Endogenous Timing in Three-Player Tullock Contests

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Abstract

We study a three-player Tullock contest in an endogenous-timing framework, focusing on the players' decisions on timing of effort exertion. In this model, there are two points in time at which the players may choose their effort levels. The players decide independently and announce simultaneously when they each will expend their effort, and then each player chooses her effort level at the point in time which she announced. We find that, given moderate asymmetries among the players, each of the players announces the *first point in time*, and thus they all choose their effort levels simultaneously at the first point in time. This finding is in sharp contrast to a well-known result obtained from two-player asymmetric contests with endogenous timing.

Keywords: Endogenous timing; Contest; Rent seeking; Two points in time

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1. Introduction

A situation in which individuals or groups of individuals or organizations compete by expending effort or resources to win a prize, which is referred to as a contest, is common in the real world. An example is an election in which candidates compete to be the president of a country. Another example is a rent-seeking contest in which firms compete to win rents created by government protection or those generated by governmental trade policies. Yet another example is a patent-seeking contest in which firms or researchers compete to obtain a patent. Other examples of a contest include litigation, various sporting contests, competition for college admission, and competition for a job or promotion to a higher rank.

A strand of the literature on contests deals with endogenous timing of effort exertion in contests. In this strand of the literature, a well-known result is that the contestants choose their effort levels sequentially. Specifically, the seminal papers of Baik and Shogren (1992) and Leininger (1993) consider two-player asymmetric contests in which there are two points in time at which the players may choose their effort levels; the players decide independently and announce simultaneously when they each will expend their effort, and then each player chooses her effort level at the point in time which she announced. They show that the weak player – in terms of the players' composite strength determined by their valuations for the prize and their relative abilities – announces the *first point in time* while the strong player announces the *second point in time*; accordingly, the weak player chooses her effort level before the strong player.[1]

The intuition behind this result is as follows. Note first the following fact obtained in the effort-expending stage: Around the intersection of the players' reaction functions, the weak player's reaction function is decreasing in the strong player's effort level (measured along the horizontal axis) while the strong player's reaction function is increasing in the weak player's effort level. To put this differently, around the intersection, the weak player regards her effort as a strategic substitute to the strong player's while the strong player regards her effort as a strategic complement to the weak player's.[2] Given this fact, the weak player wants to take the leadership role in the effort-expending stage because she, as the leader, can show the strong player her commitment – in the form of exerting low effort – to avoid a big costly fight. On the other hand, the strong player wants to concede the leadership role because she, as the follower, can compete efficiently against the weak player by responding with an appropriate level of effort to the weak player's challenge. Surprisingly, what the players want to do is perfectly compatible, and is actually carried out.

Now, a natural question that arises is: What happens if another player is present in such contests? Do the players in three-player contests choose their effort levels in some sequential manner? In particular, is there any player that wants to concede the leadership role and actually chooses her effort level after some other player?

To address these questions, we study a three-player Tullock contest in an endogenous-timing framework, focusing on the players' decisions on timing of effort exertion. In this model, as in those of Baik and Shogren (1992) and Leininger (1993), each player's valuation for the prize is exogenously fixed and publicly known, and the players' relative abilities to convert effort into probability of winning are also publicly known. There are two points in time at which the players may choose their effort levels. Each player chooses her effort level at either of the two points, but not at both points. The players play the following game. First, the players decide independently and announce simultaneously whether they each will expend their effort at the first point in time or at the second point in time. Then, knowing when the players choose their effort levels, each player independently chooses her effort level at the point in time which she announced. A player who chooses her effort level at the second point in time knows exactly what happened at the first point in time.

Three-player contests in which the players' order of moves is endogenously determined are easily observed in the real world. One example may arise in three-candidate competition for presidential office. Given two time periods, the early period and the late period, in which the candidates may hold their election campaigns, the candidates first decide and announce when they will hold their (main) campaigns, and then actually run their campaigns according to their announced campaign schedules.[3] Another example may arise in three-firm patent competition with two time periods, the early period and the late period. The firms first announce (and commit to) when they will undertake their (main) research activities, and then carry out their research activities according to their announced research plans. Indeed, recently, we observed that vaccine makers first announced their development plans for vaccines for COVID-19 and then actually expended their effort according to their announced plans.

We find that, given *moderate asymmetries* among the players, the game has a *unique* subgame-perfect equilibrium, where each of the players announces the *first point in time*, and thus they all choose their effort levels simultaneously at the first point in time. This finding is in sharp contrast to the aforementioned well-known result obtained from two-player asymmetric contests with endogenous timing. Indeed, given moderate asymmetries among the players, the presence of an additional player makes a big difference in the equilibrium timing of effort exertion.

More detailed explanations for the main result will be given later in Sections 4 and 5, but here it is desirable to give a brief explanation for it. In the three-player contest, if a player were to announce the *second point in time*, then she would suffer seriously from the second-mover disadvantage in the effort-expending stage because the leaders, if any, would not restrain themselves due to intense competition between themselves. On the other hand, if the player announces the *first point in time*, then she may exercise strategic leadership and enjoy a first-mover advantage in the effort-expending stage or, at least, she will compete with the other players on equal footing in the effort-expending stage. Consequently, the player is better off by announcing the *first point in time* rather than the *second point in time*.

Other papers which study endogenous timing in contests include Nitzan (1994), Morgan (2003), Fu (2006), Konrad and Leininger (2007), Hoffmann and Rota-Graziosi (2012), and Baik and Lee (2013). All these papers, except Konrad and Leininger (2007), study contests in which just two contestants compete for a prize. Konrad and Leininger (2007) use an all-pay-auction contest success function, Hoffmann and Rota-Graziosi (2012) use a general contest success function, and the rest use logit-form contest success functions.

In Nitzan (1994), Konrad and Leininger (2007), and Baik and Lee (2013), each player's valuation for the prize is exogenously fixed and publicly known at the start of the game. In Morgan (2003) and Fu (2006), however, it is drawn from a probability distribution – which is publicly known at the start of the game – after the players announce when they will expend their effort. The realized valuations are revealed to both players, in Morgan (2003), while the realized common valuation is revealed to only one of the two players, in Fu (2006). By contrast, in Hoffmann and Rota-Graziosi (2012), the players' common valuation for the prize is endogenously determined, depending only on their effort levels, and the "valuation function" is known to the players at the start of the game.

These papers all show that the contestants choose their effort levels sequentially. Specifically, Fu (2006) shows that the uninformed player chooses her effort level before the informed player. Considering a contest, or an all-pay auction, in which each player's cost function is a general convex function of effort, Konrad and Leininger (2007) show that the player with the lowest cost of effort typically chooses her effort level late while the other players each choose their effort levels either early or late. Hoffmann and Rota-Graziosi (2012) show that in some cases the players choose their effort levels sequentially while in others they do so simultaneously. Baik and Lee (2013) consider a two-player contest in which the players hire delegates and announce simultaneously their contracts, then the delegates decide independently and announce simultaneously whether they each will expend their effort at the first point in time or at the second point in time, and finally each delegate chooses his effort level at the point in time which he announced. They show that the weak delegate, or the delegate with less contingent compensation, announces the *first point in time* while the strong delegate announces the *second point in time*, and thus the weak delegate chooses his effort level before the strong delegate.

This paper is related also to the literature on endogenous timing in an oligopoly (model). Reinganum (1985), Gal-Or (1985), Dowrick (1986), Boyer and Moreaux (1987), and Robson (1990) study endogenous role selection in a duopoly. Hamilton and Slutsky (1990) study endogenous timing in two duopoly games, the extended game with observable delay and the extended game with action commitment. Deneckere et al. (1992) examine the incentives to lead and follow in a price-setting game in which two firms have loyal consumer segments, and then examine two games of timing of price announcements. Matsumura (1999) studies endogenous sequencing in a quantity-setting oligopoly model with n firms. In his model, there are m periods in which the firms may choose their output levels. He shows that, in every pure-strategy equilibrium, at least $n-1$ firms choose their output levels simultaneously in the first period. Hoffmann and Rota-Graziosi (2020) generalize "the extended game with observable delay" in Hamilton and Slutsky (1990), and study endogenous timing when the payoff or the marginal payoff of a player becomes non-monotonic with respect to the opponent's strategy.

The paper proceeds as follows. In Section 2, we present the model of a three-player contest and set up a two-stage game. In Section 3, we analyze the proper subgames starting at the second stage, and obtain each player's equilibrium effort levels and expected payoffs in these subgames. Section 4 looks at the first stage in which the players decide independently and announce simultaneously when they each will expend their effort, and obtains the subgame-perfect equilibrium of the full game. In Section 5, we first study a two-player contest in an endogenous-timing framework, and then compare the equilibrium timing of effort exertion in this two-player contest with that in the three-player contest. Section 6 considers variations of the main model presented in Section 2, and discusses restrictions on the parameters imposed by Assumption 2. Finally, Section 7 offers our conclusions.

2. The model

Consider a contest in which three risk-neutral players, 1 through 3, compete with each other by expending irreversible effort to win a prize. The players' valuations for the prize may differ. Their abilities to convert effort into probability of winning also may differ. There are two points in time, points 1 and 2, at which the players may choose their effort levels. Each player chooses her effort level at either of the two points, but not at both points. The players play the following game. First, the players decide independently and announce simultaneously whether they each will expend their effort at point 1 or at point 2. Then, knowing when the players choose their effort levels, each player independently chooses her effort level at the point in time which she announced. A player, if any, who chooses her effort level at point 2 observes the effort levels, if any, chosen at point 1 before she does.

Let $N \equiv \{1, 2, 3\}$ denote the set of the players. Let v_i, for $i \in N$, denote player i's valuation for the prize. We assume that each player's valuation for the prize is positive and publicly known at the start of the game. Without loss of generality, let $v_i = \beta_i v_3$, where $\beta_i > 0$ and $\beta_3 = 1$, and let $v_3 = 1$.

