Managerial Delegation Contracts in Duopolies: One Disclosed and One Hidden

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March 2024

Abstract

First, we study a quantity-setting [price-setting] duopoly with managerial delegation in which only one firm's delegation contract is disclosed to the public before the managers choose their firms' output levels [prices]. In each duopoly model, we consider two distinct cases, cases I and II, which are distinguished by the timing of the owners' moves, and compare their outcomes. Interestingly, we show in each duopoly model that the outcomes in case I are the same, respectively, as those in case II. Second, we study two general games (between two parties) with imperfect and asymmetric information which are applicable to the duopolies with managerial delegation described above and other similar situations. In both games, each party has two sequential moves; only one party discloses its chosen first action to the public before the parties simultaneously choose their second actions. The two games differ in the timing of the parties' first moves. We develop solution techniques for these games.

Keywords: Managerial incentives; Delegation contracts; Duopolies; Hidden contract; Games with imperfect and asymmetric information; Solution techniques

JEL classification: D43, C72, L13, D82

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1. Introduction

Disclosure of information on important decisions within a firm, such as compensation contracts and research and development (R&D) investments, is commonly observed. Likewise, withholding of such information is commonly observed. These both imply that, in a single market, it may occur that some firms disclose information about their important decisions, while others do not. In this case, the firms engage in quantity or price competition under asymmetric information regarding their important decisions made.

Specifically, consider a duopoly in which each firm, consisting of an owner and a manager, makes two decisions sequentially.[1] First, the owner of each firm designs and offers a delegation contract to her manager. Then, after accepting the contracts for them, the managers compete in quantities [prices]. In particular, consider a situation in which one firm (or the disclosing firm) discloses its chosen delegation contract to the public – so that the contract is observed by the managers of both firms – before the managers compete, while the other firm (or the receiving firm) does not disclose its chosen contract, so that its contract is observed only by the manager of that firm.[2]

This paper has two objectives. The first is to study the quantity-setting [price-setting] duopoly with managerial delegation described above – precisely, to examine the equilibrium contracts, output levels [prices], and profits of the firms in that duopoly. In each duopoly model, we set up and analyze two distinct games which are distinguished by the timing of the owners' moves.

The second objective is to study two general games with imperfect and asymmetric information which are applicable to the duopolies with managerial delegation described above and other similar situations.[3] In Section 3, we set up games I and II, and develop the solution techniques for these games, one for each game. Both general games deal with two-party strategic interactions in which each party has two sequential moves. One party (or the *disclosing party*) discloses its chosen first action to the public before the parties simultaneously choose their second actions, while the other party (or the *receiving party*) does not disclose its chosen first action. The two games differ in the timing of the parties' first moves. In game I, we assume that the receiving party chooses its first action before observing the disclosing party's chosen first action, and thus both parties choose their first actions without observing the first action chosen by their rival party. In game II, we assume that the receiving party chooses its first action after observing the disclosing party's chosen first action.

Note that games I and II differ from standard two-stage games in which the first action chosen by each party is observed by the players in both parties before the parties choose their second actions (see, for example, Gibbons, 1992, pp. 71-82; Osborne, 2004, pp. 205-212). Note also that games I and II differ from games dealing with two-party strategic interactions in which each party has two sequential moves; the first action chosen by each party is observed only by the players in that party before the parties choose their second actions (see Baik and Lee, 2007; Baik and Kim, 2014).

In Sections 2 and 4, we study the quantity-setting duopoly with managerial delegation described above. We consider two distinct games, games A and B, using the two solution techniques developed in Section 3, and then compare the outcomes of game A with those of game B. In game A, which is an application of game I described above, we assume that the owner of the receiving firm writes a contract with her manager *before* observing the disclosing firm's chosen contract. In game B, which is an application of game II described above, we assume that the owner of the receiving firm writes a contract with her manager *after* observing the disclosing firm's chosen contract.

Interestingly, we show that the equilibrium contracts, output levels, and profits of the firms in game A are the same, respectively, as those in game B. We show also the following. In both games, the owner of the disclosing firm makes her manager more aggressive – through a strategic commitment to her firm's contract – in the output competition, as compared with the case where her manager is given an incentive to maximize her firm's profits. By contrast, the owner of the receiving firm, in both games, makes her manager a profit maximizer in the output competition.

