Initial Winning Probabilities and Endogenous Timing of Effort Exertion in Contests with Delegation

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Abstract

We study two-player contests with delegation in which each player begins with an initial probability of winning a prize, her final probability of winning depends on both this initial probability and the delegates' effort levels, and the delegates endogenously determine when to exert effort. We show, among other things, that if player 1 has a higher initial winning probability than player 2, then player 1 offers a less generous (or lower) contract to delegate 1 than player 2 offers to delegate 2, delegate 1 exerts effort earlier than delegate 2, and delegate 1 exerts less effort than delegate 2. We also show that, despite the two delegates having equal ability in the contest, delegate 1 earns a higher economic rent than delegate 2. Finally, we show that as player 1's initial winning probability rises from one half, both players offer less generous (or lower) contracts, total effort decreases, player 1's expected payoff increases, delegate 1's expected payoff may either increase or decrease, and the expected payoffs of both player 2 and delegate 2 decrease.

Keywords: Contest with delegation; Initial winning probability; Endogenous timing of effort exertion; Delegation contract; Effort level; Economic rent

JEL classification: D82, D72, C72, K41

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1. Introduction

Contests involving delegation – where contestants compete for a prize and hire delegates to exert effort on their behalf – are common.[1] A prominent example is litigation, where plaintiffs and defendants compete to win a lawsuit and hire attorneys to represent their interests. Another example is rent-seeking contests, in which firms vie for a government-granted monopoly and hire lobbyists to advocate on their behalf. Yet another example is professional-athlete scouting contests, where athletes compete to be recruited by a professional team and hire sports agents to represent them.

An important observation from such contests is that contestants often begin with initial winning probabilities. In litigation, for instance, litigants typically begin with winning probabilities based on the objective merits of the case. In rent-seeking contests, firms often begin with winning probabilities shaped by their relationships with government officials – relationships influenced by prior rent-seeking efforts, educational backgrounds, and personal connections. In professional-athlete scouting contests, athletes start with winning probabilities determined by their talents and skills, developed through prior training and performance.

Another observation is that delegates often endogenously determine the timing of effort exertion before exerting effort. This occurs because delegates may recognize that, given delegation contracts, they can increase their expected payoffs relative to simultaneous effort exertion by strategically choosing when to exert effort.[2] For example, in litigation, attorneys – acting in their own self-interest – may choose when to exert effort and communicate their planned timing to one another. In rent-seeking contests, lobbyists similarly decide when to exert effort. In professional-athlete scouting contests, agents choose when to act on behalf of their clients based on their own strategic objectives.

Motivated by these observations, this paper studies contests with delegation in which each contestant begins with an initial probability of winning a prize, her final probability of winning depends on both this initial probability and the effort levels exerted by the delegates, and the delegates endogenously determine the timing of effort exertion. Specifically, the paper

examines how initial winning probabilities and endogenous timing shape delegation contracts, the timing of effort exertion, delegates' effort levels, and the resulting payoffs to both contestants and delegates.

Specifically, this paper considers the following two-player contest with delegation. At the beginning of the contest, both players know their initial probabilities of winning the prize and understand that their final probabilities of winning depend on both these initial probabilities and the effort levels exerted by the two delegates. Each player then hires a delegate and offers a delegation contract without observing the contract offered by the other player. The players simultaneously announce their contracts publicly. Next, each delegate independently chooses whether to exert effort in period 1 or period 2, and the delegates simultaneously announce their timing decisions. Each delegate then chooses an effort level in the period he previously selected. Finally, the prize is awarded to one of the players.

This paper presents the following results for the case in which player 1 begins with a higher initial winning probability than player 2. Facing this initial disadvantage, player 2 offers delegate 2 a more generous (or greater) contract, thereby providing stronger incentives than those offered by player 1 to delegate 1. As a result, delegate 2 exerts effort after delegate 1 and ultimately exerts a greater level of effort. Despite delegate 2's greater effort, player 1's higher initial winning probability enables her to attain a higher *equilibrium* winning probability than player 2. Combined with the less generous (or lower) contract offered to delegate 1, this allows player 1 to secure a higher expected payoff than player 2. Delegate 1 also earns a higher expected payoff than delegate 2 because he exerts less effort while benefiting from a greater probability of winning. Consequently, although the two delegates possess equal ability in the contest, delegate 1 earns a larger economic rent than delegate 2.

This paper also presents several comparative statics results. As player 1's initial winning probability increases from one half, both players offer less generous (or lower) equilibrium contracts, the equilibrium total effort level decreases, and player 1's equilibrium expected payoff

rises. Delegate 1's equilibrium expected payoff, however, may either increase or decrease. In contrast, the equilibrium expected payoffs of both player 2 and delegate 2 decline.

Finally, comparison with a corresponding contest in which the winner is determined solely by the delegates' effort levels shows that incorporating the players' initial winning probabilities leads to less generous (or lower) equilibrium contracts and a lower equilibrium total effort level. Moreover, the player with the higher initial winning probability earns a greater expected payoff than in the contest that ignores these probabilities.

Contests involving delegation have been studied by many economists. Baik and Kim (1997), Wärneryd (2000), Schoonbeek (2002, 2004, 2007, 2017), Lim and Shogren (2004), and Kang (2024) consider two-party contests with unilateral or bilateral delegation, focusing on endogenous delegation decisions, equilibrium delegation contracts, information sharing, reimbursement, or related issues. In addition, Baik and Lee (2013) investigate the endogenous timing of effort exertion in contests with delegation, while Kang (2023) examines how players' initial winning probabilities affect equilibrium contracts and effort levels. However, to the best of our knowledge, no previous studies have examined contests with delegation in which players begin with initial winning probabilities and delegates endogenously choose the timing of effort exertion.

Several studies have explored contests in which players' initial winning probabilities (or initial claims) affect the determination of the winner or the contest outcome. For example, Farmer and Pecorino (1999), Hirshleifer and Osborne (2001), and Baharad et al. (2022) study litigation contests in which outcomes depend on both players' initial winning probabilities, based on the objective merits of the case, and their effort levels. Ansink (2011) examines contests in which players' final shares of a contested resource are jointly determined by their initial claims and effort levels. Faith et al. (2008) analyze contests in which children's final bequests and gifts are jointly determined by their initial bequest shares, chosen and committed by the parents, and the children's effort levels. Baik and Jung (2019) study contests in which players' final probabilities of winning a prize depend on both their initial winning probabilities and effort

levels. Baik and Park (2022) investigate contests in which fishing companies' fractional catch shares are jointly determined by their initial catch shares and effort levels.

This paper is also related to another strand of literature that examines the endogenous timing of effort exertion in contests. Examples include Baik and Shogren (1992), Leininger (1993), Nitzan (1994), Morgan (2003), Fu (2006), Konrad and Leininger (2007), Hoffmann and Rota-Graziosi (2012, 2020), Baik (2013), and Baik and Lee (2024). However, these studies analyze the endogenous timing of effort exertion in contests without delegation or initial winning probabilities.

The remainder of the paper is organized as follows. Section 2 develops the model and describes the three-stage game. Section 3 solves for the subgame-perfect equilibria of the game and obtain its outcomes. Section 4 conducts a comparative statics analysis of these outcomes with respect to each parameter. In Section 5, we compare these outcomes with those of the corresponding contest in which the winner is determined solely by the delegates' effort levels. Finally, Section 6 concludes the paper.

2. The model

Consider a contest with two players, 1 and 2, who vie for a prize. Each player hires a delegate to exert irreversible effort on behalf of his employer to win the prize. Each player begins with an initial probability of winning the prize, and her final probability of winning depends on both this initial probability and the effort levels exerted by the two delegates. The delegates, 1 and 2, personally bear the costs associated with their own effort. There are two periods, 1 and 2, in which the delegates may exert effort. Each delegate can choose to exert effort in exactly one of these periods, but not in both.

The strategic interaction between the players and the delegates occurs as follows. First, the players hire delegates and simultaneously announce their contracts publicly. When designing these contracts, each player anticipates that the delegates will endogenously decide in which period to exert their effort – that is, which delegate exerts effort in which period. Next, the

delegates independently decide and simultaneously announce whether to exert their effort in period 1 or in period 2. Finally, each delegate independently chooses his effort level in the period he has selected.

Let $v > 0$ represent the players' valuation for the prize. This valuation is publicly known at the start of the game. A contract between player i and delegate i, for $i = 1, 2$, takes the following form: player i pays a compensation of $\alpha_i v$ to delegate i if she wins the prize, and zero if she loses it, where $0 < \alpha_i < 1$. We assume that the delegates are risk-neutral. Each delegate accepts the contract for him if it yields an equilibrium expected payoff greater than or equal to his reservation wage. Furthermore, we assume that each delegate has a reservation wage of zero.

Let θ_i represent player i's initial probability of winning the prize, where $\theta_1 \geq \theta_2 > 0$ and $\theta_1 + \theta_2 = 1$. We assume that the players' initial winning probabilities are publicly known.

Let x_i, for $i = 1, 2$, represent delegate i's effort level, and let X represent the total effort level expended by both delegates, where $x_i \geq 0$. Delegate i's effort level is measured in units commensurate with the prize, and may or may not be observable to other decision makers, including player i.

Let p_i represent the final probability that player i (or delegate i) wins the prize. We assume that p_i depends on player i's initial winning probability and the delegates' effort levels. Specifically, we assume the following contest success function for player i (or delegate i):

$$p_i = \lambda\theta_i + (1 - \lambda)f_i(x_1, x_2), \tag{1}$$

where $0 < \lambda < 1$ and

$$f_i(x_1, x_2) = \begin{cases} x_i/X & \text{for } X > 0 \\ 1/2 & \text{for } X = 0. \end{cases}$$

Contest success function (1) implies that, *ceteris paribus*, p_i is increasing in player i's initial winning probability θ_i. It also implies that, *ceteris paribus*, p_i is increasing in x_i at a

decreasing rate, but decreasing in x_j at a decreasing rate, for $i, j = 1, 2$ with $i \neq j$. We assume that the contest success function for player i (or delegate i) is publicly known.

A natural interpretation of function (1) is that p_i is determined by a weighted average of player i's initial winning probability and the value of $f_i(x_1, x_2)$, where the weights are given by λ and $(1-\lambda)$. We assume that the impact parameter λ is exogenously fixed and publicly known. An alternative interpretation of function (1) is stochastic: with probability λ, p_i is determined solely by player i's initial winning probability, and with probability $(1-\lambda)$, it is determined solely by the effort-based function $f_i(x_1, x_2)$.

We formally consider the following game. In the first stage, each player hires a delegate and writes a contract with him – concisely, player i chooses a contract α_i – without observing the contract chosen by the other player. The players simultaneously announce their contracts publicly. In the second stage, after observing both contracts, α_1 and α_2, delegates 1 and 2 independently decide when to expend effort – that is, they choose between *Period 1* and *Period 2*. They announce their choices simultaneously. In the third (or effort-expending) stage, after observing both the contracts and the period chosen by the rival delegate, each delegate independently chooses his effort level in the period he previously announced. At the end of the game, the winning player is determined.

We assume that any delegate who chooses his effort level in period 2 observes, prior to doing so, the effort level, if any, chosen by the rival delegate in period 1.

We assume that the players are risk-neutral. Let ψ_i, for $i = 1, 2$, represent the expected payoff for player i. Then the payoff function for player i is

$$\psi_i = (1 - \alpha_i)v\{\lambda\theta_i + (1 - \lambda)f_i(x_1, x_2)\}. \tag{2}$$

Let π_i, for $i = 1, 2$, represent the expected payoff for delegate i. Then the payoff function for delegate i is

$$\pi_i = \alpha_i v\{\lambda\theta_i + (1 - \lambda)f_i(x_1, x_2)\} - x_i. \tag{3}$$

We assume that all of the above is common knowledge among the players and delegates. We employ subgame-perfect equilibrium as the solution concept.