Let x_i, for $i \in N$, denote player i's effort level, where $x_i \in R_+$. Let p_i denote player i's probability of winning the prize, where $0 \leq p_i \leq 1$ and $\sum_{k=1}^{3} p_k = 1$. Let $\boldsymbol{x}$ denote a 3-tuple vector of effort levels, one for each player: $\boldsymbol{x} \equiv (x_1, x_2, x_3)$. Then we assume the following contest success function for player i:

$$p_i = p_i(\boldsymbol{x}) = \begin{pmatrix} \sigma_i x_i / X & \text{for } X>0 \\ 1/3 & \text{for } X=0 \end{pmatrix}, \tag{1}$$

where $\sigma_i > 0$, $\sigma_3 = 1$, and $X = \sigma_1 x_1 + \sigma_2 x_2 + x_3$.[4] The parameter σ_i indicates player i's ability in the contest relative to the other players. For example, $\sigma_j > \sigma_k$, for $j, k \in N$, means that player j has more ability than player k in that, *ceteris paribus*, if $x_j = x_k > 0$, then player j's probability of winning is greater than player k's. We assume that the parameter σ_i, for $i \in N$, is publicly known at the start of the game. Function (1) implies that, *ceteris paribus*, player i's probability of winning is increasing in her effort level at a decreasing rate; however, it is decreasing in a rival's effort level at a decreasing rate.

Let $w_i \equiv \beta_i \sigma_i v_3$ for $i \in N$. Then w_i is the product of player i's valuation for the prize and her ability parameter, and thus it indicates her "composite strength" in the contest relative to the other players.

Assumption 1. *We assume, without loss of generality, that* $w_1 \geq w_2 \geq w_3 = 1$.

We will make further assumptions on the parameters later, during the analysis, which may be considered as defining a three-player contest narrowly (see Assumption 2).

Let π_i denote the expected payoff for player *i*. Then the payoff function for player *i* is:[5]

$$\pi_i \;=\; v_i p_i(\boldsymbol{x}) \;-\; x_i \;=\; \begin{pmatrix} w_i x_i/X \;-\; x_i & \text{for } X>0 \\ v_i/3 & \text{for } X=0 \end{pmatrix}. \tag{2}$$

We formally consider the following game. In the first stage, each player chooses independently between *Point 1* and *Point 2*. The players announce (and commit to) their choices simultaneously (or, equivalently, without knowing their rivals' choices). In the second stage, after knowing when the players choose their effort levels, each player independently chooses her effort level at the point in time which she announced in the first stage.[6] A player, if any, who chooses her effort level at point 2 observes the effort levels, if any, chosen at point 1 before she does. At the end of this stage, the winner is determined.

We assume that the structure of the game is common knowledge among the players. We employ subgame-perfect equilibrium as the solution concept.

3. Subgames starting at the second stage

To obtain a subgame-perfect equilibrium of the game, we work backward. In this section, we analyze the proper subgames starting at the second stage, and obtain each player's equilibrium effort levels and expected payoffs in these subgames. Then, in Section 4, we look at the players' decisions, in the first stage, on when to expend their effort.[7]

There are eight proper subgames which start at the second stage, but we need to analyze the following seven subgames: a simultaneous-move subgame and six sequential-move subgames.[8] A simultaneous-move subgame arises when the three players choose and announce the same point in time, either *Point 1* or *Point 2*, in the first stage. If player i, for $i \in N$, announces *Point 1* but the rest announce *Point 2*, then the *iL* sequential-move subgame arises.[9] Finally, the *jkL* sequential-move subgame, for $j = 1$, 2 and $k = 2$, 3 with $j \neq k$, arises when players j and k announce *Point 1* but the remaining player announces *Point 2*.[10]

3.1. A simultaneous-move subgame

In this subgame, the players choose their effort levels simultaneously and independently. Accordingly, player i, for $i \in N$, seeks to maximize π_i in (2) over her effort level x_i, given her belief about the other players' effort levels.

From the first-order condition for maximizing π_i, we first derive player i's reaction function (see Appendix A).[11] Then, using the players' reaction functions, we obtain a unique Nash equilibrium, which is denoted by the vector $\boldsymbol{x^S} \equiv (x_1^S, x_2^S, x_3^S)$. Next, substituting into (2) the players' equilibrium effort levels, we obtain player i's expected payoff π_i^S at the Nash equilibrium. We report them in Lemma A in Appendix A.

It is straightforward to check that, under Assumption 1, all the players are active – that is, they expend positive effort – at the Nash equilibrium if and only if $w_1 + w_2 > w_1 w_2$.

3.2. The iL sequential-move subgame

Fix player i, for $i \in N$. In this subgame, player i first chooses her effort level at point 1. Then, after observing player i's effort level, the rest choose their effort levels simultaneously and independently at point 2.

Let x_h^{iL}, for $h \in N$, denote player h's effort level specified in an equilibrium of the *iL* sequential-move subgame. Let π_h^{iL} denote player h's expected payoff in the equilibrium. We obtain and report them in Appendix B.

It is straightforward to check that, under Assumption 1, all the players are active in the equilibrium of the *iL* sequential-move subgame if and only if $w_1(w_2 + 1) < w_2 + 2$ for $i = 1$, $w_2(w_1 + 1) < w_1 + 2$ for $i = 2$, and $w_1 + w_2 > w_1 w_2$ for $i = 3$.

3.3. The jkL sequential-move subgame

Fix players j and k, for $j = 1, 2$ and $k = 2, 3$ with $j \neq k$. In this subgame, players j and k first choose their effort levels simultaneously and independently at point 1. Then, after observing their effort levels, the remaining player chooses her effort level at point 2.

Let x_h^{jkL}, for $h \in N$, denote player h's effort level specified in an equilibrium of the jkL sequential-move subgame. Let π_h^{jkL} denote player h's expected payoff in the equilibrium. We obtain and report them in Appendix C.

It is straightforward to check that, under Assumption 1, all the players are active in the equilibrium of the jkL sequential-move subgame if and only if $w_1 + w_2 > 1.5w_1w_2$ for $jk = 12$, $w_1 < 2$ for $jk = 13$, and $w_2 < 2$ for $jk = 23$.

4. Players' decisions on when to exert effort

We consider the first stage of the full game in which the players each announce when to expend their effort. Which point in time does each player choose and announce in a subgame-perfect equilibrium of the full game?

In the first stage, each player has perfect foresight about the equilibrium of every second-stage subgame, and thus about the players' second-stage equilibrium expected payoffs reported in Lemmas A through C. Figure 1 illustrates the strategic interaction among the players in the first stage. Panel A of Figure 1 illustrates it when player 3 announces *Point 1* in the first stage, while panel B illustrates it when player 3 announces *Point 2*. For example, if player 3 announces *Point 1* but the rest announce *Point 2*, then the $3L$ sequential-move subgame will arise, which will lead to the payoff vector $(\pi_1^{3L}, \pi_2^{3L}, \pi_3^{3L})$ in panel A.

We restrict our analysis to cases where all three players are active in the equilibrium of *every* second-stage subgame (see Section 6.2). It follows from the assumptions stated in Lemmas A through C that, under Assumption 1, these cases occur if and only if Assumption 2 holds.

Assumption 2. *We assume that* $w_1(w_2+1) < w_2+2$ *and* $w_1+w_2 > 1.5w_1w_2$.

Note that these two inequalities in Assumption 2 come from the assumptions stated in part (*i*) of Lemmas B and that of Lemma C, respectively. Figure 2 illustrates the values of w_1 and w_2 (or, equivalently, $\beta_1\sigma_1$ and $\beta_2\sigma_2$), which satisfy both Assumption 1 and Assumption 2. In other words, at the values of w_1 and w_2 located in the shaded area of Figure 2, all three players are active in the equilibrium of every second-stage subgame. Note that the northeast part of the shaded area is bounded by the "active-players-in-equilibrium condition" for the $1L$ sequential-move subgame, $w_1(w_2+1) < w_2+2$, and that for the $12L$ sequential-move subgame, $w_1+w_2 > 1.5w_1w_2$.

Now, we compare, for each player, the second-stage equilibrium expected payoffs reported in Lemmas A through C. Under Assumptions 1 and 2, it is straightforward to obtain Lemma 1.

Lemma 1. *Under Assumptions* 1 *and* 2, *we obtain:* (*a*) $\pi_1^S > \pi_1^{23L}$, $\pi_1^{12L} > \pi_1^{2L}$, $\pi_1^{13L} > \pi_1^{3L}$, *and* $\pi_1^{1L} > \pi_1^S$; (*b*) $\pi_2^S > \pi_2^{13L}$, $\pi_2^{12L} > \pi_2^{1L}$, $\pi_2^{23L} > \pi_2^{3L}$, *and* $\pi_2^{2L} > \pi_2^S$; *and* (*c*) $\pi_3^S > \pi_3^{12L}$, $\pi_3^{13L} > \pi_3^{1L}$, $\pi_3^{23L} > \pi_3^{2L}$, *and* $\pi_3^{3L} > \pi_3^S$.

Suppose, for example, that players 2 and 3 each announce *Point 1*. In this case, if player 1 also announces *Point 1*, then a simultaneous-move subgame will arise, which will lead to π_1^S for her. If instead player 1 announces *Point 2*, then the $23L$ sequential-move subgame will arise, which will lead to π_1^{23L} for her. Part (*a*) says that $\pi_1^S > \pi_1^{23L}$.

Suppose, for another example, that players j and k each, for $j = 1$, 2 and $k = 2$, 3 with $j \neq k$, announce *Point 2*. In this case, if the remaining player, player i, also announces *Point 2*, then a simultaneous-move subgame will arise, which will lead to π_i^S for her. If instead player i announces *Point 1*, then the iL sequential-move subgame will arise, which will lead to x_i^{iL} for her. Lemma 1 says that $\pi_i^{iL} > \pi_i^S$, for $i \in N$. This comparison result can be easily understood because, in the *iL* sequential-move subgame, player *i* is the first mover, and thus one of her options is to receive the payoff π_i^S by choosing her effort level at the Nash equilibrium of a simultaneous-move subgame.

