compares individual and total effort levels in the endogenous-timing framework with those in the simultaneous-move framework.

PROPOSITION 4. *In a lopsided contest, both players expend less effort in the subgame-perfect equilibrium, compared with the simultaneous-move Nash equilibrium. Thus total effort level is lower in the subgame-perfect equilibrium than at the simultaneous-move Nash equilibrium. In the even contest, individual and total effort levels in the subgame-perfect equilibrium are the same as those at the simultaneous-move Nash equilibrium.*

VI. Conclusions

We have considered contests with two asymmetric players both in the simultaneous-move framework and in the endogenous-timing framework, focusing on effort expended by the players. Our model can be applied to many specific contexts, such as rent-seeking contests and R&D competition. In rent-seeking contests, effort expended by the players is interpreted as social costs. Tullock [26, 109] argues that social costs can be lowered by introducing bias into the selection process. However, his argument is based on a model in which two players value the prize equally. We have shown that if the players value the prize differently, then introducing bias into the selection process may increase social costs. More precisely, if bias is introduced in favor of the player who values the prize lower, then social costs may increase.

8. The detailed analysis of this endogenous-timing game is omitted and is available from the author upon request. Baik and Shogren [4] analyze a game similar to this endogenous-timing game.

We have established that if rent-seeking contests are lopsided, then social costs are lower in the endogenous-timing framework than in the simultaneous-move framework. However, the endogenous-timing framework requires certain conditions such as observable effort. We may then argue that one way to lower social costs is to create an environment in which such conditions are satisfied.

Another important application of our model is R&D competition. In R&D competition, effort expended by the players is interpreted as R&D expenditures and the prize is interpreted as the patent. If we introduce valuation and ability asymmetries into the models studied in Loury [17] and Dasgupta and Stiglitz [5], we obtain firms' profit functions similar to functions (4) and (5). The only difference from functions (4) and (5) is that the probability-of-winning function for firm 1 is given by: $p = \sigma h(x_1)/(\sigma h(x_1) + h(x_2) + r)$, where r is the discount rate. Using this probability-of-winning function, we obtain the same qualitative results. Our results suggest that when government designs policies to increase or decrease R&D expenditures in some industries, it must examine carefully the effects of the policies on firms' valuations of the patent and relative abilities between the firms.

We now compare our results with those in the auction literature. To do so, we replace function (1) with the following: $p = 0$ for $x_1 < x_2$, $p = 1/2$ for $x_1 = x_2$, and $p = 1$ for $x_1 > x_2$. Note that this probability-of-winning function for player 1 does not have the ability parameter. Consider first the case in which the winner's bid or effort is nonrefundable but the loser's effort is *refundable,* as in sealed first-price auctions. Riley [22] shows that in all Nash equilibria the player with higher valuation bids the valuation of his opponent and wins with probability one. Then, Proposition 1 is modified as follows: As the valuation parameter α increases from an arbitrarily small positive number, player 2's (= total nonrefundable) effort level becomes larger; this is true until $\alpha = 1$; as α increases beyond unity, player 1's (= total nonrefundable) effort level remains unchanged at v. Next, consider the case in which both players' effort is nonrefundable. From Hillman and Riley [12, 25] and Riley [22], we know the following: If $\alpha \leq 1$, then player 1's expected effort level is $\alpha^2 v/2$, player 2's expected effort level is $\alpha v/2$, and expected total effort level is $\alpha(\alpha + 1)v/2$; if $\alpha > 1$, then player 1's expected effort level is $v/2$, player 2's expected effort level is $v/2\alpha$, and expected total effort level is $(\alpha + 1)v/2\alpha$. Then, Proposition 1 is modified as follows: As the valuation parameter α increases from an arbitrarily small positive number, both players expend more effort and thus total effort level becomes larger; this is true until $\alpha = 1$; as α increases beyond unity, player 1's effort level remains constant, player 2's effort level becomes smaller, and thus total effort level becomes smaller.

We have assumed that the players are risk-neutral. Assuming that the players are risk-averse, we may obtain the same qualitative results both in the simultaneous-move framework and in the endogenous-timing framework. Suppose that player i has initial wealth of w_i^o and has the following von Neumann-Morgenstern utility function: $u_i = k_i - e^{-\lambda w_i}$, where $k_i \geq 0$, λ is a small positive number, and w_i is his final wealth. Suppose that the probability-of-winning function for player 1 is $p = \sigma x_1/(\sigma x_1 + x_2)$. Then, we obtain the same qualitative results as in Propositions 1 through 4.

In this paper, each player knows his opponent's valuation and ability. What happens to our results if the players are uncertain about their opponent's valuation or ability? In the case where they are uncertain about α but its cumulative distribution function is common knowledge, our results remain the same. In this case, we simply replace α in function (4) with the expected value of α. When the players are uncertain about σ, the analysis becomes involved. We leave this problem for future research.

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