3. Equilibrium delegation contracts

To obtain a subgame-perfect equilibrium of the game, we proceed by backward induction. We first analyze the delegates' decisions in the third (or effort-expending) stage, then their decisions in the second stage, and finally the players' decisions in the first stage.

3.1. The delegates' decisions on effort levels

At the beginning of the third (effort-expending) stage, each delegate knows the contracts, α_1 and α_2, chosen by both parties in the first stage, and also knows the timing of when the delegates will choose their effort levels.

There are four subgames that start at the third stage. If both delegates choose and announce *Period 1*, a simultaneous-move subgame arises. If delegate 1 chooses *Period 1* and delegate 2 chooses *Period 2*, the 1*L* sequential-move subgame (with delegate 1 as the leader) arises. If delegate 1 chooses *Period 2* and delegate 2 chooses *Period 1*, the 2*L* sequential-move subgame arises. If both delegates choose *Period 2*, a simultaneous-move subgame arises.

We analyze these subgames to obtain the delegates' equilibrium effort levels, their equilibrium expected payoffs, and the players' equilibrium expected payoffs, in each of the subgames.

3.1.1. The simultaneous-move subgame

In this subgame, the delegates choose their effort levels simultaneously. To obtain a Nash equilibrium of the subgame, we begin by deriving the delegates' reaction functions.

Delegate 1 seeks to maximize his expected payoff π_1 in (3) over his effort level x_1, taking delegate 2's effort level x_2 as given. From the first-order condition for maximizing π_1, we derive delegate 1's reaction function:

$$x_1 = \begin{cases} \sqrt{\alpha_1 v(1-\lambda)x_2} - x_2 & \text{for } 0 < x_2 \leq \alpha_1 v(1-\lambda) \\ 0 & \text{for } x_2 > \alpha_1 v(1-\lambda). \end{cases}$$

It is straightforward to check that the second-order condition for maximizing π_1 is satisfied.[3]

Similarly, we obtain delegate 2's reaction function:

$$x_2 = \begin{cases} \sqrt{\alpha_2 v(1-\lambda)x_1} - x_1 & \text{for } 0 < x_1 \leq \alpha_2 v(1-\lambda) \\ 0 & \text{for } x_1 > \alpha_2 v(1-\lambda). \end{cases}$$

Next, using the two reaction functions above, we obtain the Nash equilibrium of the simultaneous-move subgame: $x_1^S = \alpha_1^2 \alpha_2 v(1-\lambda)/(\alpha_1+\alpha_2)^2$ and $x_2^S = \alpha_1 \alpha_2^2 v(1-\lambda)/(\alpha_1+\alpha_2)^2$.

Finally, we obtain delegate i's expected payoff π_i^S, for $i = 1, 2$, and player i's expected payoff ψ_i^S at the Nash equilibrium by substituting the delegates' equilibrium effort levels, x_1^S and x_2^S, into (3) and (2), respectively.

Lemma 1. (*a*) $x_1^S = \alpha_1^2 \alpha_2 v(1-\lambda)/(\alpha_1+\alpha_2)^2$ *and* $x_2^S = \alpha_1 \alpha_2^2 v(1-\lambda)/(\alpha_1+\alpha_2)^2$.
(*b*) $\pi_1^S = \alpha_1 v\lambda\theta_1 + \alpha_1^3 v(1-\lambda)/(\alpha_1+\alpha_2)^2$ *and* $\pi_2^S = \alpha_2 v\lambda\theta_2 + \alpha_2^3 v(1-\lambda)/(\alpha_1+\alpha_2)^2$.
(*c*) $\psi_1^S = (1-\alpha_1)v\lambda\theta_1 + \alpha_1(1-\alpha_1)v(1-\lambda)/(\alpha_1+\alpha_2)$ *and* $\psi_2^S = (1-\alpha_2)v\lambda\theta_2 + \alpha_2(1-\alpha_2)v(1-\lambda)/(\alpha_1+\alpha_2)$.

Define the favorite [underdog] as the delegate whose probability of winning at the Nash equilibrium of the simultaneous-move subgame is greater [less] than one-half (see Dixit, 1987). Using (1) and Lemma 1, it follows that delegate 1 is the favorite and delegate 2 is the underdog if and only if $(1-2\lambda+2\lambda\theta_1)\alpha_1 > (1-2\lambda\theta_1)\alpha_2$; conversely, delegate 2 is the favorite and delegate 1 is the underdog if and only if $(1-2\lambda+2\lambda\theta_1)\alpha_1 < (1-2\lambda\theta_1)\alpha_2$. Note that $1-2\lambda+2\lambda\theta_1 \geq 1-2\lambda\theta_1$.

3.1.2. The 1L sequential-move subgame

In this subgame, delegate 1 chooses his effort level in period 1. After observing delegate 1's effort level, delegate 2 then chooses his effort level in period 2. To obtain a subgame-perfect equilibrium of the subgame, we proceed by backward induction.

In period 2, knowing delegate 1's effort level x_1, delegate 2 seeks to maximize π_2 in (3) over his effort level x_2. From the first-order condition for maximizing π_2, we derive delegate 2's reaction function:

$$x_2 = \begin{cases} \sqrt{\alpha_2 v(1-\lambda)x_1} - x_1 & \text{for } 0 < x_1 \leq \alpha_2 v(1-\lambda) \\ 0 & \text{for } x_1 > \alpha_2 v(1-\lambda). \end{cases} \tag{4}$$

Next, consider period 1 in which delegate 1 chooses his effort level. Let $\pi_1(x_1)$ be delegate 1's expected payoff – computed in period 1 of the subgame – which takes into account delegate 2's reaction function (4) in period 2. We consider only the case where $\alpha_1 \leq 2\alpha_2$.[4] Substituting (4) into (3), we obtain

$$\pi_1(x_1) = \alpha_1 v\lambda\theta_1 + \alpha_1 v(1-\lambda)x_1/\sqrt{\alpha_2 v(1-\lambda)x_1} - x_1.$$

In period 1, delegate 1 has perfect foresight about $\pi_1(x_1)$ for any value of x_1. He chooses a value of x_1 which maximizes $\pi_1(x_1)$. From the first-order condition for maximizing $\pi_1(x_1)$, we obtain delegate 1's equilibrium effort level in the $1L$ sequential-move subgame: $x_1^{1L} = \alpha_1^2 v(1-\lambda)/4\alpha_2$.

Next, substituting x_1^{1L} into (4), we obtain delegate 2's equilibrium effort level in the $1L$ sequential-move subgame: $x_2^{1L} = \alpha_1(2\alpha_2 - \alpha_1)v(1-\lambda)/4\alpha_2$.

Finally, we obtain delegate i's expected payoff π_i^{1L}, for $i = 1, 2$, and player i's expected payoff ψ_i^{1L} in the subgame-perfect equilibrium of the $1L$ sequential-move subgame by substituting the delegates' equilibrium effort levels, x_1^{1L} and x_2^{1L}, into (3) and (2), respectively.

Lemma 2. *Consider the case where* $\alpha_1 \leq 2\alpha_2$. (*a*) $x_1^{1L} = \alpha_1^2 v(1-\lambda)/4\alpha_2$ *and* $x_2^{1L} = \alpha_1(2\alpha_2 - \alpha_1)v(1-\lambda)/4\alpha_2$. (*b*) $\pi_1^{1L} = \alpha_1 v\lambda\theta_1 + \alpha_1^2 v(1-\lambda)/4\alpha_2$ *and* $\pi_2^{1L} = \alpha_2 v\lambda\theta_2 + (2\alpha_2 - \alpha_1)^2 v(1-\lambda)/4\alpha_2$. (*c*) $\psi_1^{1L} = (1-\alpha_1)v\lambda\theta_1 + \alpha_1(1-\alpha_1)v(1-\lambda)/2\alpha_2$ *and* $\psi_2^{1L} = (1-\alpha_2)v\lambda\theta_2 + (1-\alpha_2)(2\alpha_2 - \alpha_1)v\,(1-\lambda)/2\alpha_2$.

3.1.3. The 2L sequential-move subgame

In this subgame, delegate 2 chooses his effort level in period 1. After observing delegate 2's effort level, delegate 1 then chooses his effort level in period 2. To obtain a subgame-perfect equilibrium of the subgame, we proceed by backward induction.

In period 2, knowing delegate 2's effort level x_2, delegate 1 seeks to maximize π_1 in (3) over his effort level x_1. From the first-order condition for maximizing π_1, we derive delegate 1's reaction function:

$$x_1 = \begin{cases} \sqrt{\alpha_1 v(1-\lambda)x_2} - x_2 & \text{for } 0 < x_2 \leq \alpha_1 v(1-\lambda) \\ 0 & \text{for } x_2 > \alpha_1 v(1-\lambda). \end{cases} \tag{5}$$

Next, consider period 1 in which delegate 2 chooses his effort level. Let $\pi_2(x_2)$ be delegate 2's expected payoff – computed in period 1 of the subgame – which takes into account delegate 1's reaction function (5) in period 2. We consider only the case where $\alpha_2 \leq 2\alpha_1$.[5] Substituting (5) into (3), we obtain

$$\pi_2(x_2) = \alpha_2 v\lambda\theta_2 + \alpha_2 v(1-\lambda)x_2/\sqrt{\alpha_1 v(1-\lambda)x_2} - x_2.$$

In period 1, delegate 2 has perfect foresight about $\pi_2(x_2)$ for any value of x_2. He chooses a value of x_2 which maximizes $\pi_2(x_2)$. From the first-order condition for maximizing $\pi_2(x_2)$, we obtain delegate 2's equilibrium effort level in the $2L$ sequential-move subgame: $x_2^{2L} = \alpha_2^2 v(1-\lambda)/4\alpha_1$.

Next, substituting x_2^{2L} into (5), we obtain delegate 1's equilibrium effort level in the $2L$ sequential-move subgame: $x_1^{2L} = \alpha_2(2\alpha_1 - \alpha_2)v(1-\lambda)/4\alpha_1$.

Finally, we obtain delegate *i*'s expected payoff π_i^{2L}, for $i = 1, 2$, and player *i*'s expected payoff ψ_i^{2L} in the subgame-perfect equilibrium of the $2L$ sequential-move subgame by substituting the delegates' equilibrium effort levels, x_1^{2L} and x_2^{2L}, into (3) and (2), respectively.

Lemma 3. *Consider the case where* $\alpha_2 \leq 2\alpha_1$. (*a*) $x_1^{2L} = \alpha_2(2\alpha_1 - \alpha_2)v(1-\lambda)/4\alpha_1$ *and* $x_2^{2L} = \alpha_2^2 v(1-\lambda)/4\alpha_1$. (*b*) $\pi_1^{2L} = \alpha_1 v\lambda\theta_1 + (2\alpha_1 - \alpha_2)^2 v(1-\lambda)/4\alpha_1$ *and* $\pi_2^{2L} = \alpha_2 v\lambda\theta_2 + \alpha_2^2 v(1-\lambda)/4\alpha_1$. (*c*) $\psi_1^{2L} = (1-\alpha_1)v\lambda\theta_1 + (1-\alpha_1)(2\alpha_1 - \alpha_2)v(1-\lambda)/2\alpha_1$ *and* $\psi_2^{2L} = (1-\alpha_2)v\lambda\theta_2 + \alpha_2(1-\alpha_2)v(1-\lambda)/2\alpha_1$.

3.2. The delegates' decisions in the second stage

We now analyze the delegates' second-stage decisions (that is, their announcements) regarding the timing of effort exertion. At the beginning of the second stage, each delegate knows the contracts, α_1 and α_2, chosen by both parties in the first stage.

Figure 1 illustrates the strategic interaction between the delegates in this stage. Since each delegate announces either the action *Period 1* or the action *Period 2*, there are four possible combinations of actions resulting from the delegates' announcements: (*Period 1*, *Period 1*), (*Period 1*, *Period 2*), (*Period 2*, *Period 1*), and (*Period 2*, *Period 2*).