Note that Lemma 1 holds even for the case where the three players have equal composite strength: $w_1 = w_2 = w_3 = 1$.[12] Using Lemma 1, we obtain the following result regarding the players' decisions on timing of effort exertion.

Proposition 1. *Under Assumptions* 1 *and* 2, *for every player*, Point 1 *leads to a higher second-stage equilibrium expected payoff than does* Point 2, *no matter what the other players announce in the first stage*. *Consequently*, *under Assumptions* 1 *and* 2, *the game has a unique subgame-perfect equilibrium in which every player announces* Point 1 *in the first stage.*

The result that player *i*, for $i \in N$, announces *Point 1* in the first stage, no matter what the other players announce, can be explained as follows. Consider first the case where the other players both announce *Point 1*. Player *i* has two options: either to announce *Point 1* or to announce *Point 2*. If she announces *Point 1*, then she will compete with the other players on equal footing in the effort-expending stage: The three players will choose their effort levels simultaneously and independently at point 1. On the other hand, if player *i* announces *Point 2*, then she, as the only follower, will suffer seriously from the second-mover disadvantage in the effort-expending stage: She will face the aggressive leaders because the leaders themselves will compete and choose high effort levels at point 1.[13] Accordingly, given that the other players both announce *Point 1*, player *i* also announces *Point 1*.

Next, consider the case where the other players announce different points in time: Player *j* announces *Point 1*, while player *k* announces *Point 2*, for $j, k \in N \setminus \{i\}$ with $j \neq k$. In this case, if player *i* announces *Point 1*, then she will be one of the two leaders in the effort-expending stage; however, if she announces *Point 2*, then she will be one of the two followers. Since a first-mover advantage exists in the effort-expending stage, player i is better off by announcing *Point 1* rather than *Point 2*.

Finally, consider the case where the other players both announce *Point 2*. In this case, too, player i is better off by announcing *Point 1* rather than *Point 2*. The intuition is obvious. Given that the other players both announce *Point 2*, if player i announces *Point 1*, then she will exercise strategic leadership and enjoy a first-mover advantage in the effort-expending stage. However, if player i announces *Point 2*, then she will play a simultaneous-move game with the other players in the effort-expending stage, which will result in a smaller expected payoff to player i as compared to player i's announcing *Point 1*.[14]

5. Comparison with two-player asymmetric contests

In this section, we first study briefly a two-player contest in an endogenous-timing framework. Then we compare the equilibrium timing of effort exertion in this two-player contest with that in the three-player contest analyzed in the preceding sections.

Consider a contest which is the same as the one in Section 2 with the exception that now only player h, for $h = 1$ or 2, and player 3 compete to win a prize. Specifically, consider the following game. In the first stage, each player chooses independently between *Point 1* and *Point 2*. The players announce (and commit to) their choices simultaneously. In the second stage, after knowing when the players choose their effort levels, each player independently chooses her effort level at the point in time which she announced in the first stage. A player who chooses her effort level at point 2 observes the effort levels chosen at point 1 before she does.

Now that the analysis is similar to that in Sections 3 and 4, it is relegated to Appendix D. As shown in Appendix D, under the assumption that $1 < w_h < 2$, we obtain the following result on the players' decisions on timing of effort exertion.[15]

Remark 1. *Under Assumption* 1 *and the assumption that* $1 < w_h < 2$, *in equilibrium*, *player* 3 *announces* Point 1, *while player h announces* Point 2.

Remark 1 implies that the weak player (in terms of composite strength) chooses her effort level before the strong player. The intuition behind this well-known result has been explained in Section 1 (see also Baik and Lee, 2013), and therefore is omitted.

What happens if another player is present in the contest? According to Proposition 1, in the three-player contest with the restrictions on the parameters by Assumption 2, each of the players announces *Point 1*, and thus they all choose their effort levels simultaneously at point 1. Since we have provided, below Proposition 1, the detailed intuition behind this result, here we briefly explain why we do not obtain a result similar to Remark 1. Suppose, similarly to Remark 1, that the weakest player announces *Point 1* – that is, she chooses to be the leader in the effort-expending stage – while the remaining players announce *Point 2*. Given the restrictions on the parameters by Assumption 2, unlike in the two-player contest, each of the followers could not compete efficiently against the weakest player (or the leader) – in other words, the weakest player would not trigger a softened response from each of the followers – because the followers themselves engage in intense competition. This would make each of the followers not gain such advantage that the follower in the two-player contest can obtain. Further, this would lead to each of the followers deviating from *Point* 2 to *Point* 1.[16]

Indeed, in the three-player contest with the restrictions on the parameters by Assumption 2, none of the players announce that they will be (possible) followers. A follower, if any, would compete with either two leaders or one leader and another follower. In both cases, she would suffer seriously from the second-mover disadvantage, facing the opponents who would not restrain themselves due to more intense competition, compared to the two-player contest.

6. Discussion

6.1. More than three players

We consider a model (modified from the main model presented in Section 2) in which there are n players, where $n > 3$. Let $N^+ \equiv \{1, \dots, n\}$ denote the set of the players. Without loss of generality, let $v_i = \beta_i v_n$ for $i \in N^+$, where $\beta_i > 0$ and $\beta_n = 1$, and let $v_n = 1$. We assume the following contest success function for player i:

$$p_i = \begin{pmatrix} \sigma_i x_i / X & \text{for } X>0 \\ 1/n & \text{for } X=0 \end{pmatrix},$$

where $\sigma_i > 0$, $\sigma_n = 1$, and $X = \sum_{z=1}^{n} \sigma_z x_z$. Let $w_i \equiv \beta_i \sigma_i v_n$. Assumption 1 in Section 2 is now replaced with Assumption 3 below.

Assumption 3. *We assume*, *without loss of generality*, *that* $w_1 \geq w_2 \geq \ \ldots \ \geq w_n = 1$.

As in Section 2, we consider the following game. In the first stage, the players each choose independently between *Point 1* and *Point 2*, and announce their choices simultaneously. In the second stage, after knowing when the players choose their effort levels, each player independently chooses her effort level at the point in time which she announced in the first stage. A player who chooses her effort level at point 2 observes the effort levels chosen at point 1 before she does.

We relegate the analysis to Appendix E. As shown in Appendix E, we obtain the following result on the players' decisions on timing of effort exertion.

Remark 2. *Under Assumption* 3 *and the assumption that* $n - 0.5 < \sum_{z=1}^{n} (1/w_z) \leq 2(n-1)/w_1$, *a subgame-perfect equilibrium occurs in which each of the n players announces* Point 1.

Note that the second assumption in Remark 2 reflects the "active-players-in-equilibrium condition" in (E14) and the condition (E15) under which no player has an incentive to deviate from her first-stage action, *Point 1*. Note also that, due to computational intractability, we merely find a subgame-perfect equilibrium, without checking its possible *uniqueness*, in which each of the n players announces *Point 1*.

6.2. Restrictions on the parameters

In Section 4, we have assumed (Assumption 2) that all three players are active in the equilibrium of every second-stage subgame. Several explanations for this assumption are in order since it appears restrictive.

First, a three-player contest may be defined as one in which all three players are active at the Nash equilibrium of a simultaneous-move game (or, in this paper, a simultaneous-move subgame). If so, the values of w_1 and w_2 that are relevant to our analysis are constrained by Assumption 1 and the "active-players-in-equilibrium condition" for a simultaneous-move subgame, $w_1 + w_2 > w_1 w_2$ (see Lemma A). Then, in Figure 2, these values are located in the area "bounded" by the horizontal line at $w_2 = 1$, the 45° line, and curve S that represents the equation $w_1 + w_2 = w_1 w_2$.

Second, Assumption 2, which may be considered as defining a three-player contest narrowly, enables us to *complete* the analysis of the first stage (see Lemma 1). Indeed, due to Assumption 2, we are able to compare, for each player, the second-stage equilibrium expected payoffs, and thereby to show explicitly that the game has a *unique* subgame-perfect equilibrium in which each of the three players announces *Point 1* in the first stage.

Third, it is straightforward to check that Remark 2 holds true for $n = 3$. That is, it is straightforward to see that, under Assumption 1 and the assumption that $2.5 < \sum_{z=1}^{3}(1/w_z) \leq 4/w_1$ or, equivalently, $1.5 w_1 w_2 < w_1 + w_2 \leq (4 - w_1) w_2$, a subgame-perfect equilibrium occurs in which each of the three players announces *Point 1* in the first stage. In Figure 2, the values of w_1 and w_2 that satisfy these two assumptions are located in the shaded area and in area T. (The solid border line of area T represents the equation $w_1 + w_2 = (4 - w_1) w_2$.) Note that, unlike Assumption 2, the second assumption ensures only that all three players are active at the Nash equilibrium of a simultaneous-move subgame and in the equilibrium of the *jkL* sequential-move subgame, for $j = 1, 2$ and $k = 2, 3$ with $j \neq k$ (see (E14) in Appendix E).

Fourth, the values of w_1 and w_2 located in area K of Figure 2 satisfy the following strict inequalities: $1.5w_1w_2 < w_1 + w_2$ and $w_1 + w_2 > (4 - w_1)w_2$. The first inequality ensures that all three players are active at the Nash equilibrium of a simultaneous-move subgame and in the equilibrium of the *jkL* sequential-move subgame, for $j = 1, 2$ and $k = 2, 3$ with $j \neq k$. Satisfying the second inequality implies that there is at least one player who does not announce *Point 1* in the first stage (see (E15) in Appendix E). Hence, at the values of w_1 and w_2 located in area K, the game has no subgame-perfect equilibrium in which each player announces *Point 1* in the first stage.