In Section 5, we study the price-setting duopoly with managerial delegation described above, in which only one firm's chosen contract is disclosed before the managers of both firms choose their firms' prices. We consider two distinct games, games C and D, and then compare the outcomes of game C with those of game D. In game C, which is an application of game I described above, we assume that the owner of the receiving firm writes a contract with her manager *before* observing the disclosing firm's chosen contract. In game D, which is an application of game II described above, we assume that the owner of the receiving firm writes a contract with her manager *after* observing the disclosing firm's chosen contract.

We show that the equilibrium contracts, prices, and profits of the firms in game C are the same, respectively, as those in game D. We show also the following. In both games, the owner of the disclosing firm makes her manager less aggressive – through a strategic commitment to her firm's contract – in the price competition, as compared with the case where her manager is given an incentive to maximize her firm's profits. By contrast, the owner of the receiving firm, in both games, makes her manager a profit maximizer in the price competition.

This paper is related to papers which study duopolies with managerial delegation, in which the owner of each firm first writes a delegation contract with the firm's manager, and then the managers compete in quantities [prices]. Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), Theilen (2007), and Manasakis et al. (2010) study oligopolies with strategic managerial delegation in which both firms disclose their delegation contracts to the public – so that their chosen contracts are observed by the managers of both firms – before the managers compete in quantities [prices].

Baik and Lee (2020) extend Fershtman and Judd (1987) and Sklivas (1987) by incorporating firms' decisions on disclosing their contract information. More specifically, they study a quantity-setting [price-setting] duopoly with managerial delegation in which each firm has the option of disclosing or not to the public the contract information between its owner and manager. They show in both duopoly models that both firms disclose their contract information.

Kopel and Putz (2021a) consider downstream duopolies – each in a vertically related market – with managerial delegation in which each downstream firm has the option of disclosing or not to the public its managerial contract information. They show that, under quantity competition, a partial information-sharing equilibrium may occur in which one of the downstream firms keeps its contract information private; under price competition, both downstream firms disclose their managerial contract information.

Kopel and Putz (2021b) consider a Cournot-Bertrand duopoly with managerial delegation in which each firm has the option of disclosing or not to the public its managerial contract information. They show that both firms disclose their contract information if their products are sufficiently differentiated; however, either the Cournot firm or the Bertrand firm keeps its contract information private if their products are poorly differentiated.

The remainder of this paper proceeds as follows. In Section 2, we introduce a quantity-setting duopoly with managerial delegation in which only one firm's chosen contract is disclosed to the public before the managers of both firms simultaneously choose their firms' output levels, and set up two distinct games which are distinguished by the timing of the owners' moves. In Section 3, we set up two general games between two parties which are applicable to the duopolies with managerial delegation studied in the current paper, and develop solution techniques for the games. In Section 4, using the solution techniques developed in Section 3, we analyze the two games which we set up in Section 2, and then compare the outcomes of those two games. In Section 5, we study a price-setting duopoly with managerial delegation. Finally, Section 6 offers our conclusions.

2. Quantity-setting duopoly with delegation: one disclosed and one hidden contract

We consider a duopoly in which firms 1 and 2 sell homogeneous products and compete in quantities. Each firm consists of an owner and a manager. The owner of each firm hires the firm's manager, and writes a delegation contract with him that specifies how he will be rewarded. The manager of each firm chooses the firm's output level. Hence, in this duopoly, each firm has two sequential moves: The first move is the owner's decision on the firm's contract, and the second the manager's decision on the firm's output level.

We assume that firm 1 discloses to the public the (true) information about the chosen contract between owner 1 and manager 1 before the managers of both firms choose their firms' output levels. However, firm 2's chosen contract is observed only by owner 2 and manager 2; it is hidden from owner 1 and manager 1. We assume that the managers choose their firms' output levels simultaneously and independently.

The cost function of firm i, for $i = 1, 2$, is given by $c(y_i) = cy_i$ for all $y_i \in R_+$, where y_i denotes firm i's output level and c is a positive constant. The market price P is determined by

$$P = \begin{cases} a - bY & \text{for } 0 \leq Y \leq a/b \\ 0 & \text{for } Y > a/b, \end{cases}$$

where $Y = y_1 + y_2$ and a and b are positive constants. We assume that $a > c$.

Owner i, for $i = 1, 2$, uses a compensation scheme in which manager i's compensation depends on his performance measured by a linear combination of firm i's profits and its sales. More specifically, manager i is given an incentive to maximize

$$H_i = \beta_i \psi_i + (1 - \beta_i) S_i, \tag{1}$$

where ψ_i denotes firm i's profits (without subtracting manager i's compensation), S_i denotes firm i's sales, and β_i is a parameter whose value is chosen when owner i writes a contract with manager i.[4] We make no restrictions on the parameter β_i.