In this stage, each delegate has perfect foresight regarding the equilibrium outcomes of all third-stage subgames, and thus about the delegates' third-stage equilibrium expected payoffs described in Lemmas 1 through 3. For example, delegate 1 knows that if delegate 2 announces *Period 1* and he announces *Period 2*, the combination (*Period 2*, *Period 1*) will occur, triggering the $2L$ sequential-move subgame analyzed in Section 3.1.3 and yielding the equilibrium payoffs, (π_1^{2L}, π_2^{2L}), reported in Lemma 3.

We determine the equilibrium combinations of actions by comparing each delegate's equilibrium expected payoffs across the four third-stage subgames. Using Lemmas 1 through 3, we obtain the following comparison results: (*a*) If $\alpha_1 = \alpha_2$, then we have that $\pi_i^S = \pi_i^{1L} = \pi_i^{2L}$ for $i = 1, 2$; (*b*) if $\alpha_1 > \alpha_2$, then we have that $\pi_1^{2L} > \pi_1^S$, $\pi_2^{2L} > \pi_2^S$, and $\pi_2^S > \pi_2^{1L}$; and (*c*) if

$\alpha_1 < \alpha_2$, then we have that $\pi_1^{1L} > \pi_1^S$, $\pi_2^{1L} > \pi_2^S$, and $\pi_1^S > \pi_1^{2L}$. Part (*a*) implies that all four combinations of actions can arise in equilibrium (see Figure 1). Part (*b*) implies that only the combination (*Period 2*, *Period 1*) arises in equilibrium. Part (*c*) implies that only the combination (*Period 1*, *Period 2*) arises in equilibrium.

Lemma 4. (*a*) *If* $\alpha_1 = \alpha_2$, *then each delegate chooses and announces either the action* Period 1 *or the action* Period 2. (*b*) *If* $\alpha_1 > \alpha_2$, *then delegate* 1 *chooses and announces the action* Period 2 *while delegate* 2 *chooses and announces the action* Period 1. (*c*) *If* $\alpha_1 < \alpha_2$, *then delegate* 1 *chooses and announces the action* Period 1 *while delegate* 2 *chooses and announces the action* Period 2.

In the literature on the endogenous timing of effort exertion in contests, a well-known result is that, in equilibrium, the underdog chooses his effort level before the favorite (see, for example, Baik and Shogren, 1992; Leininger, 1993). This standard result may not hold in Lemma 4. Part (*c*) of Lemma 4 states that delegate 1 announces the action *Period 1* while delegate 2 announces the action *Period 2* if $\alpha_1 < \alpha_2$. However, according to the result stated below Lemma 1, delegate 1 is the underdog if and only if $\alpha_1 < \{(1 - 2\lambda\theta_1)/(1 - 2\lambda + 2\lambda\theta_1)\}\alpha_2$. Recall that $(1 - 2\lambda\theta_1)/(1 - 2\lambda + 2\lambda\theta_1) \leq 1$.

3.3. The players' decisions in the first stage

We now examine the players' first-stage decisions regarding their delegation contracts. In this stage, given the other player's delegation contract, each player chooses her own delegation contract to maximize her expected payoff, with perfect foresight regarding the equilibrium outcomes in all subgames beginning in the second stage of the game.

Suppose that, given player 2's contract α_2, player 1 chooses the contract α_1 such that $\alpha_1 < \alpha_2$. Then, by Lemma 4, delegate 1 will choose and announce *Period 1* in the second stage while delegate 2 will choose and announce *Period 2*. This leads to the $1L$ sequential-move

subgame analyzed in Section 3.1.2. It then follows from Lemma 2 that player 1's expected payoff is $\psi_1^{1L} = (1-\alpha_1)v\lambda\theta_1 + \alpha_1(1-\alpha_1)v(1-\lambda)/2\alpha_2$.

Suppose that, given player 2's contract α_2, player 1 chooses the contract α_1 such that $\alpha_1 = \alpha_2$. Then, by Lemma 4, each delegate will choose and announce either *Period 1* or *Period 2*, leading to the simultaneous-move subgame (see Section 3.1.1), the $1L$ sequential-move subgame (Section 3.1.2), or the $2L$ sequential-move subgame (Section 3.1.3). It then follows from Lemmas 1 through 3 that player 1's expected payoff is $\psi_1^S = \psi_1^{1L} = \psi_1^{2L} = (1-\alpha_2)v\lambda\theta_1 + \alpha_2(1-\alpha_2)v(1-\lambda)/2\alpha_2$.[6]

Suppose finally that, given player 2's contract α_2, player 1 chooses the contract α_1 such that $\alpha_1 > \alpha_2$. Then, by Lemma 4, delegate 1 will choose and announce *Period 2* in the second stage while delegate 2 will choose and announce *Period 1*, leading to the $2L$ sequential-move subgame analyzed in Section 3.1.3. It then follows from Lemma 3 that player 1's expected payoff is $\psi_1^{2L} = (1-\alpha_1)v\lambda\theta_1 + (1-\alpha_1)(2\alpha_1 - \alpha_2)v(1-\lambda)/2\alpha_1$.

Similarly, given player 1's contract α_1, if player 2 chooses the contract α_2 such that $\alpha_2 < \alpha_1$, then it follows from Lemmas 3 and 4 that player 2's expected payoff is $\psi_2^{2L} = (1-\alpha_2)v\lambda\theta_2 + \alpha_2(1-\alpha_2)v(1-\lambda)/2\alpha_1$. If player 2 chooses the contract α_2 such that $\alpha_2 = \alpha_1$, then it follows from Lemmas 1 through 4 that player 2's expected payoff is $\psi_2^S = \psi_2^{1L} = \psi_2^{2L} = (1-\alpha_1)v\lambda\theta_2 + \alpha_1(1-\alpha_1)v(1-\lambda)/2\alpha_1$.[7] If player 2 chooses the contract α_2 such that $\alpha_2 > \alpha_1$, then it follows from Lemmas 2 and 4 that player 2's expected payoff is $\psi_2^{1L} = (1-\alpha_2)v\lambda\theta_2 + (1-\alpha_2)(2\alpha_2 - \alpha_1)v(1-\lambda)/2\alpha_2$.

3.3.1 The players' reaction functions

We begin by deriving player 1's reaction function. Recall from the above that, given player 2's contract α_2, if player 1 chooses α_1 in the first stage, then her expected payoff is given by

$$\psi_1 = \psi_1^{1L} = (1-\alpha_1)v\lambda\theta_1 + \alpha_1(1-\alpha_1)v(1-\lambda)/2\alpha_2 \quad \text{for } 0 < \alpha_1 \leq \alpha_2$$

and (6)

$$\psi_1 = \psi_1^{2L} = (1-\alpha_1)v\lambda\theta_1 + (1-\alpha_1)(2\alpha_1 - \alpha_2)v(1-\lambda)/2\alpha_1 \quad \text{for } \alpha_2 \leq \alpha_1 < 1.$$

Note that, in (6), we combine the case where $\alpha_1 = \alpha_2$ with the case where $\alpha_1 > \alpha_2$ and also with the case where $\alpha_1 < \alpha_2$.

Player 1 faces the following maximization problem: Given player 2's contract α_2, maximize ψ_1 in (6) over her contract α_1. Let $\alpha_1 = b_1(\alpha_2)$ represent player 1's reaction function. In Appendix A, we obtain the following:

$$b_1(\alpha_2) = \begin{cases} \sqrt{(1-\lambda)\alpha_2/2(1-\lambda+\lambda\theta_1)} & \text{for } 0 < \alpha_2 \leq (1-\lambda)/2(1-\lambda+\lambda\theta_1) \\ 1/2 - \lambda\theta_1\alpha_2/(1-\lambda) & \\ \qquad\qquad \text{for } (1-\lambda)/2(1-\lambda+\lambda\theta_1) < \alpha_2 < \min\{(1-\lambda)/2\lambda\theta_1, 1\} & \\ \emptyset \quad \text{for } \min\{(1-\lambda)/2\lambda\theta_1, 1\} \leq \alpha_2 < 1. \end{cases} \tag{7}$$

Next, we derive player 2's reaction function. Recall from the above that, given player 1's contract α_1, if player 2 chooses α_2 in the first stage, then her expected payoff is given by

$$\psi_2 = \psi_2^{2L} = (1-\alpha_2)v\lambda\theta_2 + \alpha_2(1-\alpha_2)v(1-\lambda)/2\alpha_1 \quad \text{for } 0 < \alpha_2 \leq \alpha_1$$

and (8)

$$\psi_2 = \psi_2^{1L} = (1-\alpha_2)v\lambda\theta_2 + (1-\alpha_2)(2\alpha_2 - \alpha_1)v(1-\lambda)/2\alpha_2 \quad \text{for } \alpha_1 \leq \alpha_2 < 1.$$

Player 2 faces the following maximization problem: Given player 1's contract α_1, maximize ψ_2 in (8) over her contract α_2. Let $\alpha_2 = b_2(\alpha_1)$ represent player 2's reaction function. In Appendix A, we obtain the following:

$$b_2(\alpha_1) = \begin{cases} \sqrt{(1-\lambda)\alpha_1/2(1-\lambda+\lambda\theta_2)} & \text{for } 0 < \alpha_1 \leq (1-\lambda)/2(1-\lambda+\lambda\theta_2) \\ 1/2 - \lambda\theta_2\alpha_1/(1-\lambda) & \\ \qquad\qquad \text{for } (1-\lambda)/2(1-\lambda+\lambda\theta_2) < \alpha_1 < \min\{(1-\lambda)/2\lambda\theta_2, 1\} & \\ \emptyset \quad \text{for } \min\{(1-\lambda)/2\lambda\theta_2, 1\} \leq \alpha_1 < 1. \end{cases} \tag{9}$$

Figure 2 illustrates the players' reaction functions when $\theta_1 > \theta_2$ and $\theta_1 < (1-\lambda)/2\lambda$. In Figure 2, $\alpha_1 < \alpha_2$ holds at any point on the left (or upper) side of the 45° line or the $\alpha_2 = \alpha_1$ line, while $\alpha_1 > \alpha_2$ holds at any point on the right (or lower) side of the line. This, together with

Lemma 4, implies that any contract pair on the left (or upper) side of the line leads to the players' expected payoffs, ψ_1^{1L} and ψ_2^{1L}, while any contract pair on the right (or lower) side of the line leads to the players' expected payoffs, ψ_1^{2L} and ψ_2^{2L}.

3.3.2 The players' equilibrium contracts

Let (α_1^*, α_2^*) represent the equilibrium contract pair which the players choose in the first stage – that is, the contract pair specified in a subgame-perfect equilibrium of the full game. It must satisfy both player 1's reaction function (7) and player 2's reaction function (9). Geometrically, it corresponds to the intersection of the players' reaction functions (see point E in Figure 2). It is straightforward to verify that the reaction functions intersect at a unique point, implying the existence of a unique equilibrium contract pair. Solving the system of simultaneous equations, $\alpha_1 = 1/2 - \lambda\theta_1\alpha_2/(1-\lambda)$ from (7) and $\alpha_2 = \sqrt{(1-\lambda)\alpha_1/2(1-\lambda+\lambda\theta_2)}$ from (9), we obtain Lemma 5. Recall from Section 2 that $\theta_2 = (1-\theta_1)$.

Lemma 5. (*a*) *If* $\theta_1 = \theta_2 = 1/2$, *then* $\alpha_1^* = \alpha_2^* = (1-\lambda)/(2-\lambda) < 1/2$. (*b*) *If* $\theta_1 > \theta_2$, *then* $\alpha_1^* = \{2(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2 - \lambda\theta_1 H\}/4(1-\lambda)(1-\lambda\theta_1) < 1/2$ *and* $\alpha_2^* = (H - \lambda\theta_1)/4(1-\lambda\theta_1) < 1/2$, *where* $H \equiv \sqrt{4(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2}$.

It is straightforward to verify that α_i^*, for $i = 1, 2$, is positive under the parameter assumptions in Section 2.