6.3. Optimal timing of effort exertion

We have shown in Section 4 that, in the endogenous-timing framework, the three players all choose their effort levels simultaneously at point 1. An interesting question that arises is: If a contest organizer or the government could decide and enforce when the players will each expend their effort, what would the optimal timing of effort exertion be?

We consider a variation of the main model in which the contest organizer chooses and enforces the timing of effort exertion which maximizes (or, alternatively, minimizes) the total effort expended by the players (see Baik, 2013; Hinnosaar, 2019).[17] Formally, we consider the following game. In the first stage, the contest organizer decides and announces when the players will each choose their effort levels. In the second stage, after knowing who moves when, each player independently chooses her effort level at the point in time which the contest organizer assigned to her in the first stage. A player who chooses her effort level at point 2 observes the effort levels chosen at point 1 before she does.

It seems to be computationally intractable to analyze this game with general values of the parameters, β_1, β_2, σ_1, and σ_2. Accordingly, we assume that $\beta_1\sigma_1 = \beta_2\sigma_2 = 1$ or, equivalently, $w_1 = w_2 = 1$.

Table 1 shows, in this case, under which timing assumptions the total effort level is maximized and minimized, respectively. We obtain it from Lemmas A through C. In Table 1, X^S denotes the total effort level resulting when the players choose their effort levels simultaneously. In the table, X^{iL}, for $i \in N$, denotes the total effort level resulting when player i chooses her effort level at point 1 and the rest choose their effort levels simultaneously at point 2. Finally, X^{jkL}, for $j = 1, 2$ and $k = 2, 3$ with $j \neq k$, denotes the total effort level resulting when players j and k choose their effort levels simultaneously at point 1 and the remaining player chooses her effort level at point 2.

Suppose, for example, that $\sigma_1 > \sigma_2 > \sigma_3$. Table 1 shows, in this case, that the total effort level is maximized when player 3 chooses her effort level at point 1 and the rest choose their effort levels simultaneously at point 2. Therefore, if the contest organizer desires to maximize the total effort level, then she would announce this timing of effort exertion in the first stage.

Suppose, for another example, that $\sigma_1 = \sigma_2 = \sigma_3$. Table 1 shows, in this case, that the total effort level is minimized if the players choose their effort levels simultaneously. Therefore, if the contest organizer desires to minimize the total effort level, then she would announce this timing of effort exertion in the first stage.

Hinnosaar (2019) studies sequential contests with identical players in which, at each point in time, the sum of the effort levels expended by the earlier movers is publicly disclosed. He shows that the total effort level is minimized in the simultaneous contest, and is maximized in the fully sequential contest. However, Table 1 indicates that the total effort level may not be minimized in the simultaneous contest if players are asymmetric.

6.4. Three points in time

We have so far assumed that there are only two points in time at which the players may choose their effort levels. This two-point-in-time model has a limitation in that it excludes the possibility that each player announces a point in time which is different from the other players' choices. Hence, the model is vulnerable to criticism on the grounds that it examines sequentiality versus simultaneity without actually allowing full sequentiality.

Here we briefly consider a model (modified from the main model presented in Section 2) in which there are three points in time (points 1, 2, and 3) at which the three players may choose their effort levels.

Formally, we consider the following game. In the first stage, each of the three players chooses independently among *Point 1*, *Point 2*, and *Point 3*. The players announce (and commit to) their choices simultaneously. In the second stage, after knowing when the players choose their effort levels, each player independently chooses her effort level at the point in time which she announced in the first stage. A player who chooses her effort level at point 2 or point 3 observes the effort levels previously chosen. At the end of this stage, the winner is determined.

It is immediate from Remark 2 that, under Assumption 1 and the assumption that $1.5w_1w_2 < w_1 + w_2 \leq (4 - w_1)w_2$ or, equivalently, $2.5w_1 < 1 + w_1 + w_1/w_2 \leq 4$, there exists a subgame-perfect equilibrium in which each of the three players announces *Point 1* in the first stage – that is, no player has an incentive to deviate from *Point* 1 to *Point* 2 or *Point* 3.[18]

Due to computational intractability, we are unable to complete the full analysis. However, on the basis of the results obtained in Section 4 and those described in the following paragraph, we believe that, under Assumption 1 and the assumption that all three players are active in the equilibrium of every subgame starting at the second stage, the subgame-perfect equilibrium in which each of the three players announces *Point 1* in the first stage is the only one.

Assuming that $\sigma_1 = \sigma_2 = 1$, we find the following (see also Lee, 2007). First, there exists no subgame-perfect equilibrium in which every player announces *Point 3*. Second, there exists no subgame-perfect equilibrium in which each player announces a point in time which is different from the other players' choices.

7. Conclusions

We have studied a three-player Tullock contest in an endogenous-timing framework, focusing on the players' decisions on timing of effort exertion. In this model, there are two points in time, 1 and 2, at which the players may choose their effort levels. The players decide independently and announce simultaneously when they each will expend their effort, and then each player chooses her effort level at the point in time which she announced.

Restricting our analysis to cases where all three players are active in the equilibrium of every second-stage subgame, we have shown that, in equilibrium, each of the players announces *Point 1*, and thus they all choose their effort levels simultaneously at point 1. This is in sharp contrast to a well-known result from two-player asymmetric contests studied in an endogenous-timing framework: In equilibrium, the weak player announces *Point 1* and chooses her effort level at point 1 while the strong player announces *Point 2* and chooses her effort level at point 2. We have elaborated on this contrast in Section 5.

In Section 6, we have considered variations of the main model presented in Section 2: a model in which there are more than three players, a model in which a contest organizer decides and enforces when the three players will each expend their effort, and a model in which there are three points in time at which the three players may choose their effort levels.

We have focused on the players' equilibrium decisions on timing of effort exertion. However, it would be interesting to examine how the players' effort levels in the subgame-perfect equilibrium respond when player k's valuation parameter β_k or her ability parameter σ_k or her composite strength $\beta_k\sigma_k$, for $k = 1, 2$, changes, *ceteris paribus* (see Baik, 1994). One could, no doubt, perform these comparative statics using Lemma A.

We have assumed that player i's valuation parameter β_i and her ability parameter σ_i, for $i \in N^+$, are publicly known. It would be interesting to study a model in which the players have incomplete information about these parameters. Perhaps more importantly, it would be interesting to experimentally investigate the main qualitative predictions of the three-player contest – specifically, the equilibrium timing of effort exertion and the equilibrium effort levels of the players. Further, it would be interesting to compare these experimental results with existing experimental studies of two-player asymmetric contests (see, for example, Baik et al., 1999). We leave them for future research.

Footnotes

1. This result can also be stated as follows: The underdog chooses her effort level before the favorite. Here the underdog (the favorite) is defined as the player who has a probability of winning less (greater) than one-half at the Nash equilibrium of a two-player asymmetric contest in which the players choose their effort levels simultaneously (see Dixit, 1987).

This result is empirically supported by Boyd and Boyd (1995), who use data from the track and field events held at the 1992 Summer Olympics in Barcelona, Spain. Also, it is experimentally supported by Baik et al. (1999), who examine subjects' behavior in a laboratory environment in which subjects are given enough time to think about their strategies.

2. The terms *strategic substitute* and *strategic complement* are defined in Bulow et al. (1985).

3. The candidates may choose from and announce one of three campaigning periods before they campaign. This situation can be described by the model in Section 6.4. However, due to computational intractability, we are unable to complete the full analysis.

4. This contest success function is extensively used in the literature on contests. Examples include Tullock (1980), Hillman and Riley (1989), Leininger (1993), Baik et al. (1999), Morgan (2003), Epstein and Nitzan (2007), Konrad (2009), Baik and Lee (2013), Vojnović (2015), and Balart et al. (2016).

5. We can use the following payoff function for player i, for $i \in N$, to obtain the players' equilibrium effort levels in Section 3: $\hat{\pi}_i = w_i z_i / Z - z_i$ for $Z > 0$ and $\hat{\pi}_i = v_i/3$ for $Z = 0$, where $z_i = \sigma_i x_i$ and $Z = z_1 + z_2 + z_3$. Note that the function $\hat{\pi}_i$ is an increasing affine transformation of the function π_i.

6. Unless the three players announce the same point in time in the first stage, one may say that the game has three stages. But we do not break the effort-expending stage into two stages. Instead, we say that the effort-expending stage has two points in time at which the players may choose their effort levels.

7. We will impose severe restrictions on the values of the parameters such that all three players expend positive effort in the equilibria of the second-stage subgames. Given these restrictions, the full game has a unique subgame-perfect equilibrium in which each of the three players announces *Point 1* in the first stage.

8. Note that, in each of the six sequential-move subgames, two players choose their effort levels *simultaneously* either at point 1 or at point 2.

9. We use *iL* as a shorthand for "with player *i* as the leader."

10. We use *jkL* as a shorthand for "with players *j* and *k* as the leaders."

11. Note that player *i*'s payoff function in (2) is strictly concave in x_i, which implies that the second-order condition for maximizing π_i is satisfied and player *i*'s best response to the other players' effort levels is unique. Note also that every maximization problem in this paper satisfies globally its second-order condition and thus has a unique maximum. Henceforth, for brevity, we do not state it explicitly.

12. This implies that the game has a unique subgame-perfect equilibrium even in the case where $w_1 = w_2 = w_3 = 1$ (see footnote 15).

13. In this context, we mean that $x_j^{jkL} > x_j^S$ and $x_k^{jkL} > x_k^S$, for $j, k \in N \setminus \{i\}$ with $j \neq k$. It is straightforward to check that these inequalities hold in the case where $w_1 = w_2 = 1$.

14. Note that, if player *i* announces *Point 1*, one of her options in the effort-expending stage is to choose her effort level at the Nash equilibrium of the simultaneous-move subgame.

15. Part (*iii*) of Lemma D implies that, if $w_h = 1$, then each player announces either *Point 1* or *Point 2* in equilibrium, which means that the game has multiple subgame-perfect equilibria.