We assume that owner i's objective is to maximize firm i's profits net of manager i's compensation. Because any delegation contract provides manager i with equilibrium compensation exactly equal to his reservation wage (see footnote 4), this assumption is mathematically equivalent to assuming that owner i seeks to maximize firm i's profits

$$\psi_i = (a - bY)\, y_i - cy_i.$$

By contrast, given the incentive structure (1), manager *i* seeks to maximize

$$H_i \;=\; (a - bY)\, y_i \;-\; \beta_i c y_i. \tag{2}$$

We consider the following two cases separately, which are distinguished by the timing of the owners' moves. In the first case (or case I), owner 2 writes a contract with manager 2 *before* observing firm 1's chosen contract. This implies that, in this case, each owner writes a contract with her manager without observing the contract chosen by the rival firm. In the second case (or case II), owner 2 writes a contract with manager 2 *after* observing firm 1's chosen contract.

Case I is modeled by the following game, called game A. First, the owner of each firm writes a contract with her manager – concisely, owner *i* (or firm *i*) chooses a value of β_i – without observing the contract chosen by the other firm. Next, owner 1 (or firm 1) discloses the chosen contract between owner 1 and manager 1 to the public, but owner 2 (or firm 2) does not disclose to the public the chosen contract between owner 2 and manager 2. As a result, manager 1 knows only the value of β_1, whereas manager 2 knows the values of both parameters, β_1 and β_2. Finally, the managers choose their firms' output levels simultaneously and independently. (At the end of the game, the owner of each firm observes the firm's profits and its sales, and pays compensation to her manager according to the firm's chosen contract.)

Case II is modeled by the following game, called game B. First, owner 1 writes a contract with manager 1 and discloses it to the public. Next, after observing the value of β_1, owner 2 writes a contract with manager 2, but does not disclose it to the public. As a result, manager 1 knows only the value of β_1, whereas manager 2 knows the values of both parameters, β_1 and β_2. Finally, the managers choose their firms' output levels simultaneously and independently.

We assume that all of the above is common knowledge among the owners and managers.

3. Solution techniques: general games between two parties, each with two sequential moves, in which only one of the first actions chosen by the parties is publicly observed

Consider a two-party game in which parties 1 and 2 each have two sequential moves. Party 1 discloses its chosen first action to the public before the parties choose their second actions. However, the first action chosen by party 2 is observed only by the players in that party; it is hidden from the players in party 1. The parties choose their second actions simultaneously.

For expositional convenience, let leader i, for $i = 1, 2$, represent the player (or subset of players) in party i who chooses party i's first action; let follower i represent the player (or subset of players) in party i who chooses party i's second action. Note that leader i and follower i may be the same player (or subset of players).

Let a_i, for $i = 1, 2$, represent leader i's action from A_i, where A_i denotes the set of all actions available to leader i. Let x_i represent follower i's action from X_i, where X_i denotes the set of all actions available to follower i.

Let π_i, for $i = 1, 2$, represent the (expected) payoff for leader i. The payoff function for leader i is given by $\pi_i = \pi_i(a_i, x_1, x_2)$. Let f_i represent the (expected) payoff for follower i. The payoff function for follower i is given by $f_i = f_i(a_i, x_1, x_2)$. Note that, since a_j, for $j = 1, 2$ with $j \neq i$, is absent in the payoff function for leader i and that for follower i, the payoffs to leader i and follower i do not depend directly on the action a_j chosen by leader j.[5]

3.1. Two games

We consider the following two games: game I in which party 2 chooses its first action *before* observing the first action chosen by party 1, and game II in which party 2 chooses its first action *after* observing the first action chosen by party 1.

3.1.1. Game I

In this game, we assume that party 2 chooses its first action before observing the first action chosen by party 1, and thus both parties choose their first actions without observing the first action chosen by their rival party. This assumption may involve assuming that party 2 commits to its chosen first action and cannot change it after observing the first action chosen by party 1.

We formally consider the following game. First, leaders 1 and 2 choose actions $a_1 \in A_1$ and $a_2 \in A_2$, respectively. Leader i chooses her action without observing leader j's chosen action. Next, leader 1 discloses her chosen action to the public, but leader 2 does not disclose her chosen action to the public. As a result, follower 1 knows only the action a_1 chosen by leader 1, whereas follower 2 knows both the action a_1 chosen by leader 1 and the action a_2 chosen by leader 2. Finally, followers 1 and 2 simultaneously choose actions $x_1 \in X_1$ and $x_2 \in X_2$, respectively.