Note that the players' equilibrium contracts, α_1^* and α_2^*, do not depend on their valuation v for the prize. This result follows from the fact that, given player j's contract α_j, player i's best response $b_i(\alpha_j)$, for $i, j = 1, 2$ with $i \neq j$, is independent of the valuation v because both players have the same valuation for the prize (see (7) and (9)).[8]

It is also worth noting that each player pays her delegate a compensation that is less than half of her prize valuation if she wins. In other words, in equilibrium, her contract is always less

than one-half, regardless of the players' initial winning probabilities or the value of the impact parameter λ.

3.4. The subgame-perfect equilibrium outcomes

Let x_i^*, for $i = 1, 2$, represent delegate i's effort level in a subgame-perfect equilibrium of the full game. Let p_i^* represent the probability that player i (or delegate i) wins the prize in a subgame-perfect equilibrium. Let π_i^* and ψ_i^* represent the expected payoffs for delegate i and player i, respectively, in a subgame-perfect equilibrium. Then, using (1) through (3) and Lemmas 1 through 5, we obtain Lemmas 6 and 7.

Lemma 6. *Consider the case where* $\theta_1 = \theta_2 = 1/2$. (*a*) *Each delegate announces either the action* Period 1 *or the action* Period 2. (*b*) $x_1^* = x_2^* = (1-\lambda)^2 v/4(2-\lambda)$. (*c*) $p_1^* = p_2^* = 1/2$. (*d*) $\pi_1^* = \pi_2^* = (1-\lambda^2)v/4(2-\lambda)$. (*e*) $\psi_1^* = \psi_2^* = v/2(2-\lambda)$.

Lemma 7. *Consider the case where* $\theta_1 > \theta_2$. (*a*) *Delegate* 1 *announces the action* Period 1 *while delegate* 2 *announces the action* Period 2. (*b*) $x_1^* = \{2(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2 - \lambda\theta_1 H\}^2 v/16(1-\lambda)(1-\lambda\theta_1)(H-\lambda\theta_1)$, *where* $H \equiv \sqrt{4(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2}$, *and* $x_2^* = \{2(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2 - \lambda\theta_1 H\}\{(2-2\lambda+\lambda\theta_1)H - 2(1-\lambda) - \lambda^2\theta_1^2\}v/16(1-\lambda)(1-\lambda\theta_1)(H-\lambda\theta_1)$. (*c*) $p_1^* = \{\lambda\theta_1 H + 2(1-\lambda)(1-\lambda\theta_1) - \lambda^2\theta_1^2\}/2(H-\lambda\theta_1)$ *and* $p_2^* = \{(2-\lambda\theta_1)H - 2(1-\lambda) - \lambda^2\theta_1(2-\theta_1)\}/2(H-\lambda\theta_1)$. (*d*) $\pi_1^* = \{2(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2 - \lambda\theta_1 H\}\{2(1-\lambda)(1-\lambda\theta_1) - 3\lambda^2\theta_1^2 + 3\lambda\theta_1 H\}v/16(1-\lambda)(1-\lambda\theta_1)(H-\lambda\theta_1)$ *and* $\pi_2^* = [4\lambda(1-\lambda)(1-\theta_1)(H-\lambda\theta_1)^2 + \{(2-2\lambda+\lambda\theta_1)H - 2(1-\lambda) - \lambda^2\theta_1^2\}^2]v/16(1-\lambda)(1-\lambda\theta_1)(H-\lambda\theta_1)$. (*e*) $\psi_1^* = \{2(1-\lambda)(1-\lambda\theta_1) - \lambda^2\theta_1^2 + \lambda\theta_1 H\}^2 v/8(1-\lambda)(1-\lambda\theta_1)(H-\lambda\theta_1)$ *and* $\psi_2^* = \{4 - 3\lambda\theta_1 - H\}\{(2-\lambda\theta_1)H - 2(1-\lambda) - \lambda^2\theta_1(2-\theta_1)\}v/8(1-\lambda\theta_1)(H-\lambda\theta_1)$.

It is straightforward to derive Lemmas 6 and 7. Moreover, it is straightforward to verify that x_i^*, p_i^*, π_i^*, and ψ_i^*, for $i = 1, 2$, are positive under the parameter assumptions specified in Section 2.

Applying Lemmas 5 through 7, we obtain Propositions 1 and 2.

Proposition 1. *In the case where* $\theta_1 = \theta_2 = 1/2$, *we have:* (*a*) $\alpha_1^* = \alpha_2^*$, (*b*) $x_1^* = x_2^*$, (*c*) $p_1^* = p_2^*$, (*d*) $\pi_1^* = \pi_2^*$, *and* (*e*) $\psi_1^* = \psi_2^*$.

Proposition 2. *In the case where* $\theta_1 > \theta_2$, *we have:* (*a*) $\alpha_1^* < \alpha_2^*$, (*b*) $x_1^* < x_2^*$, (*c*) $p_1^* > p_2^*$, (*d*) $\pi_1^* > \pi_2^*$, *and* (*e*) $\psi_1^* > \psi_2^*$.

Appendix B provides the proof of Proposition 2. Proposition 1 shows that if the players have the same initial winning probability – that is, $\theta_1 = \theta_2 = 1/2$ – then, for any values of the impact parameter λ and the players' valuation v for the prize, the players offer their delegates identical contracts in equilibrium. Consequently, the delegates exert the same level of effort, earn the same expected payoff, and the players themselves receive the same expected payoff. These results are intuitive because, when $\theta_1 = \theta_2$, the players are symmetric, and so are the delegates.

Proposition 2 shows that if player 1 has a higher initial winning probability than player 2 – that is, if $\theta_1 > \theta_2$ – then the equilibrium outcomes are asymmetric. In particular, player 1 offers delegate 1 a lower (or less generous) contract than player 2 offers delegate 2; delegate 1 exerts less effort than delegate 2; delegate 1 earns a higher expected payoff than delegate 2; and player 1 obtains a higher expected payoff than player 2.

Table 1 summarizes the subgame-perfect equilibrium outcomes of the full game when $\theta_1 > \theta_2$. In this case, player 2, who begins with a lower initial winning probability, attempts to offset this disadvantage by offering her delegate a greater (or more generous) contract, thereby providing stronger incentives than those offered by player 1. As a result, delegate 2 becomes the

follower in the effort-expending stage and exerts greater effort than delegate 1. In contrast, delegate 1, who receives a lower (or less generous) contract, softens the competition by moving first and expending less effort than delegate 2. Despite delegate 2's greater effort, player 1's higher initial winning probability enables her to achieve a higher *equilibrium* winning probability than player 2. Delegate 1 earns a higher expected payoff than delegate 2, despite receiving the lower (or less generous) contract, because he exerts less effort while benefiting from the higher probability of winning. Finally, player 1, who begins with the higher *initial* winning probability, earns a higher expected payoff than player 2 because she offers the lower (or less generous) contract to her delegate while enjoying a higher *equilibrium* winning probability.

It follows from part (*d*) of Lemma 6 and part (*d*) of Lemma 7 that both delegates' equilibrium expected payoffs exceed their reservation wage of zero, implying that both earn positive economic rents.[9] Interestingly, although the delegates have identical abilities, their economic rents differ. Delegate 1, who is employed by the player with the higher initial winning probability, earns a greater economic rent than delegate 2.

4. Comparative statics

In this section, we examine how changes in each of the parameters, θ_1, λ, and v, affect the outcomes of the game derived in Lemmas 5 through 7; that is, we perform comparative statics with respect to these parameters.

We first analyze how the players' equilibrium contracts respond, *ceteris paribus*, to changes in θ_1, λ, and v. Proposition 3 summarizes the results (see also Table 2).

Proposition 3. (*a*) *As player* 1*'s initial probability* θ_1 *of winning the prize increases from* $1/2$, *both* α_1^* *and* α_2^* *decrease*. (*b*) *As the impact parameter* λ *increases*, *both* α_1^* *and* α_2^* *decrease*. (*c*) *As the players' valuation v for the prize increases*, *both* α_1^* *and* α_2^* *remain unchanged*.

The proof of Proposition 3 is provided in Appendix C. Part (*a*) of Proposition 3 implies that, for any given value of λ, both α_1^* and α_2^* are maximized when $\theta_1 = 1/2$. Part (*b*) implies that, for any given value of θ_1, both α_1^* and α_2^* approach their maxima as λ approaches zero. Note that, for $\theta_1 = 1/2$, the limit of α_i^*, for $i = 1, 2$, as λ approaches zero, is $1/2$.

Part (*a*) of Proposition 3 states that, as θ_1 increases – and thus θ_2 decreases – from $1/2$, each player makes her delegate weaker or less motivated (to win the prize) by offering him lower compensation (in the event of winning).

This can be explained as follows. If $\theta_1 = \theta_2 = 1/2$, then the players are symmetric and so are the delegates; hence the players offer identical contracts in equilibrium: $\alpha_1^* = \alpha_2^*$. As θ_1 increases from $1/2$, given player 2's contract α_2, player 1's reaction function shifts inward (or leftward) (see (7) and Figure 2). This is because, as θ_1 increases, *ceteris paribus*, the final probability that player 1 (or delegate 1) wins the prize increases (see (1)), which allows player 1 to reduce her best response to α_2.

By contrast, as θ_1 increases – and thus θ_2 decreases – from $1/2$, given player 1's contract α_1, player 2's reaction function shifts outward (or upward) (see (9) and Figure 2). This is because, as θ_2 decreases, *ceteris paribus*, the final probability that player 2 (or delegate 2) wins the prize falls, which prompts player 2 to increase her best response to α_1 in order to make her delegate more motivated to win.

Since player 1's reaction function is decreasing in α_2 and player 2's reaction function is increasing in α_1 around their point of intersection, an inward shift in player 1's reaction function together with an outward shift in player 2's reaction function implies that α_1^* decreases, while α_2^* may either decrease or increase depending on the relative magnitudes of the shifts. However, it turns out that both α_1^* and α_2^* decrease as θ_1 increases.

Next, we consider the intuition behind part (*b*). As the impact parameter λ increases, given player *j*'s contract α_j, player *i*'s reaction function shifts inward, for $i, j = 1, 2$ with $i \neq j$ (see (7), (9), and Figure 2). This is because, as λ increases, *ceteris paribus*, the final probability that player *i* (or delegate *i*) wins the prize places greater weight on the initial winning probability

θ_i and less weight on the value of $f_i(x_1, x_2)$, thereby reducing player i's best response to α_j. Since player 1's reaction function is decreasing in α_2 and player 2's reaction function is increasing in α_1 around their point of intersection, inward shifts in both reaction functions imply that α_1^* may either decrease or increase, depending on the relative magnitudes of the shifts, whereas α_2^* decreases. However, it turns out that both α_1^* and α_2^* decrease as λ increases.

Next, we examine how the equilibrium total effort level, $X^* = x_1^* + x_2^*$, expended by both delegates responds, *ceteris paribus*, to changes in the parameters, θ_1, λ, and v. Proposition 4 summarizes the results (see also Table 2).

Proposition 4. (*a*) *As* θ_1 *increases from* $1/2$, *the equilibrium total effort level* X^* *expended by both delegates decreases*. (*b*) *As* λ *increases*, X^* *decreases*. (*c*) *As v increases*, X^* *increases*.

The proof of Proposition 4 is provided in Appendix D. Part (*a*) of Proposition 4 implies that, given any values of λ and v, X^* is maximized when $\theta_1 = 1/2$, and approaches its minimum as θ_1 approaches unity. Part (*b*) implies that, given any values of θ_1 and v, X^* approaches its maximum [resp. minimum] as λ approaches zero [resp. unity]. Note that, for $\theta_1 = 1/2$, the limit of X^* as λ approaches zero is $v/4$ for any value of v.

The results in parts (*a*) and (*b*) of Proposition 4 are intuitive because, as shown in parts (*a*) and (*b*) of Proposition 3, each player makes her delegate less motivated to win the prize, in both cases, by offering him lower compensation (in the event of winning).

The result in part (*c*) can be attributed to the fact that, as v increases, both $\alpha_1^* v$ and $\alpha_2^* v$ increase (see part (*c*) of Proposition 3), which intensifies competition between the delegates.