16. Consider a three-player contest in which there are three points in time at which the players may choose their effort levels (see Section 6.4). Suppose that the weakest player announces *Point 1*, while each of the remaining players announces *Point 2* or *Point 3*. In this case, too, each of the two followers would be better off by deviating to *Point 1*.

17. The effort expended by the players in contests is of great importance. This is because it accounts for various outcomes of the contests, and, in some cases, it is related to economic performance. In competition for presidential office, the effort expended by the players may represent the candidates' campaign spendings. In rent-seeking contests, it may represent bribes given to government officials, and it is viewed as social costs due to rent-seeking activities. In patent competition, it may represent R&D expenditures of the firms, thereby determining the expected date of invention.

18. Note that Remark 2 still holds true if we consider a model (modified from the main model presented in Section 2) in which there are n players and t points in time, where $n > 3$ and $3 \leq t \leq n$.

Appendix A: A simultaneous-move subgame

Player 1 seeks to maximize π_1 in (2) over her effort level x_1, taking the other players' effort levels as given. From the first-order condition for maximizing π_1, we derive player 1's reaction function:

$$x_1 = \begin{pmatrix} \{\sqrt{w_1(\sigma_2 x_2+x_3)} - (\sigma_2 x_2+x_3)\}/\sigma_1 & \text{for } 0<\sigma_2 x_2+x_3\leq w_1 \\ 0 & \text{for } \sigma_2 x_2+x_3>w_1 \end{pmatrix}.$$

It is straightforward to see that π_1 in (2) is strictly concave in x_1, which implies that the second-order condition for maximizing π_1 is satisfied and player 1's best response to the other players' effort levels is unique.

Similarly, we derive the reaction functions for players 2 and 3:

$$x_2 = \begin{pmatrix} \{\sqrt{w_2(\sigma_1 x_1+x_3)} - (\sigma_1 x_1+x_3)\}/\sigma_2 & \text{for } 0<\sigma_1 x_1+x_3\leq w_2 \\ 0 & \text{for } \sigma_1 x_1+x_3>w_2 \end{pmatrix}$$

and

$$x_3 = \begin{pmatrix} \sqrt{w_3(\sigma_1 x_1+\sigma_2 x_2)} - (\sigma_1 x_1+\sigma_2 x_2) & \text{for } 0<\sigma_1 x_1+\sigma_2 x_2\leq w_3 \\ 0 & \text{for } \sigma_1 x_1+\sigma_2 x_2>w_3 \end{pmatrix}.$$

Using these three reaction functions, we obtain a unique Nash equilibrium. Next, substituting into (2) the players' effort levels at the Nash equilibrium, we obtain their equilibrium expected payoffs.

Lemma A. *Assume that Assumption* 1 *holds and that* $w_1 + w_2 > w_1 w_2$. (a) *The players' (positive) effort levels at a unique Nash equilibrium are:* $x_1^S = 2\beta_1 w_2(w_1 w_2 + w_1 - w_2)/(w_1 w_2 + w_1 + w_2)^2$, $x_2^S = 2\beta_2 w_1(w_1 w_2 + w_2 - w_1)/(w_1 w_2 + w_1 + w_2)^2$, *and* $x_3^S = 2w_1 w_2(w_1 +$

$w_2 - w_1w_2)/(w_1w_2 + w_1 + w_2)^2$. ($b$) *The players' expected payoffs at the Nash equilibrium are:* $\pi_1^S = \beta_1(w_1w_2 + w_1 - w_2)^2/(w_1w_2 + w_1 + w_2)^2$, $\quad \pi_2^S = \beta_2(w_1w_2 + w_2 - w_1)^2/(w_1w_2 + w_1 + w_2)^2$, *and* $\pi_3^S = (w_1 + w_2 - w_1w_2)^2/(w_1w_2 + w_1 + w_2)^2$.

It is immediate from part (a) that, under Assumption 1, players 1 and 2 are always active at the Nash equilibrium, but player 3 is active at the Nash equilibrium if and only if $w_1 + w_2 > w_1w_2$. It is also immediate from part (a) that all the players are active at the Nash equilibrium if players 2 and 3 have the same composite strength: $w_2 = w_3 = 1$.

Appendix B: The *iL* sequential-move subgame

Fix player i, for $i \in N$. In this subgame, player i first chooses her effort level at point 1. Then, after observing player i's effort level, the rest choose their effort levels simultaneously and independently at point 2.

To solve for an equilibrium of the 1L sequential-move subgame, we work backward. At point 2, players 2 and 3 know player 1's effort level x_1. Player 2 seeks to maximize π_2 in (2) over her effort level x_2, taking player 3's effort level as given. From the first-order condition for maximizing π_2, we derive player 2's reaction function:

$$x_2 = \begin{pmatrix} \{\sqrt{w_2(\sigma_1x_1+x_3)} - (\sigma_1x_1+x_3)\}/\sigma_2 & \text{for } 0<\sigma_1x_1+x_3\leq w_2 \\ 0 & \text{for } \sigma_1x_1+x_3>w_2 \end{pmatrix}.$$

Similarly, we derive the reaction function for player 3:

$$x_3 = \begin{pmatrix} \sqrt{w_3(\sigma_1x_1+\sigma_2x_2)} - (\sigma_1x_1+\sigma_2x_2) & \text{for } 0<\sigma_1x_1+\sigma_2x_2\leq w_3 \\ 0 & \text{for } \sigma_1x_1+\sigma_2x_2>w_3 \end{pmatrix}.$$

Using these two reaction functions, we obtain the Nash equilibrium at point 2:

$$x_2^N(x_1) = \{w_2^2 - 2(w_2+1)\sigma_1x_1 + w_2\sqrt{w_2^2 + 4w_2(w_2+1)\sigma_1x_1}\}/2(w_2+1)^2\sigma_2$$

and

$$x_3^N(x_1) \;=\; \{w_2 - 2w_2(w_2+1)\sigma_1 x_1 + \sqrt{w_2^2 + 4w_2(w_2+1)\sigma_1 x_1}\}/2(w_2+1)^2.$$

Next, consider point 1 at which player 1 chooses her effort level. Let $\pi_1(x_1)$ be player 1's expected payoff – computed at point 1 of the subgame – which takes into account the Nash equilibrium at point 2. Substituting $x_2^N(x_1)$ and $x_3^N(x_1)$ into (2), we obtain

$$\pi_1(x_1) \;=\; 2w_1(w_2+1)x_1/\{w_2 + \sqrt{w_2^2 + 4w_2(w_2+1)\sigma_1 x_1}\} \;-\; x_1.$$

At point 1, player 1 has perfect foresight about $\pi_1(x_1)$ for any value of x_1. She chooses a value of x_1 which maximizes $\pi_1(x_1)$. From the first-order condition for maximizing $\pi_1(x_1)$, we obtain player 1's equilibrium effort level x_1^{1L}.

Substituting x_1^{1L} into $x_2^N(x_1)$ and $x_3^N(x_1)$ above, we obtain the equilibrium effort levels of players 2 and 3, x_2^{1L} and x_3^{1L}, respectively. Substituting into (2) the players' equilibrium effort levels, we obtain player h's equilibrium expected payoff π_h^{1L}, for $h \in N$.

Similarly, we obtain the players' equilibrium effort levels and their equilibrium expected payoffs in the 2L and the 3L sequential-move subgame.

Lemma B. *Assume that Assumption* 1 *holds*. (*i*) *Assuming that* $w_1(w_2+1) < w_2+2$, *we obtain the following in a unique equilibrium of the* 1*L sequential-move subgame*. (*a*) *The players' (positive) effort levels are:* $x_1^{1L} = \{w_1^2(w_2+1)^2 - w_2^2\}/4\sigma_1 w_2(w_2+1)$, $x_2^{1L} = \{2w_2^3 + w_2^2 - w_1^2(w_2+1)^2 + 2w_1 w_2^2(w_2+1)\}/4\sigma_2 w_2(w_2+1)^2$, *and* $x_3^{1L} = \{w_2^2 + 2w_2 - w_1^2(w_2+1)^2 + 2w_1 (w_2+1)\}/4(w_2+1)^2$. (*b*) *The players' expected payoffs are:* $\pi_1^{1L} = \{w_1(w_2+1) - w_2\}^2/4\sigma_1 w_2(w_2+1)$, $\pi_2^{1L} = \{2w_2^2 + w_2 - w_1(w_2+1)\}^2/4\sigma_2 w_2(w_2+1)^2$, *and* $\pi_3^{1L} = \{w_2 + 2 - w_1(w_2 + 1)\}^2/4(w_2+1)^2$.

(*ii*) *Assuming that* $w_2(w_1+1) < w_1+2$, *we obtain the following in a unique equilibrium of the* 2*L sequential-move subgame*. (*a*) *The players' (positive) effort levels are:* $x_1^{2L} = \{2w_1^3 + w_1^2 - w_2^2(w_1+1)^2 + 2w_1^2 w_2(w_1+1)\}/4\sigma_1 w_1(w_1+1)^2$, $x_2^{2L} = \{w_2^2(w_1+1)^2 - w_1^2\}/4\sigma_2 w_1(w_1+1)$, *and* $x_3^{2L} = \{w_1^2 + 2w_1 - w_2^2(w_1+1)^2 + 2w_2(w_1+1)\}/4(w_1+1)^2$. (*b*) *The players' expected payoffs are:* $\pi_1^{2L} = \{2w_1^2 + w_1 - w_2(w_1+1)\}^2/4\sigma_1 w_1(w_1+1)^2$, $\pi_2^{2L} = \{w_2(w_1+1) - w_1\}^2/4\sigma_2 w_1(w_1+1)$, *and* $\pi_3^{2L} = \{w_1 + 2 - w_2(w_1+1)\}^2/4(w_1+1)^2$.