Next, we examine how the players' (or delegates') equilibrium probabilities of winning the prize respond, *ceteris paribus*, to changes in the parameters, θ_1, λ, and v. Proposition 5 summarizes the results (see also Table 2).

Proposition 5. (*a*) *As* θ_1 *increases from* $1/2$, *player* 1*'s* (*or delegate* 1*'s*) *equilibrium winning probability* p_1^* *increases from* $1/2$. (*b*) *As* λ *increases*, p_1^* *remains unchanged at* $1/2$ *when* $\theta_1 = \theta_2 = 1/2$*; but it increases when* $\theta_1 > \theta_2$. (*c*) *As v increases*, p_1^* *remains unchanged.*

The proof of Proposition 5 is provided in Appendix E. Note from part (*b*) that, when $\theta_1 > \theta_2$, p_1^* approaches its minimum value of $1/2$ as λ approaches zero.

Part (*a*) of Proposition 5 can be explained as follows. First, as θ_1 increases from $1/2$, $\lambda\theta_1$ in (1) increases. Second, as θ_1 increases, each player reduces her delegate's incentive to win the prize by offering him lower compensation in the event of victory, as shown in part (*a*) of Proposition 3. This tends to reduce the equilibrium effort levels, x_1^* and x_2^*, and thereby decreases $f_1(x_1^*, x_2^*)$ in (1).[10] Finally, the increase in $\lambda\theta_1$ outweighs the decrease in $(1 - \lambda)f_1(x_1^*, x_2^*)$, resulting in an increase in p_1^* as θ_1 increases.

Part (*b*) of Proposition 5 can be explained as follows. First, as λ increases, $\lambda\theta_1$ in (1) increases. Second, as λ increases, each player makes her delegate less motivated to win the prize by offering him lower compensation (in the event of winning), as shown in part (*b*) of Proposition 3. This decreases $f_1(x_1^*, x_2^*)$ in (1) when $\theta_1 > \theta_2$. Finally, when $\theta_1 > \theta_2$, p_1^* increases as λ increases because the increase in $\lambda\theta_1$ is greater than the decrease in $(1 - \lambda)f_1(x_1^*, x_2^*)$.

Finally, we analyze how the equilibrium expected payoffs for both the delegates and the players respond, *ceteris paribus*, to changes in the parameters, θ_1, λ, and v. Proposition 6 and Table 3 summarize the results.

Proposition 6. (*a*) *As* θ_1 *increases from* $1/2$, ψ_1^* *increases*, *while* π_2^* *and* ψ_2^* *both decrease. However*, π_1^* *may increase or decrease*, *depending on the values of* λ and θ_1. (*b*) *As* λ *increases*, ψ_1^* *increases*. *However*, π_1^*, π_2^*, *and* ψ_2^* *each may increase or decrease*, *depending on the values of* θ_1 *and* λ. (*c*) *As v increases*, π_1^*, ψ_1^*, π_2^*, *and* ψ_2^* *all increase.*

Utilizing (2) and Propositions 3 and 5, it is straightforward to obtain that $\partial\psi_1^*/\partial\theta_1 > 0$ and $\partial\psi_1^*/\partial\lambda > 0$. Part ($c$) follows immediately from Lemmas 6 and 7. However, due to algebraic intractability, we cannot analytically derive the remaining comparative statics results for arbitrary values of λ and θ_1. Consequently, Table 3, which elaborates on Proposition 6, is obtained by *graphically* examining each result while fixing either λ or θ_1 at a specific value.

The symbol $+\ -$ in Table 3 indicates that the partial derivative with respect to the parameter is positive for low values of the parameter but becomes negative for higher values. For example, in the first panel, when $\lambda = 0.5$, as θ_1 increases from $1/2$, π_1^* increases for low values of θ_1 but decreases for higher values.

An interesting finding in Proposition 6 and Table 3 is that, for any value of λ, player 1's equilibrium expected payoff ψ_1^* increases as θ_1 increases from $1/2$. In contrast, delegate 1's equilibrium expected payoff π_1^* increases with θ_1 for low values of λ, but decreases with θ_1 for high values of λ. The former result follows immediately from two facts, together with (2): As θ_1 increases from $1/2$, (i) α_1^* decreases (see Proposition 3), and (ii) p_1^* increases from $1/2$ (see Proposition 5). The latter result can be explained by two facts, together with (3): As θ_1 increases from $1/2$, (i) delegate 1's equilibrium gross expected payoff, $\alpha_1^* v p_1^*$ in (3), decreases, and (ii) his equilibrium effort level x_1^* decreases.[11] Since a decrease in $\alpha_1^* v p_1^*$ lowers π_1^*, while a decrease in x_1^* raises π_1^*, the effect of an increase in θ_1 on π_1^* depends on the relative magnitudes of these opposing forces. For instance, in the first panel of Table 3, when $\lambda = 0.1$, as θ_1 increases from $1/2$, the decrease in $\alpha_1^* v p_1^*$ is smaller than the decrease in x_1^*, so that π_1^* increases with θ_1. However, when $\lambda = 0.9$, as θ_1 increases from $1/2$, the decrease in $\alpha_1^* v p_1^*$ exceeds the decrease in x_1^*, so that π_1^* decreases with θ_1.

5. Comparison with the corresponding contest in which $\lambda = 0$

In this section, we compare the outcomes of the full game, as described in Lemmas 5 through 7, with those of the corresponding contest in which the winner is determined solely by the delegates' effort levels, without any influence from the initial winning probabilities. The

outcomes of the latter contest, reported in Remark 1, can be obtained by setting $\lambda = 0$ in Lemmas 5 through 7.[12] We represent these outcomes by the superscript E.

Remark 1. (a) $\alpha_1^E = \alpha_2^E = 1/2$. ($b$) *Each delegate announces either the action* Period 1 *or the action* Period 2. (c) $x_1^E = x_2^E = v/8$ *and* $X^E = v/4$. (d) $p_1^E = p_2^E = 1/2$. (e) $\pi_1^E = \pi_2^E = v/8$. (f) $\psi_1^E = \psi_2^E = v/4$.

Now, using Propositions 3 through 6 and Remark 1, we obtain Remark 2 and Table 4.

Remark 2. (a) $\alpha_i^* < \alpha_i^E$ *for* $i = 1, 2$. (b) $X^* < X^E$. (c) $p_i^* = p_i^E$ *for* $i = 1, 2$ *when* $\theta_1 = \theta_2 = 1/2$*;* $p_1^* > p_1^E$ *and* $p_2^* < p_2^E$ *when* $\theta_1 > \theta_2$. (d) $\psi_1^* > \psi_1^E$. (e) ψ_2^*, π_1^*, *and* π_2^* *may be greater than*, *equal to*, *or less than* ψ_2^E, π_1^E, *and* π_2^E, *respectively*, *depending on the values of* θ_1 *and* λ.

Table 4 provides details for part (e) of Remark 2. Unless otherwise noted, each cell reports an equilibrium expected payoff that exceeds its counterpart. For example, in the first panel, when $\theta_1 = 0.5$ and $\lambda = 0.1$, ψ_2^* exceeds ψ_2^E; however, when $\theta_1 = 0.9$ and $\lambda = 0.9$, ψ_2^E exceeds ψ_2^*. In the second panel, when $\theta_1 = 0.5$ and $\lambda = 0.1$, π_1^* exceeds π_1^E; however, when $\theta_1 = 0.5$ and $\lambda = 0.9$, π_1^E exceeds π_1^*. In the third panel, when $\theta_1 = 0.5$ and $\lambda = 0.1$, π_2^* exceeds π_2^E; however, when $\theta_1 = 0.5$ and $\lambda = 0.9$, π_2^E exceeds π_2^*.

Remark 2 states that the players' equilibrium contracts, α_1^* and α_2^*, and the equilibrium total effort level X^* are lower for any positive value of the impact parameter λ than in the corresponding contest in which $\lambda = 0$. It also states that player 1's equilibrium expected payoff ψ_1^* is higher for any positive value of λ than in the corresponding contest in which $\lambda = 0$. The intuition underlying Remark 2 is similar to that provided for Propositions 3 through 6, and is therefore omitted.

6. Conclusions

We have studied a two-player contest with delegation in which each player begins with an initial probability of winning a prize, her final probability of winning depends on both this initial probability and the effort levels exerted by the two delegates, and the delegates endogenously determine when to exert effort. After the delegation contracts are signed, each delegate independently chooses the period – either period 1 or period 2 – in which he will exert effort, and the delegates simultaneously announce their choices. Finally, in the chosen periods, the delegates independently choose their effort levels.

We have obtained the outcomes of the full game in Section 3 and then performed comparative statics of these outcomes with respect to the parameters θ_1, λ, and v in Section 4. Finally, in Section 5, we have compared the outcomes of the full game with those of the corresponding contest in which the winner is determined solely by the delegates' effort levels, without the influence of initial winning probabilities.

In Section 3, we have shown that if player 1 has a *higher* initial winning probability than player 2, then in equilibrium the following hold: (i) player 1 offers a lower (or less generous) contract to her delegate, delegate 1, than player 2 offers to delegate 2; (ii) delegate 1 exerts effort in period 1, while delegate 2 exerts effort in period 2; (iii) delegate 1 exerts less effort than delegate 2; (iv) delegate 1 obtains a higher expected payoff than delegate 2; and (v) player 1 obtains a higher expected payoff than player 2. Additionally, we have shown that, despite the two delegates having equal ability in the contest, delegate 1 – who is hired by the player with the higher initial winning probability – earns a greater economic rent than delegate 2.

In Section 4, we have derived the following comparative statics results. First, as player 1's initial winning probability θ_1 increases from one half, (i) both players' equilibrium contracts decrease (or become less generous), (ii) the equilibrium total effort level decreases, (iii) player 1's equilibrium expected payoff increases, whereas delegate 1's equilibrium expected payoff may either increase or decrease, and (iv) both player 2's and delegate 2's equilibrium expected payoffs decrease. Second, as the impact parameter λ increases, (i) both players' equilibrium contracts

decrease (or become less generous), (ii) the equilibrium total effort level decreases, (iii) player 1's equilibrium expected payoff increases, whereas delegate 1's equilibrium expected payoff may either increase or decrease, and (iv) the equilibrium expected payoffs of player 2 and delegate 2 may each either increase or decrease.

In Section 5, we have shown that, for any positive value of the impact parameter λ, the equilibrium contracts in Lemma 5 are lower (or less generous) and the equilibrium total effort level is lower than in the corresponding contest in which $\lambda = 0$. At the same time, player 1's equilibrium expected payoff in Lemmas 6 and 7 is higher compared to the corresponding contest in which $\lambda = 0$.

It would be interesting to explore several modifications of the model, such as allowing for more than two players, introducing asymmetric valuations for the prize, or permitting heterogeneity in delegate ability. In addition, several extensions merit consideration, including a setting in which players' valuations are private information, one in which players have incomplete information about the impact parameter λ, one in which delegation contracts are unobservable to rival parties when delegates choose their effort levels, and one in which players' initial winning probabilities are endogenous. We leave these modifications and extensions for future research.

Footnotes

1. Important contributions to the literature on contest theory include Tullock (1980), Dixit (1987), Hillman and Riley (1989), Baye et al. (1993), Epstein and Nitzan (2007), Konrad (2009), Vojnović (2015), and Bevià and Corchón (2024).

2. Baik and Shogren (1992), Leininger (1993), and Baik (2013) demonstrate that, in every lopsided two-player contest without delegation, both players achieve higher equilibrium expected payoffs under endogenous timing than under simultaneous moves.

3. Under the parameter assumptions outlined in Section 2, the second-order condition is satisfied for every maximization problem considered in this paper.

4. It is straightforward to verify that, when $\alpha_1 > 2\alpha_2$, the delegates' effort levels in the subgame-perfect equilibrium of the $1L$ sequential-move subgame are $x_1^{1L} = \alpha_2 v(1-\lambda)$ and $x_2^{1L} = 0$. Since this case does not affect the main results, it is omitted for brevity.

5. In the case where $\alpha_2 > 2\alpha_1$, the delegates' effort levels in the subgame-perfect equilibrium of the $2L$ sequential-move subgame are $x_1^{2L} = 0$ and $x_2^{2L} = \alpha_1 v(1-\lambda)$. Since this case does not affect the main analysis, it is omitted for brevity.

6. Recall that $\alpha_1 = \alpha_2$.

7. Recall that $\alpha_2 = \alpha_1$.

8. If the players had different valuations for the prize, then α_1^* and α_2^* would depend on these valuations.

9. Schoonbeek (2002) and Baik and Lee (2013) establish this result in their respective studies of contests with delegation.

10. Using (1) and Lemma 7, we obtain $f_1(x_1^*, x_2^*) = (H - \lambda\theta_1)/4(1-\lambda)$, where $H \equiv \sqrt{4(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2}$. It is straightforward to obtain that $\{\partial f_1(x_1^*, x_2^*)/\partial\theta_1\} < 0$, and that $\{\partial f_1(x_1^*, x_2^*)/\partial\lambda\} < 0$ when $\theta_1 > \theta_2$.

11. It is straightforward to verify graphically that, for any value of λ, $\alpha_1 v p_1^*$ and x_1^* each decrease as θ_1 increases from $1/2$.

12. Remark 1 can also be obtained by setting $v_i = v$, for $i = 1, 2$, in Proposition 1 of Baik and Lee (2013).

Appendix A: Deriving the players' reaction functions

A1. Player 1's reaction function

We obtain:

$$b_1(\alpha_2) = \begin{cases} \sqrt{(1-\lambda)\alpha_2/2(1-\lambda+\lambda\theta_1)} & \text{for } 0 < \alpha_2 \leq (1-\lambda)/2(1-\lambda+\lambda\theta_1) \\ 1/2 - \lambda\theta_1\alpha_2/(1-\lambda) & \\ \qquad\qquad \text{for } (1-\lambda)/2(1-\lambda+\lambda\theta_1) < \alpha_2 < \min\{(1-\lambda)/2\lambda\theta_1, 1\} & \\ \emptyset \quad \text{for } \min\{(1-\lambda)/2\lambda\theta_1, 1\} \leq \alpha_2 < 1. & \end{cases}$$

Proof. First, consider the case where $0 < \alpha_2 < (1-\lambda)/2(1-\lambda+\lambda\theta_1)$. Fix a value of α_2.

(i) The first-order condition, $\partial\psi_1^{1L}/\partial\alpha_1 = 0$, for maximizing ψ_1^{1L} yields that $\alpha_1^{o} = 1/2 - \lambda\theta_1\alpha_2/(1-\lambda)$, which is greater than the given value of α_2. That is, α_1^{o} is located on the right side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_1^{1L}/\partial\alpha_1^2 < 0$, leads to $\partial\psi_1^{1L}/\partial\alpha_1 > 0$ for $\alpha_1 < \alpha_2$.

(ii) We have that $\psi_1^{1L} = \psi_1^{2L}$ for $\alpha_1 = \alpha_2$.

(iii) The first-order condition, $\partial\psi_1^{2L}/\partial\alpha_1 = 0$, for maximizing ψ_1^{2L} yields that $\alpha_1^{oo} = \sqrt{(1-\lambda)\alpha_2/2(1-\lambda+\lambda\theta_1)}$, which is greater than the given value of α_2. That is, α_1^{oo} is located on the right side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_1^{2L}/\partial\alpha_1^2 < 0$, implies that ψ_1^{2L} increases as α_1 increases from the given value of α_2 to α_1^{oo}, and decreases as α_1 increases from α_1^{oo}.

It follows from (i), (ii), and (iii) that, given $0 < \alpha_2 < (1-\lambda)/2(1-\lambda+\lambda\theta_1)$, player 1's best response to α_2 is $b_1(\alpha_2) = \sqrt{(1-\lambda)\alpha_2/2(1-\lambda+\lambda\theta_1)}$.

Next, consider the case where $\alpha_2 = (1-\lambda)/2(1-\lambda+\lambda\theta_1)$.

(iv) The first-order condition, $\partial\psi_1^{1L}/\partial\alpha_1 = 0$, for maximizing ψ_1^{1L} yields that $\alpha_1^{o} = (1-\lambda)/2(1-\lambda+\lambda\theta_1)$, which is equal to the given value of α_2. That is, α_1^{o} is located on the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_1^{1L}/\partial\alpha_1^2 < 0$, leads to $\partial\psi_1^{1L}/\partial\alpha_1 > 0$ for $\alpha_1 < \alpha_2$.

(v) We have that $\psi_1^{1L} = \psi_1^{2L}$ for $\alpha_1 = \alpha_2$.

(vi) The first-order condition, $\partial\psi_1^{2L}/\partial\alpha_1 = 0$, for maximizing ψ_1^{2L} yields that $\alpha_1^{oo} = (1-\lambda)/2(1-\lambda+\lambda\theta_1)$, which is equal to the given value of α_2. That is, α_1^{oo} is located on the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_1^{2L}/\partial\alpha_1^2 < 0$, implies that ψ_1^{2L} decreases as α_1 increases from the given value of α_2.

It follows from (iv), (v), and (vi) that, given $\alpha_2 = (1-\lambda)/2(1-\lambda+\lambda\theta_1)$, player 1's best response to α_2 is $b_1(\alpha_2) = (1-\lambda)/2(1-\lambda+\lambda\theta_1)$.

Next, consider the case where $(1-\lambda)/2(1-\lambda+\lambda\theta_1) < \alpha_2 < \min\{(1-\lambda)/2\lambda\theta_1,\ 1\}$. Fix a value of α_2.

(vii) The first-order condition, $\partial\psi_1^{1L}/\partial\alpha_1 = 0$, for maximizing ψ_1^{1L} yields that $\alpha_1^{o} = 1/2 - \lambda\theta_1\alpha_2/(1-\lambda)$, which is less than the given value of α_2. That is, α_1^{o} is located on the left side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_1^{1L}/\partial\alpha_1^2 < 0$, implies that ψ_1^{1L} increases as α_1 increases from 0 to α_1^{o}, and decreases as α_1 increases from α_1^{o} to the given value of α_2.

(viii) We have that $\psi_1^{1L} = \psi_1^{2L}$ for $\alpha_1 = \alpha_2$.

(ix) The first-order condition, $\partial\psi_1^{2L}/\partial\alpha_1 = 0$, for maximizing ψ_1^{2L} yields that $\alpha_1^{oo} = \sqrt{(1-\lambda)\alpha_2/2(1-\lambda+\lambda\theta_1)}$, which is less than the given value of α_2. That is, α_1^{oo} is located on the left side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_1^{2L}/\partial\alpha_1^2 < 0$, leads to $\partial\psi_1^{2L}/\partial\alpha_1 < 0$ for $\alpha_1 > \alpha_2$.

It follows from (vii), (viii), and (ix) that, given $(1-\lambda)/2(1-\lambda+\lambda\theta_1) < \alpha_2 < \min\{(1-\lambda)/2\lambda\theta_1,\ 1\}$, player 1's best response to α_2 is $b_1(\alpha_2) = 1/2 - \lambda\theta_1\alpha_2/(1-\lambda)$.

Finally, consider the case where $\min\{(1-\lambda)/2\lambda\theta_1,\ 1\} \le \alpha_2 < 1$. Fix a value of α_2.

(x) The first-order condition, $\partial\psi_1^{1L}/\partial\alpha_1 = 0$, for maximizing ψ_1^{1L} yields that $\alpha_1^{o} = 1/2 - \lambda\theta_1\alpha_2/(1-\lambda)$, which is negative and is less than the given value of α_2. That is, α_1^{o} is located on the left side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_1^{1L}/\partial\alpha_1^2 < 0$, implies that ψ_1^{1L} decreases as α_1 increases from 0 to the given value of α_2.

(xi) We have that $\psi_1^{1L} = \psi_1^{2L}$ for $\alpha_1 = \alpha_2$.

(xii) The first-order condition, $\partial\psi_1^{2L}/\partial\alpha_1 = 0$, for maximizing ψ_1^{2L} yields that $\alpha_1^{oo} = \sqrt{(1-\lambda)\alpha_2/2(1-\lambda+\lambda\theta_1)}$, which is less than the given value of α_2. That is, α_1^{oo} is located on the left side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_1^{2L}/\partial\alpha_1^2 < 0$, leads to $\partial\psi_1^{2L}/\partial\alpha_1 < 0$ for $\alpha_1 > \alpha_2$.

It follows from (x), (xi), and (xii) that, given $\min\{(1-\lambda)/2\lambda\theta_1, 1\} \leq \alpha_2 < 1$, player 1's best response to α_2 is $b_1(\alpha_2) = \emptyset$.

A2. Player 2's reaction function

We obtain:

$$b_2(\alpha_1) = \begin{cases} \sqrt{(1-\lambda)\alpha_1/2(1-\lambda+\lambda\theta_2)} & \text{for } 0 < \alpha_1 \leq (1-\lambda)/2(1-\lambda+\lambda\theta_2) \\ 1/2 - \lambda\theta_2\alpha_1/(1-\lambda) & \\ \qquad\qquad \text{for } (1-\lambda)/2(1-\lambda+\lambda\theta_2) < \alpha_1 < \min\{(1-\lambda)/2\lambda\theta_2, 1\} & \\ \emptyset \quad \text{for } \min\{(1-\lambda)/2\lambda\theta_2, 1\} \leq \alpha_1 < 1. & \end{cases}$$

Proof. First, consider the case where $0 < \alpha_1 < (1-\lambda)/2(1-\lambda+\lambda\theta_2)$. Fix a value of α_1.

(i) The first-order condition, $\partial\psi_2^{2L}/\partial\alpha_2 = 0$, for maximizing ψ_2^{2L} yields that $\alpha_2^{o} = 1/2 - \lambda\theta_2\alpha_1/(1-\lambda)$, which is greater than the given value of α_1. That is, α_2^{o} is located on the upper side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_2^{2L}/\partial\alpha_2^2 < 0$, leads to $\partial\psi_2^{2L}/\partial\alpha_2 > 0$ for $\alpha_2 < \alpha_1$.

(ii) We have that $\psi_2^{2L} = \psi_2^{1L}$ for $\alpha_2 = \alpha_1$.

(iii) The first-order condition, $\partial\psi_2^{1L}/\partial\alpha_2 = 0$, for maximizing ψ_2^{1L} yields that $\alpha_2^{oo} = \sqrt{(1-\lambda)\alpha_1/2(1-\lambda+\lambda\theta_2)}$, which is greater than the given value of α_1. That is, α_2^{oo} is located on the upper side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_2^{1L}/\partial\alpha_2^2 < 0$, implies that ψ_2^{1L} increases as α_2 increases from the given value of α_1 to α_2^{oo}, and decreases as α_2 increases from α_2^{oo}.

It follows from (i), (ii), and (iii) that, given $0 < \alpha_1 < (1-\lambda)/2(1-\lambda+\lambda\theta_2)$, player 2's best response to α_1 is $b_2(\alpha_1) = \sqrt{(1-\lambda)\alpha_1/2(1-\lambda+\lambda\theta_2)}$.

Next, consider the case where $\alpha_1 = (1-\lambda)/2(1-\lambda+\lambda\theta_2)$.

(iv) The first-order condition, $\partial\psi_2^{2L}/\partial\alpha_2 = 0$, for maximizing ψ_2^{2L} yields that $\alpha_2^{o} = (1-\lambda)/2(1-\lambda+\lambda\theta_2)$, which is equal to the given value of α_1. That is, α_2^{o} is located on the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_2^{2L}/\partial\alpha_2^2 < 0$, leads to $\partial\psi_2^{2L}/\partial\alpha_2 > 0$ for $\alpha_2 < \alpha_1$.

(v) We have that $\psi_2^{2L} = \psi_2^{1L}$ for $\alpha_2 = \alpha_1$.