(*iii*) *Assuming that* $w_1 + w_2 > w_1 w_2$, *we obtain the following in a unique equilibrium of the* 3*L sequential-move subgame.* (*a*) *The players' (positive) effort levels are:* $x_1^{3L} = \{2w_1^3 w_2 + w_1^2 w_2^2 - (w_1+w_2)^2 + 2w_1^2(w_1+w_2)\}/4\sigma_1 w_1(w_1+w_2)^2$, $x_2^{3L} = \{2w_1 w_2^3 + w_1^2 w_2^2 - (w_1+w_2)^2 + 2w_2^2(w_1+w_2)\}/4\sigma_2 w_2(w_1+w_2)^2$, *and* $x_3^{3L} = \{(w_1+w_2)^2 - w_1^2 w_2^2\}/4w_1 w_2(w_1+w_2)$. (*b*) *The players' expected payoffs are:* $\pi_1^{3L} = \{2w_1^2 + w_1 w_2 - (w_1+w_2)\}^2/4\sigma_1 w_1(w_1+w_2)^2$, $\pi_2^{3L} = \{2w_2^2 + w_1 w_2 - (w_1+w_2)\}^2/4\sigma_2 w_2(w_1+w_2)^2$, *and* $\pi_3^{3L} = (w_1 + w_2 - w_1 w_2)^2/4w_1 w_2(w_1+w_2)$.

Appendix C: The *jkL* sequential-move subgame

Fix players j and k, for $j = 1, 2$ and $k = 2, 3$ with $j \neq k$. In this subgame, players j and k first choose their effort levels simultaneously and independently at point 1. Then, after observing their effort levels, the remaining player chooses her effort level at point 2.

To solve for an equilibrium of the 12*L* sequential-move subgame, we work backward. At point 2, player 3 knows player 1's effort level, x_1, and player 2's effort level, x_2. Player 3 seeks to maximize π_3 in (2) over her effort level x_3. From the first-order condition for maximizing π_3, we derive player 3's strategy in any equilibrium:

$$x_3(x_1, x_2) = \left(\begin{array}{ll} \sqrt{w_3(\sigma_1 x_1 + \sigma_2 x_2)} - (\sigma_1 x_1 + \sigma_2 x_2) & \text{for } 0 < \sigma_1 x_1 + \sigma_2 x_2 \leq w_3 \\ 0 & \text{for } \sigma_1 x_1 + \sigma_2 x_2 > w_3 \end{array} \right). \quad \text{(C1)}$$

Next, consider point 1 at which players 1 and 2 choose their effort levels simultaneously and independently. Let $\pi_j(x_1, x_2)$, for $j = 1, 2$, be player j's expected payoff – computed at point 1 of the subgame – which takes into account player 3's equilibrium strategy in (C1). Substituting $x_3(x_1, x_2)$ in (C1) into (2), we obtain

$$\pi_1(x_1, x_2) \;=\; w_1 x_1 / \sqrt{(\sigma_1 x_1 + \sigma_2 x_2)} \;-\; x_1$$

and (C2)

$$\pi_2(x_1, x_2) \;=\; w_2 x_2 / \sqrt{(\sigma_1 x_1 + \sigma_2 x_2)} \;-\; x_2.$$

At point 1, players 1 and 2 have perfect foresight about $\pi_1(x_1, x_2)$ and $\pi_2(x_1, x_2)$ for any values of x_1 and x_2.

We first derive the reaction function for player 1. Player 1 seeks to maximize $\pi_1(x_1, x_2)$ in (C2) over her effort level x_1, taking player 2's effort level x_2 as given. The first-order condition for maximizing $\pi_1(x_1, x_2)$ reduces to

$$4(\sigma_1 x_1 + \sigma_2 x_2)^3 \;-\; w_1^2(\sigma_1 x_1 + 2\sigma_2 x_2)^2 \;=\; 0. \tag{C3}$$

Solving equation (C3) for x_1, we obtain player 1's best response to player 2's effort level x_2, which is denoted by $x_1(x_2)$. Player 1's reaction function is then $x_1 = x_1(x_2)$, the implicit form of which is given in equation (C3).

Similarly, the implicit form of player 2's reaction function, $x_2 = x_2(x_1)$, is

$$4(\sigma_1 x_1 + \sigma_2 x_2)^3 \;-\; w_2^2(2\sigma_1 x_1 + \sigma_2 x_2)^2 \;=\; 0. \tag{C4}$$

Using the reaction functions for players 1 and 2, or rather equations (C3) and (C4), we obtain the equilibrium effort levels of players 1 and 2, x_1^{12L} and x_2^{12L}, in the $12L$ sequential-move subgame.

Substituting x_1^{12L} and x_2^{12L} into $x_3(x_1, x_2)$ in (C1), we obtain player 3's equilibrium effort level x_3^{12L}. Substituting into (2) the players' equilibrium effort levels, we obtain player h's equilibrium expected payoff π_h^{12L}, for $h \in N$.

Similarly, we obtain the players' equilibrium effort levels and their equilibrium expected payoffs in the $13L$ and the 23L sequential-move subgame.

Lemma C. *Assume that Assumption* 1 *holds*. (*i*) *Assuming that* $w_1 + w_2 > 1.5w_1w_2$, *we obtain the following in a unique equilibrium of the* 12*L sequential-move subgame*. (*a*) *The players' (positive) effort levels are:* $x_1^{12L} = 9w_1^2w_2^2(2w_1 - w_2)/4\sigma_1(w_1 + w_2)^3$, $x_2^{12L} = 9w_1^2w_2^2(2w_2 - w_1)/4\sigma_2(w_1 + w_2)^3$, *and* $x_3^{12L} = 3w_1w_2(2w_1 + 2w_2 - 3w_1w_2)/4(w_1 + w_2)^2$. (*b*) *The players' expected payoffs are:* $\pi_1^{12L} = 3w_1^2w_2(2w_1 - w_2)^2/4\sigma_1(w_1 + w_2)^3$, $\pi_2^{12L} = 3w_1w_2^2(2w_2 - w_1)^2/4\sigma_2(w_1 + w_2)^3$, *and* $\pi_3^{12L} = (2w_1 + 2w_2 - 3w_1w_2)^2/4(w_1 + w_2)^2$.

(*ii*) *Assuming that* $w_1 < 2$, *we obtain the following in a unique equilibrium of the* 13*L sequential-move subgame*. (*a*) *The players' (positive) effort levels are:* $x_1^{13L} = 9w_1^2(2w_1 - 1)/4\sigma_1w_2(w_1 + 1)^3$, $x_2^{13L} = 3w_1(2w_1w_2 + 2w_2 - 3w_1)/4\sigma_2w_2(w_1 + 1)^2$, *and* $x_3^{13L} = 9w_1^2(2 - w_1)/4w_2(w_1 + 1)^3$. (*b*) *The players' expected payoffs are:* $\pi_1^{13L} = 3w_1^2(2w_1 - 1)^2/4\sigma_1w_2(w_1 + 1)^3$, $\pi_2^{13L} = (2w_1w_2 + 2w_2 - 3w_1)^2/4\sigma_2w_2(w_1 + 1)^2$, *and* $\pi_3^{13L} = 3w_1(2 - w_1)^2/ 4w_2(w_1 + 1)^3$.

(*iii*) *Assuming that* $w_2 < 2$, *we obtain the following in a unique equilibrium of the* 23*L sequential-move subgame*. (*a*) *The players' (positive) effort levels are:* $x_1^{23L} = 3w_2(2w_1w_2 + 2w_1 - 3w_2)/4\sigma_1w_1(w_2 + 1)^2$, $x_2^{23L} = 9w_2^2(2w_2 - 1)/4\sigma_2w_1(w_2 + 1)^3$, *and* $x_3^{23L} = 9w_2^2(2 - w_2)/4w_1(w_2 + 1)^3$. (*b*) *The players' expected payoffs are:* $\pi_1^{23L} = (2w_1w_2 + 2w_1 - 3w_2)^2/4\sigma_1w_1(w_2 + 1)^2$, $\pi_2^{23L} = 3w_2^2(2w_2 - 1)^2/4\sigma_2w_1(w_2 + 1)^3$, *and* $\pi_3^{23L} = 3w_2(2 - w_2)^2/4w_1(w_2 + 1)^3$.

Appendix D: Endogenous timing in a two-player contest

In this appendix, we consider endogenous timing of effort exertion in a two-player contest in which player *h*, for $h = 1$ or 2, and player 3 compete to win a prize. Specifically, we consider the following game. In the first stage, each player chooses independently between *Point 1* and *Point 2*. The players announce (and commit to) their choices simultaneously. In the second stage, after knowing when the players choose their effort levels, each player independently chooses her effort level at the point in time which she announced in the first stage. A player who chooses her effort level at point 2 observes the effort levels chosen at point 1 before she does. At the end of this stage, the winner is determined.

Let y_j, for $j = h$, 3, denote player j's effort level. We assume the following contest success function for player j:

$$p_j = \begin{pmatrix} \sigma_j y_j/(\sigma_h y_h + y_3) & \text{for } y_h + y_3 > 0 \\ 1/2 & \text{for } y_h + y_3 = 0 \end{pmatrix}.$$

Let ψ_j denote the expected payoff for player j. Then the payoff function for player j is given by

$$\psi_j = v_j p_j - y_j = \begin{pmatrix} w_j y_j/(\sigma_h y_h + y_3) - y_j & \text{for } y_h + y_3 > 0 \\ v_j/2 & \text{for } y_h + y_3 = 0 \end{pmatrix}.$$

To obtain a subgame-perfect equilibrium of the game, we work backward. We first analyze the subgames starting at the second stage, and then consider the players' first-stage decisions on timing of effort exertion.

There are four subgames starting at the second stage, but we need to analyze the following three subgames: a simultaneous-move subgame, the hL sequential-move subgame, and the $3L$ sequential-move subgame (see footnote 9). Conducting an analysis similar to the one in Section 3, we obtain the players' equilibrium expected payoffs in the three subgames. Let ψ_j^S represent player j's equilibrium expected payoff in a simultaneous-move subgame. Let ψ_j^{hL} represent player j's equilibrium expected payoff in the hL sequential-move subgame, and let ψ_j^{3L} represent her equilibrium expected payoff in the $3L$ sequential-move subgame. Lemma D below reports these expected payoffs, and compares them. Note that, in addition to Assumption 1, we further assume that $w_h < 2$, which makes both players active in the equilibrium of every second-stage subgame.

Lemma D. (i) *Under Assumption* 1 *and the assumption that* $1 \leq w_h < 2$, *we obtain:* $\psi_h^S = \beta_h w_h^2/(w_h + 1)^2$, $\psi_3^S = 1/(w_h + 1)^2$, $\psi_h^{hL} = \beta_h w_h/4$, $\psi_3^{hL} = (2 - w_h)^2/4$, $\psi_h^{3L} = (2w_h - 1)^2/4\sigma_h w_h$, *and* $\psi_3^{3L} = 1/4w_h$. (ii) *If* $1 < w_h < 2$, *then we obtain that* $\psi_3^S > \psi_3^{hL}$, $\psi_3^{3L} > \psi_3^S$,

$\psi_h^{3L} > \psi_h^S$, *and* $\psi_h^{hL} > \psi_h^S$. (*iii*) *If* $w_h = 1$, *then we obtain that* $\psi_3^S = \psi_3^{hL} = \psi_3^{3L}$ *and* $\psi_h^S = \psi_h^{hL} = \psi_h^{3L}$.

Now we consider the players' first-stage decisions. We restrict our analysis to cases where $1 < w_h < 2$. Part (*ii*) of Lemma D implies that, in the asymmetric contest, *Point 1* leads to a higher second-stage equilibrium expected payoff for player 3 than does *Point 2*, no matter what player *h* announces in the first stage. This in turn implies that player 3 announces *Point 1* in the first stage. Next, it is immediate from part (*ii*) that player *h* announces *Point 2* in the first stage, forming her belief that player 3 announces *Point 1*.

Appendix E: Endogenous timing in an *n*-player contest

In this appendix, we consider endogenous timing of effort exertion in an *n*-player contest, where $n > 3$. Specifically, we consider the following game. In the first stage, the players each choose independently between *Point 1* and *Point 2*, and announce their choices simultaneously. In the second stage, after knowing when the players choose their effort levels, each player independently chooses her effort level at the point in time which she announced in the first stage. A player who chooses her effort level at point 2 observes the effort levels chosen at point 1 before she does.

To obtain a subgame-perfect equilibrium of the game, we work backward. We first analyze the proper subgames that start at the second stage, and then look at the players' first-stage decisions on timing of effort exertion.

Due to computational intractability, we aim to merely find a subgame-perfect equilibrium, without checking its possible *uniqueness*, in which each of the *n* players announces *Point 1*. Accordingly, we need to analyze only $n+1$ subgames among the 2^n ones (starting at the second stage): a simultaneous-move subgame and the $(-i)L$ sequential-move subgame, for each $i \in N^+$, where $(-i)L$ is used as a shorthand for "with all players except player *i* as the leaders."

A simultaneous-move subgame

Player i, for $i \in N^+$, seeks to maximize π_i over her effort level x_i, taking the other players' effort levels as given, where $\pi_i = w_i x_i / X - x_i$. From the first-order condition for maximizing π_i, we derive player i's reaction function:

$$x_i = \begin{pmatrix} \left(\sqrt{w_i \sum_{z \neq i} \sigma_z x_z} - \sum_{z \neq i} \sigma_z x_z\right)/\sigma_i & \text{for } 0 < \sum_{z \neq i} \sigma_z x_z \leq w_i \\ 0 & \text{for } \sum_{z \neq i} \sigma_z x_z > w_i \end{pmatrix}.$$

Let x_i^S, for $i \in N^+$, represent player i's effort level at the Nash equilibrium, and let $X^S = \sum_{z=1}^{n} \sigma_z x_z^S$. Then they satisfy player i's reaction function, so that we have

$$(X^S)^2 / w_i = X^S - \sigma_i x_i^S \quad \text{for each } i \in N^+. \tag{E1}$$

Adding these equations together, we have

$$\{\sum_{z=1}^{n} (1/w_z)\}(X^S)^2 = nX^S - X^S.$$

This yields

$$X^S = (n-1) / \sum_{z=1}^{n} (1/w_z).$$

Substituting this expression for X^S into (E1), we obtain the equilibrium effort levels of the n active players:

$$x_i^S = (n-1)\{w_i \sum_{z=1}^{n} (1/w_z) - n + 1\} / \sigma_i w_i \{\sum_{z=1}^{n} (1/w_z)\}^2 \quad \text{for } i \in N^+. \tag{E2}$$

Next, substituting the players' equilibrium effort levels in (E2) into their payoff functions, we obtain their expected payoffs at the Nash equilibrium:

$$\pi_i^S \;=\; \{w_i\sum_{z=1}^{n}(1/w_z) - n + 1\}^2/\sigma_i w_i\{\sum_{z=1}^{n}(1/w_z)\}^2 \quad \text{for } i \in N^+. \tag{E3}$$

The $(-i)L$ *sequential-move subgame*

Fix player i, for $i \in N^+$. In this subgame, all the players except player i first choose their effort levels simultaneously and independently at point 1, and then after observing their effort levels, player i chooses her effort level at point 2. To solve for an equilibrium of this subgame, we work backward.

At point 2, knowing the list $\boldsymbol{x}_{-i}$ of the other players' effort levels, player i seeks to maximize π_i over her effort level x_i. From the first-order condition for maximizing π_i, we derive player i's strategy in any equilibrium:

$$x_i(\boldsymbol{x}_{-i}) \;=\; \left(\begin{array}{ll} (\sqrt{w_i\sum_{z\neq i}\sigma_z x_z} - \sum_{z\neq i}\sigma_z x_z)/\sigma_i & \text{for } 0<\sum_{z\neq i}\sigma_z x_z \leq w_i \\ 0 & \text{for } \sum_{z\neq i}\sigma_z x_z > w_i \end{array} \right). \tag{E4}$$

Next, consider point 1 at which the $n-1$ leaders choose their effort levels simultaneously and independently. Let $\pi_j(\boldsymbol{x}_{-i})$, for $j \in N^+ \setminus \{i\}$, be player j's expected payoff – computed at point 1 of the subgame – which takes into account player i's equilibrium strategy in (E4). Substituting $x_i(\boldsymbol{x}_{-i})$ in (E4) into player j's payoff function, we obtain

$$\pi_j(\boldsymbol{x}_{-i}) \;=\; w_j x_j/\sqrt{w_i\sum_{z\neq i}\sigma_z x_z} \;-\; x_j.$$

At point 1, player j, for $j \in N^+ \setminus \{i\}$, seeks to maximize $\pi_j(\boldsymbol{x}_{-i})$ over her effort level x_j, taking the other leaders' effort levels as given. The first-order condition for maximizing $\pi_j(\boldsymbol{x}_{-i})$ reduces to

$$w_j^2(2Q - \sigma_j x_j)^2 \;=\; 4w_i Q^3, \tag{E5}$$

where $Q \equiv \sum_{z \neq i} \sigma_z x_z$.

The leaders' equilibrium effort levels satisfy the $n-1$ first-order conditions in (E5) simultaneously. In order to obtain these $n-1$ equilibrium effort levels, we take the following four steps.

Step 1. Using (E5), we have for any h, $k \in N^+ \setminus \{i\}$: $w_h^2(2Q - \sigma_h x_h)^2 = 4w_i Q^3$ and $w_k^2(2Q - \sigma_k x_k)^2 = 4w_i Q^3$. These two equations yield

$$w_h(2Q - \sigma_h x_h) \;=\; w_k(2Q - \sigma_k x_k). \tag{E6}$$

Step 2. Fix player k, for $k \in N^+ \setminus \{i\}$. Using (E6), we have for each $h \in N^+ \setminus \{i\}$: $(2Q - \sigma_h x_h) = w_k(2Q - \sigma_k x_k)\,/w_h$. Then, adding these $n-1$ equations together, we obtain

$$(2n-3)Q \;=\; \{\sum_{z \neq i}(1/w_z)\} w_k(2Q - \sigma_k x_k). \tag{E7}$$

Step 3. Using (E7), we have for $j \in N^+ \setminus \{i\}$: $(2n-3)Q = \{\sum_{z \neq i}(1/w_z)\} w_j(2Q - \sigma_j x_j)$. Squaring both sides of this equation and rearranging the terms, we obtain

$$w_j^2(2Q - \sigma_j x_j)^2 \;=\; (2n-3)^2 Q^2/\{\sum_{z \neq i}(1/w_z)\}^2.$$

Next, substituting this equation into (E5), we obtain

$$(2n-3)^2 Q^2/\{\sum_{z \neq i}(1/w_z)\}^2 \;=\; 4w_i Q^3,$$

which yields

$$Q \;=\; (2n-3)^2/4w_i\{\sum_{z \neq i}(1/w_z)\}^2. \tag{E8}$$

Step 4. Let $x_j^{(-i)L}$, for $j \in N^+ \setminus \{i\}$, represent player *j*'s effort level in the equilibrium. Substituting (E8) into (E5) and doing some algebra, we obtain

$$x_j^{(-i)L} \;=\; (2n-3)^2\{2w_j\sum_{z\neq i}(1/w_z) - 2n + 3\}/4\sigma_j w_i w_j\{\sum_{z\neq i}(1/w_z)\}^3. \tag{E9}$$

Next, substituting (E9) into $x_i(\boldsymbol{x}_{-i})$ in (E4), we obtain player *i*'s effort level $x_i^{(-i)L}$ in the equilibrium:

$$x_i^{(-i)L} \;=\; (2n-3)\{2w_i\sum_{z\neq i}(1/w_z) - 2n + 3\}/4\sigma_i w_i\{\sum_{z\neq i}(1/w_z)\}^2. \tag{E10}$$