(vi) The first-order condition, $\partial\psi_2^{1L}/\partial\alpha_2 = 0$, for maximizing ψ_2^{1L} yields that $\alpha_2^{oo} = (1-\lambda)/2(1-\lambda+\lambda\theta_2)$, which is equal to the given value of α_1. That is, α_2^{oo} is located on the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_2^{1L}/\partial\alpha_2^2 < 0$, implies that ψ_2^{1L} decreases as α_2 increases from the given value of α_1.

It follows from (iv), (v), and (vi) that, given $\alpha_1 = (1-\lambda)/2(1-\lambda+\lambda\theta_2)$, player 2's best response to α_1 is $b_2(\alpha_1) = (1-\lambda)/2(1-\lambda+\lambda\theta_2)$.

Next, consider the case where $(1-\lambda)/2(1-\lambda+\lambda\theta_2) < \alpha_1 < \min\{(1-\lambda)/2\lambda\theta_2,\ 1\}$. Fix a value of α_1.

(vii) The first-order condition, $\partial\psi_2^{2L}/\partial\alpha_2 = 0$, for maximizing ψ_2^{2L} yields that $\alpha_2^{o} = 1/2 - \lambda\theta_2\alpha_1/(1-\lambda)$, which is less than the given value of α_1. That is, α_2^{o} is located on the lower side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_2^{2L}/\partial\alpha_2^2 < 0$, implies that ψ_2^{2L} increases as α_2 increases from 0 to α_2^{o}, and decreases as α_2 increases from α_2^{o} to the given value of α_1.

(viii) We have that $\psi_2^{2L} = \psi_2^{1L}$ for $\alpha_2 = \alpha_1$.

(ix) The first-order condition, $\partial\psi_2^{1L}/\partial\alpha_2 = 0$, for maximizing ψ_2^{1L} yields that $\alpha_2^{oo} = \sqrt{(1-\lambda)\alpha_1/2(1-\lambda+\lambda\theta_2)}$, which is less than the given value of α_1. That is, α_2^{oo} is located on the lower side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_2^{1L}/\partial\alpha_2^2 < 0$, leads to $\partial\psi_2^{1L}/\partial\alpha_2 < 0$ for $\alpha_2 > \alpha_1$.

It follows from (vii), (viii), and (ix) that, given $(1-\lambda)/2(1-\lambda+\lambda\theta_2) < \alpha_1 < \min\{(1-\lambda)/2\lambda\theta_2,\ 1\}$, player 2's best response to α_1 is $b_2(\alpha_1) = 1/2 - \lambda\theta_2\alpha_1/(1-\lambda)$.

Finally, consider the case where $\min\{(1-\lambda)/2\lambda\theta_2,\ 1\} \le \alpha_1 < 1$. Fix a value of α_1.

(x) The first-order condition, $\partial\psi_2^{2L}/\partial\alpha_2 = 0$, for maximizing ψ_2^{2L} yields that $\alpha_2^{o} = 1/2 - \lambda\theta_2\alpha_1/(1-\lambda)$, which is negative and is less than the given value of α_1. That is, α_2^{o} is located on the lower side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_2^{2L}/\partial\alpha_2^2 < 0$, implies that ψ_2^{2L} decreases as α_2 increases from 0 to the given value of α_1.

(xi) We have that $\psi_2^{2L} = \psi_2^{1L}$ for $\alpha_2 = \alpha_1$.

(xii) The first-order condition, $\partial\psi_2^{1L}/\partial\alpha_2 = 0$, for maximizing ψ_2^{1L} yields that $\alpha_2^{oo} = \sqrt{(1-\lambda)\alpha_1/2(1-\lambda+\lambda\theta_2)}$, which is less than the given value of α_1. That is, α_2^{oo} is located on the lower side of the 45° line or the $\alpha_2 = \alpha_1$ line. This, together with the second-order condition that $\partial^2\psi_2^{1L}/\partial\alpha_2^2 < 0$, leads to $\partial\psi_2^{1L}/\partial\alpha_2 < 0$ for $\alpha_2 > \alpha_1$.

It follows from (x), (xi), and (xii) that, given $\min\{(1-\lambda)/2\lambda\theta_2,\ 1\} \leq \alpha_1 < 1$, player 2's best response to α_1 is $b_2(\alpha_1) = \emptyset$.

Appendix B: Proof of Proposition 2

It is straightforward to verify parts (*a*) through (*c*). We first show that $\pi_1^* > \pi_2^*$ in part (*d*), and then that $\psi_1^* > \psi_2^*$ in part (*e*).

(*d*) In the case where $\theta_1 > \theta_2$, we have: $\pi_1^* = \{2(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2 - \lambda\theta_1 H\}$ $\{2(1-\lambda)(1-\lambda\theta_1) - 3\lambda^2\theta_1^2 + 3\lambda\theta_1 H\}v/16(1-\lambda)(1-\lambda\theta_1)(H-\lambda\theta_1)$ and $\pi_2^* = [4(1-\lambda)\lambda(1-\theta_1)(H-\lambda\theta_1)^2 + \{(2-2\lambda+\lambda\theta_1)H - 2(1-\lambda) - \lambda^2\theta_1^2\}^2]v/16(1-\lambda)(1-\lambda\theta_1)(H-\lambda\theta_1)$, where $H \equiv \sqrt{4(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2}$.

(i) It is straightforward to obtain: $\pi_1^* > \pi_2^*$ if and only if $[2(1-\lambda)(1-\lambda+\lambda\theta_1)(1-\lambda\theta_1) + \lambda\theta_1\{\lambda^2\theta_1^2 + (1-\lambda)\}] > \{\lambda^2\theta_1^2 + (1-\lambda)\}H$.

(ii) Squaring both sides and simplifying yields: $\pi_1^* > \pi_2^*$ if and only if $\theta_1 > 1/2$ (or, equivalently, $\theta_1 > \theta_2$).

(*e*) We obtain that $\psi_1^* > \psi_2^*$, using the following facts: (i) $\psi_1^* = (1-\alpha_1^*)vp_1^*$ and $\psi_2^* = (1-\alpha_2^*)vp_2^*$ from (1) and (2), (ii) in the case where $\theta_1 > \theta_2$, we have that $\alpha_1^* < \alpha_2^*$ from

part (a) of Proposition 2, and (iii) in the case where $\theta_1 > \theta_2$, we have that $p_1^* > p_2^*$ from part (c) of Proposition 2.

Appendix C: Proof of Proposition 3

Recall from Lemma 5 that

$$\alpha_1^* = \{2(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2 - \lambda\theta_1 H\}/4(1-\lambda)(1-\lambda\theta_1) \tag{C1}$$

and

$$\alpha_2^* = (H - \lambda\theta_1)/4(1-\lambda\theta_1), \tag{C2}$$

where $H \equiv \sqrt{4(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2}$.

(a) Differentiating (C1) and (C2) with respect to θ_1, we obtain:

$$\partial\alpha_1^*/\partial\theta_1 = [\lambda^2\theta_1(2-\lambda\theta_1)H - \lambda(2-\lambda\theta_1)\{2(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2\}]/ 4(1-\lambda)(1-\lambda\theta_1)^2 H$$

and

$$\partial\alpha_2^*/\partial\theta_1 = \lambda\{\lambda\theta_1 + 2(1-\lambda)(1-\lambda\theta_1) - H\}/4(1-\lambda\theta_1)^2.$$

It is straightforward to obtain that $\partial\alpha_1^*/\partial\theta_1 < 0$ and $\partial\alpha_2^*/\partial\theta_1 < 0$.

(b) Differentiating (C1) and (C2) with respect to λ, we obtain:

$$\partial\alpha_1^*/\partial\lambda = \theta_1(2-\lambda-\lambda\theta_1)\{\lambda\theta_1 H - \lambda^2\theta_1^2 - 2(1-\lambda)(1-\lambda\theta_1)\}/ 4(1-\lambda)^2(1-\lambda\theta_1)^2 H$$

and

$$\partial\alpha_2^*/\partial\lambda = \{\lambda\theta_1^2 - 2(1-\theta_1)(1-\lambda\theta_1) - \theta_1 H\}/4(1-\lambda\theta_1)^2 H.$$

It is straightforward to obtain that $\partial\alpha_1^*/\partial\lambda < 0$ and $\partial\alpha_2^*/\partial\lambda < 0$.

(c) Since α_1^* and α_2^* are independent of the players' valuation v for the prize, we obtain that $\partial\alpha_1^*/\partial v = 0$ and $\partial\alpha_2^*/\partial v = 0$.

Appendix D: Proof of Proposition 4

Using Lemmas 6 and 7, we obtain the following. If $\theta_1 \geq \theta_2$, then

$$X^* = \{2(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2 - \lambda\theta_1 H\}v/8(1-\lambda\theta_1), \tag{D1}$$

where $H \equiv \sqrt{4(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2}$.

(a) Differentiating (D1) with respect to θ_1, we obtain:

$$\partial X^*/\partial\theta_1 = (2-\lambda\theta_1)\{\lambda^2\theta_1 H - 2\lambda(1-\lambda)(1-\lambda\theta_1) - \lambda^3\theta_1^2\}v/8(1-\lambda\theta_1)^2 H.$$

It is straightforward to obtain that $\partial X^*/\partial\theta_1 < 0$.

(b) Differentiating (D1) with respect to λ, we obtain:

$$\begin{aligned}\partial X^*/\partial\lambda = {} & [\{\lambda\theta_1^2(2-\lambda\theta_1) - 2(1-\lambda\theta_1)^2\}H - 2\theta_1(1-\lambda)(1-\lambda\theta_1)(2-\lambda\theta_1) \\ & + 2\lambda\theta_1(1-\lambda\theta_1)^2 - \lambda^2\theta_1^3(2-\lambda\theta_1)]v/8(1-\lambda\theta_1)^2 H.\end{aligned}$$

We have that $(1-\lambda\theta_1)^2 H > \lambda\theta_1(1-\lambda\theta_1)^2$. We also have:

$$\begin{aligned}& \lambda\theta_1^2(2-\lambda\theta_1)H - 2\theta_1(1-\lambda)(1-\lambda\theta_1)(2-\lambda\theta_1) - \lambda^2\theta_1^3(2-\lambda\theta_1) \\ & = \theta_1(2-\lambda\theta_1)\{\lambda\theta_1 H - 2(1-\lambda)(1-\lambda\theta_1) - \lambda^2\theta_1^2\} < 0.\end{aligned}$$

Hence, we obtain that $\partial X^*/\partial\lambda < 0$.

(c) Differentiating (D1) with respect to v, we obtain:

$$\partial X^*/\partial v = \{2(1-\lambda)(1-\lambda\theta_1) + \lambda^2\theta_1^2 - \lambda\theta_1 H\}/8(1-\lambda\theta_1).$$

It is straightforward to obtain that $\partial X^*/\partial v > 0$.

Appendix E: Proof of Proposition 5

Recall from part (c) of Lemma 7 that

$$p_1^* = \{\lambda\theta_1 H + 2(1-\lambda)(1-\lambda\theta_1) - \lambda^2\theta_1^2\}/2(H-\lambda\theta_1), \tag{E1}$$

where $H \equiv \sqrt{4(1-\lambda)(1-\lambda\theta_1)+\lambda^2\theta_1^2}$.

(a) Differentiating (E1) with respect to θ_1, we obtain:

$$\partial p_1^*/\partial\theta_1 \;=\; Q/2(H-\lambda\theta_1)^2, \tag{E2}$$

where $Q \equiv 6\lambda(1-\lambda) - 4\lambda^2\theta_1(1-\lambda) + 2\lambda^3\theta_1^2 - 2\lambda(1-\lambda)H - 2\lambda^2\theta_1 H$
$- 2(1-\lambda)(1-\lambda\theta_1)\{\lambda^2\theta_1 - 2\lambda(1-\lambda)\}/H.$

We take the following steps to show that $\partial p_1^*/\partial\theta_1 > 0$.