Finally, substituting the players' equilibrium effort levels in (E9) and (E10) into player *i*'s payoff function, we obtain player *i*'s expected payoff $\pi_i^{(-i)L}$ in the equilibrium:

$$\pi_i^{(-i)L} \;=\; \{2w_i\sum_{z\neq i}(1/w_z) - 2n + 3\}^2/4\sigma_i w_i\{\sum_{z\neq i}(1/w_z)\}^2. \tag{E11}$$

The active-players-in-equilibrium conditions

Using (E2), it is straightforward to check that, under Assumption 3, all the players are active at the Nash equilibrium of a simultaneous-move subgame if and only if the following condition is satisfied:

$$w_n\sum_{z=1}^{n}(1/w_z) - n + 1 \;>\; 0. \tag{E12}$$

Next, using (E9) and (E10), it is straightforward to check that, under Assumption 3, all the players are active in the equilibrium of the $(-i)L$ sequential-move subgame, for a fixed $i \in N^+$, if and only if the following condition is satisfied:

$$2w_n\sum_{z\neq i}(1/w_z) - 2n + 3 \;>\; 0.$$

This indicates that, under Assumption 3, all the players are active in the equilibrium of the $(-i)L$ sequential-move subgame, for all $i \in N^+$, if and only if the following condition is satisfied:

$$2w_n\sum_{z\neq n}(1/w_z) - 2n + 3 \ > \ 0. \tag{E13}$$

Now, note that satisfying (E13) implies satisfying (E12). (Recall from Assumption 3 that $w_n = 1$.) We can rewrite (E13) as

$$2\sum_{z=1}^{n}(1/w_z) - 2n + 1 \ > \ 0$$

or, equivalently,

$$\sum_{z=1}^{n}(1/w_z) \ > \ n - 0.5.$$

Therefore, under Assumption 3, all the players are active in the equilibrium of each of those $n+1$ subgames if and only if

$$\sum_{z=1}^{n}(1/w_z) \ > \ n - 0.5. \tag{E14}$$

Players' first-stage decisions

We look at the players' decisions, in the first stage, on when to expend their effort. As mentioned above, we aim to merely find a subgame-perfect equilibrium in which each of the n players announces *Point 1*. To this end, it suffices to show that π_i^S in (E3) $\geq \pi_i^{(-i)L}$ in (E11), for all $i \in N^+$. Indeed, in such a case, a subgame-perfect equilibrium occurs in which no player has an incentive to deviate from her first-stage action, *Point 1*.

Using (E3) and (E11), we obtain under Assumption 3 that $\pi_i^S \geq \pi_i^{(-i)L}$ for a fixed $i \in N^+$ if the following condition is satisfied:

$$w_i\sum_{z=1}^{n}(1/w_z) \ - \ 2(n-1) \ \leq \ 0.$$

Accordingly, we obtain under Assumption 3 that $\pi_i^S \geq \pi_i^{(-i)L}$ for all $i \in N^+$ if the following condition is satisfied:

$$w_1 \sum_{z=1}^{n} (1/w_z) \; - \; 2(n-1) \; \leq \; 0$$

or, equivalently,

$$\sum_{z=1}^{n} (1/w_z) \; \leq \; 2(n-1)/w_1. \tag{E15}$$

We now end this appendix by stating the result we have obtained: If we assume Assumption 3, (E14) and (E15), then a subgame-perfect equilibrium occurs in which each of the n players announces *Point 1*. Note that (E14) and (E15) can be combined and rewritten as

$$n - 0.5 \; < \; \sum_{z=1}^{n} (1/w_z) \; \leq \; 2(n-1)/w_1.$$

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TABLE 1

Comparison of the Total Effort Levels Obtained under Different Timing Assumptions

When $\beta_1\sigma_1 = \beta_2\sigma_2 = 1$

	Highest Effort Level	Lowest Effort Level
$\sigma_1 = \sigma_2 > \sigma_3$	X^{3L}	$X^{1L} = X^{2L} = X^{12L}$ if $\sigma_1 \geq 17/5$
		X^S if $1 < \sigma_1 \leq 17/5$
$\sigma_1 = \sigma_2 = \sigma_3$	$X^{1L} = X^{2L} = X^{3L}$	X^S
	$= X^{12L} = X^{13L} = X^{23L}$	
$\sigma_i > \sigma_j > \sigma_3$	X^{3L}	X^{iL} if $G \leq 0$
		X^S if $G \geq 0$
$\sigma_i > \sigma_j = \sigma_3$	$X^{3L} = X^{jL} = X^{j3L}$	X^{iL} if $\sigma_i \geq 2.2$
		X^S if $1 < \sigma_i \leq 2.2$
$\sigma_i > \sigma_3 > \sigma_j$	X^{jL}	X^{iL} if $G \leq 0$
		X^S if $G \geq 0$
$\sigma_3 > \sigma_1 = \sigma_2$	$X^{1L} = X^{2L} = X^{12L}$	X^{3L} if $0 < \sigma_1 \leq 5/11$
		X^S if $5/11 \leq \sigma_1 < 1$
$\sigma_3 > \sigma_i > \sigma_j$	X^{jL}	X^{3L} if $H \leq 0$
		X^S if $H \geq 0$
$\sigma_3 = \sigma_i > \sigma_j$	X^{jL}	$X^{3L} = X^{iL} = X^{i3L}$ if $0 < \sigma_j \leq 5/17$
		X^S if $5/17 \leq \sigma_j < 1$

Notes: Let $i, j = 1, 2$ with $i \neq j$. Let $G \equiv 22\sigma_j - 5\sigma_i - 5\sigma_1\sigma_2$, and let $H \equiv -5\sigma_2 - 5\sigma_1 + 22\sigma_1\sigma_2$. Recall that $\sigma_3 = 1$.

Panel A. The players' expected payoffs with player 3 announcing *Point 1*

		Player 2	
		Point 1	*Point 2*
Player 1	*Point 1*	$\pi_1^S, \pi_2^S, \pi_3^S$	$\pi_1^{13L}, \pi_2^{13L}, \pi_3^{13L}$
	Point 2	$\pi_1^{23L}, \pi_2^{23L}, \pi_3^{23L}$	$\pi_1^{3L}, \pi_2^{3L}, \pi_3^{3L}$

Panel B. The players' expected payoffs with player 3 announcing *Point 2*

		Player 2	
		Point 1	*Point 2*
Player 1	*Point 1*	$\pi_1^{12L}, \pi_2^{12L}, \pi_3^{12L}$	$\pi_1^{1L}, \pi_2^{1L}, \pi_3^{1L}$
	Point 2	$\pi_1^{2L}, \pi_2^{2L}, \pi_3^{2L}$	$\pi_1^S, \pi_2^S, \pi_3^S$

Figure 1. The Strategic Interaction among the Players in the First Stage

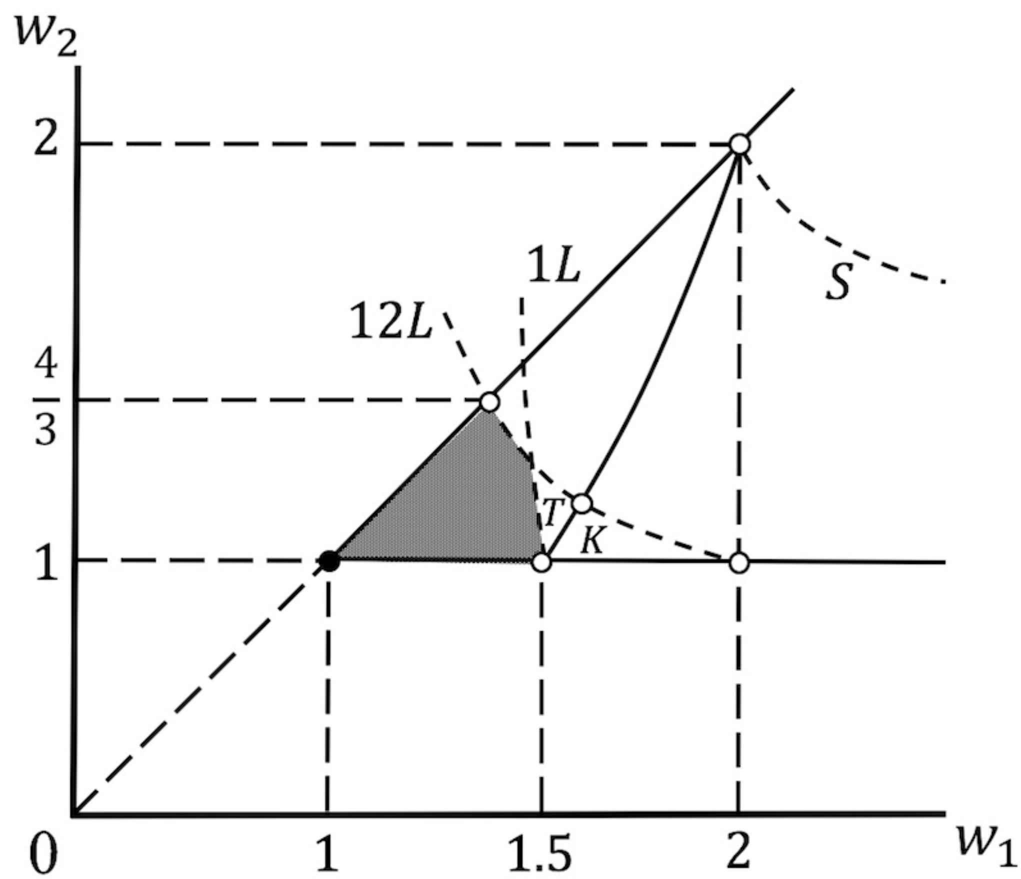

Figure 2. The Values of the Parameters
at Which All Three Players Are Always Active in Equilibrium