(i) It follows immediately from (E2) that $\partial p_1^*/\partial\theta_1 > 0$ if and only if $W > 0$, where $W \equiv \{3(1-\lambda) - 2\lambda(1-\lambda)\theta_1 + \lambda^2\theta_1^2\}H - 2(1-\lambda)^2(1-\lambda\theta_1) - \lambda^2\theta_1^2(1-\lambda) - 5\lambda\theta_1(1-\lambda)$
$(1-\lambda\theta_1) - \lambda^3\theta_1^3$.

(ii) We have that $W > 0$ if and only if

$$U \;>\; (1-\lambda+\lambda\theta_1)H, \tag{E3}$$

where $U \equiv (\theta_1^2+5\theta_1)\lambda^2 - (5\theta_1+6)\lambda + 6$.

(iii) It is straightforward to obtain that $U > 0$, and that the right-hand side of (E3) is positive.

(iv) Squaring both sides of (E3) and simplifying yields:

$$4(1-\lambda)(1-\lambda\theta_1)\,G \;>\; 0, \tag{E4}$$

where $G \equiv 2\lambda^2\theta_1^2 + 8\lambda^2\theta_1 - \lambda^2 - 8\lambda\theta_1 - 7\lambda + 8$. Now, since $(1-\lambda)$ and $(1-\lambda\theta_1)$ in (E4) are positive, we have that $W > 0$ if and only if $G > 0$.

(v) Finally, we show that $G > 0$ as follows:

$$\begin{aligned} G \;&\equiv\; 2\lambda^2\theta_1^2 + 8\lambda^2\theta_1 - \lambda^2 - 8\lambda\theta_1 - 7\lambda + 8 \\ &=\; 2(\lambda\theta_1 + 2\lambda - 2)^2 + 9\lambda(1-\lambda) \;>\; 0. \end{aligned}$$

(b) Differentiating (E1) with respect to λ, we obtain:

$$\partial p_1^*/\partial\lambda \;=\; Z/2(H-\lambda\theta_1)^2, \tag{E5}$$

where $Z \equiv 6\theta_1(1-\lambda) - 4\lambda\theta_1^2(1-\lambda) + 2\lambda^2\theta_1^3 - 2(1-\lambda\theta_1)H + 2\lambda\theta_1(1-\lambda\theta_1) - 2(1-\lambda)\theta_1 H$ $- 2\lambda\theta_1^2 H - 2(1-\lambda)(1-\lambda\theta_1)\{\lambda\theta_1^2 - 2(1-\lambda\theta_1) - 2(1-\lambda)\theta_1\}/H$.

We take the following steps to show that $\partial p_1^*/\partial\lambda > 0$ when $\theta_1 > \theta_2$.

(i) It follows immediately from (E5) that $\partial p_1^*/\partial\lambda > 0$ if and only if $K > 0$, where $K \equiv \{3\theta_1(1-\lambda) - 2\lambda(1-\lambda)\theta_1^2 + \lambda^2\theta_1^3 + \lambda\theta_1(1-\lambda\theta_1)\}H - 2(1-\lambda)(1-\lambda\theta_1)^2$ $- \lambda^2\theta_1^2(1-\lambda\theta_1) - 2\theta_1(1-\lambda)^2(1-\lambda\theta_1) - \lambda^2\theta_1^3(1-\lambda) - 5\lambda\theta_1^2(1-\lambda)(1-\lambda\theta_1) - \lambda^3\theta_1^4$.

(ii) We have that $K > 0$ if and only if

$$[\{2(\lambda^2\theta_1^3 + \lambda^2\theta_1^2 - 2\lambda\theta_1^2 - 2\lambda\theta_1 + 3\theta_1)/(\lambda\theta_1^2 - 2\lambda\theta_1 + \theta_1 + 1)\} - \lambda\theta_1] > H. \quad \text{(E6)}$$

(iii) It is straightforward to obtain that the left-hand side of (E6) is positive.

(iv) Squaring both sides of (E6) and simplifying yields:

$$\{4(1-\lambda)(1-\lambda\theta_1)(2\theta_1 - 1)T/(\lambda\theta_1^2 - 2\lambda\theta_1 + \theta_1 + 1)^2\} > 0, \quad \text{(E7)}$$

where $T \equiv \theta_1^2(\theta_1 + 4)\lambda^2 - 4\theta_1(\theta_1 + 1)\lambda + 4\theta_1 + 1$. Now, since $(1-\lambda)$, $(1-\lambda\theta_1)$, and $(2\theta_1 - 1)$ in (E7) are positive, we have that $K > 0$ if and only if $T > 0$.

(v) Finally, we show that $T > 0$ as follows. First, it is straightforward to see that T achieves its global minimum at $\lambda = 2(\theta_1 + 1)/\theta_1(\theta_1 + 4)$. Second, the global minimum of T is positive: $V = 9\theta_1/(\theta_1 + 4) > 0$.

(*c*) Since player 1's (or delegate 1's) equilibrium winning probability p_1^* is independent of the players' valuation v for the prize, we obtain that $\partial p_1^*/\partial v = 0$.

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TABLE 1

The Subgame-Perfect Equilibrium Outcomes of the Full Game When $\theta_1 > \theta_2$

	First Stage	Second Stage	Third Stage	Probabilities of Winning	Expected Payoffs
Players	$\alpha_1^* < \alpha_2^*$			$p_1^* > p_2^*$	$\psi_1^* > \psi_2^*$
Delegates		Period 1 (Del. 1) Period 2 (Del. 2)	$x_1^* < x_2^*$	$p_1^* > p_2^*$	$\pi_1^* > \pi_2^*$

TABLE 2

Comparative Statics Results of the Outcomes of the Game When $\theta_1 > \theta_2$

	α_1^*	α_2^*	X^*	p_1^*
$\theta_1 \uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$
$\lambda \uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$
$v \uparrow$	—	—	$\uparrow$	—

TABLE 3

Comparative Statics Results of the Equilibrium Expected Payoffs

λ	$\partial\pi_1^*/\partial\theta_1$	$\partial\pi_2^*/\partial\theta_1$	$\partial\psi_1^*/\partial\theta_1$	$\partial\psi_2^*/\partial\theta_1$
0.1	+	−	+	−
0.3	+	−	+	−
0.5	+ −	−	+	−
0.7	−	−	+	−
0.9	−	−	+	−

θ_1	$\partial\pi_1^*/\partial\lambda$	$\partial\pi_2^*/\partial\lambda$	$\partial\psi_1^*/\partial\lambda$	$\partial\psi_2^*/\partial\lambda$
0.5	+ −	+ −	+	+
0.7	+ −	+ −	+	+ −
0.9	+ −	−	+	+ −

TABLE 4

Comparison of the Equilibrium Expected Payoffs

$\theta_1 \setminus \lambda$	0.1	0.3	0.5	0.7	0.9
0.5	ψ_2^*	ψ_2^*	ψ_2^*	ψ_2^*	ψ_2^*
0.7	ψ_2^*	ψ_2^*	ψ_2^*	ψ_2^*	ψ_2^*
0.9	ψ_2^*	ψ_2^*	ψ_2^*	ψ_2^E	ψ_2^E

$\theta_1 \setminus \lambda$	0.1	0.3	0.5	0.7	0.9
0.5	π_1^*	π_1^*	$\pi_1^* = \pi^E$	π_1^E	π_1^E
0.7	π_1^*	π_1^*	$\pi_1^* = \pi^E$	π_1^E	π_1^E
0.9	π_1^*	π_1^*	π_1^E	π_1^E	π_1^E

$\theta_1 \setminus \lambda$	0.1	0.3	0.5	0.7	0.9
0.5	π_2^*	π_2^*	$\pi_2^* = \pi_2^E$	π_2^E	π_2^E
0.7	π_2^*	π_2^E	π_2^E	π_2^E	π_2^E
0.9	π_2^E	π_2^E	π_2^E	π_2^E	π_2^E

Delegate 2

Delegate 1		*Period 1*	*Period 2*
	Period 1	π_1^S, π_2^S	π_1^{1L}, π_2^{1L}
	Period 2	π_1^{2L}, π_2^{2L}	π_1^S, π_2^S

Figure 1. The Strategic Interaction between the Delegates in the Second Stage

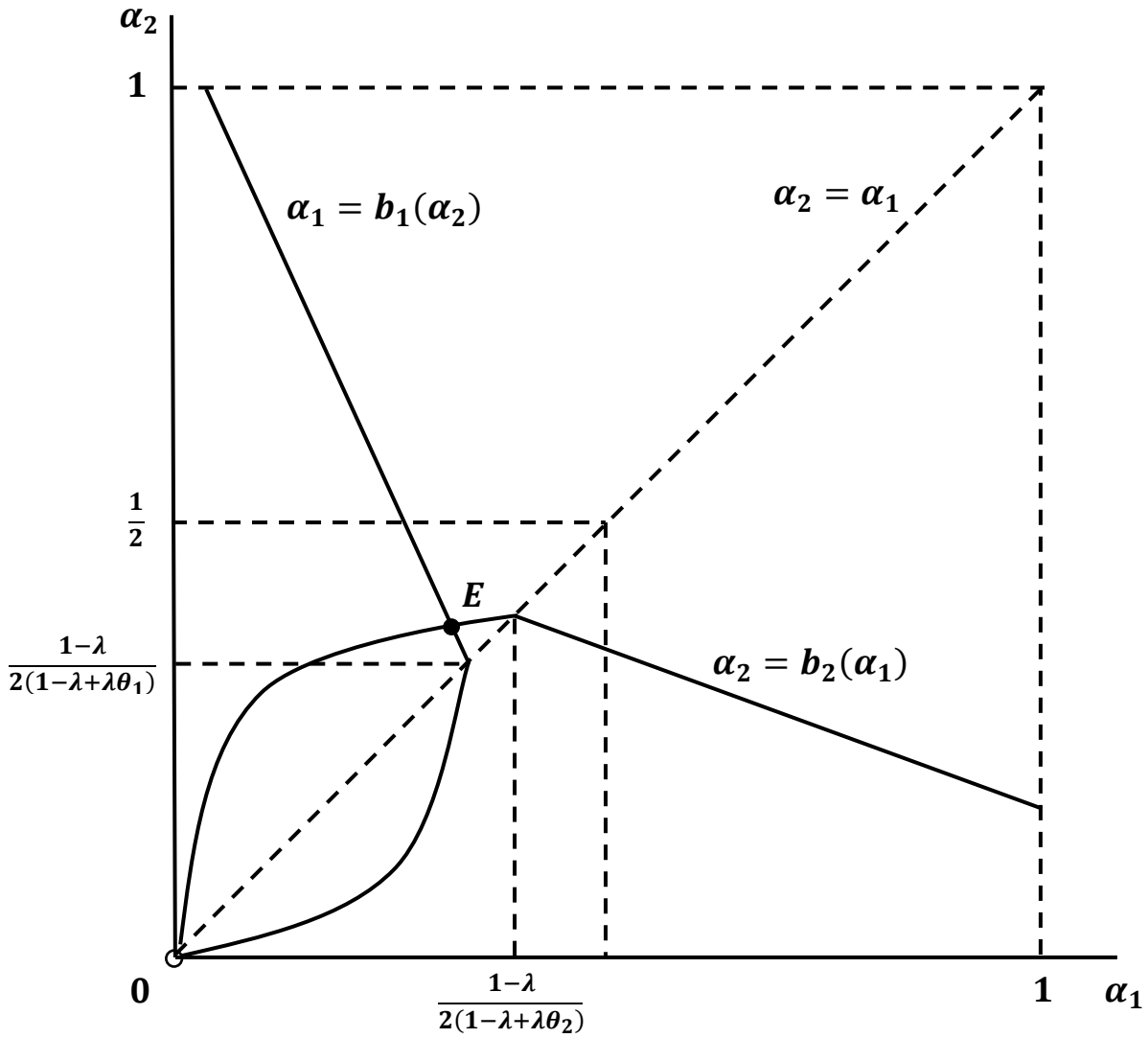

Figure 2. The Players' Reaction Functions and the Equilibrium Contracts