3.1.2. Game II

In this game, we assume that party 2 chooses its first action after observing the first action chosen by party 1. This assumption may reflect the following possibilities. The first possibility is that the two parties have an opportunity to announce (and commit to) when to choose their first actions. In this case, if party 2 announces it will delay choosing its first action until after party 1's chosen first action is observed, then the assumption is satisfied. The second possibility is that party 2 can change its first action – chosen before observing the first action chosen by party 1 – after observing the first action chosen by party 1, and party 1 knows it.

We formally consider the following game. First, leader 1 chooses an action $a_1 \in A_1$, and discloses it to the public. Next, after observing the action a_1 chosen by leader 1, leader 2 chooses an action $a_2 \in A_2$, but does not disclose it to the public. As a result, follower 1 knows only the action a_1 chosen by leader 1, whereas follower 2 knows both the action a_1 chosen by leader 1 and the action a_2 chosen by leader 2. Finally, followers 1 and 2 simultaneously choose actions $x_1 \in X_1$ and $x_2 \in X_2$, respectively.

3.2. Equilibrium actions of the games

3.2.1. Game I

To solve this game, or to obtain the equilibrium actions of game I, we take the following steps. First, we consider the followers' decisions in choosing their actions. Then, we consider the leaders' decisions in choosing their actions. Finally, using the results from the previous two steps, we obtain the equilibrium actions of the leaders and the followers.

The followers' decisions

We begin by considering follower i's maximization problem, for $i = 1, 2$. After observing the action a_i chosen by leader i, follower i seeks to maximize his payoff over his action x_i, taking follower j's action x_j as given:

$$\max_{x_i \in X_i} \quad f_i(a_i, x_1, x_2). \tag{3}$$

We assume that, for each $a_i \in A_i$ and $x_j \in X_j$, maximization problem (3) has a unique interior solution, which is denoted by $x_i(x_j; a_i)$.

Now that follower i's reaction function shows his best response $x_i(x_j; a_i)$ to every possible action that follower j might choose, we denote it by

$$x_i \;=\; x_i(x_j; a_i). \tag{4}$$

Next, our analysis of the followers' decisions goes further to obtain the followers' strategies. Follower 1 knows his own reaction function $x_1 = x_1(x_2; a_1)$ for the action a_1 chosen by leader 1. After forming his belief a_2^{o} about leader 2's chosen action (see footnote 6), follower 1 also knows follower 2's reaction function $x_2 = x_2(x_1; a_2^{\mathrm{o}})$. Note that this comes from follower 2's reaction function in (4) by substituting a_2^{o} for a_2. Using these two reaction functions, we obtain follower 1's action at their intersection:

$$x_1(a_1;\, a_2^{\mathrm{o}}).$$

Then, given his belief a_2^{o}, follower 1's strategy, which shows his action given every possible action that leader 1 might choose, is given by:

$$x_1 \;=\; x_1(a_1;\, a_2^{\mathrm{o}}). \tag{5}$$

By contrast, follower 2 knows both the action a_1 chosen by leader 1 and the action a_2 chosen by leader 2, and thus knows follower 1's reaction function $x_1 = x_1(x_2;\, a_1)$ and his own reaction function $x_2 = x_2(x_1;\, a_2)$. Using these two reaction functions, we obtain follower 2's action at their intersection:

$$x_2(a_1, a_2).$$

Then, follower 2's equilibrium strategy, which shows his action given every possible pair of actions that leader 1 and leader 2 might choose, is given by:

$$x_2 \;=\; x_2(a_1, a_2). \tag{6}$$

The leaders' decisions

We consider leader i's maximization problem, for $i = 1, 2$. Given leader j's action a_j, leader i seeks to maximize her payoff over her action a_i, taking into account the followers' strategies:

$$\max_{a_i \,\in\, A_i} \quad \pi_i(a_i, x_1(a_1;\, a_2^{\mathrm{o}}), x_2(a_1, a_2)). \tag{7}$$

We assume that, for each $a_j \in A_j$, maximization problem (7) has a unique interior solution, which is denoted by $a_i(a_j;\, a_2^{\mathrm{o}})$.

Then, leader i's reaction function is given by:

$$a_i \;=\; a_i(a_j;\, a_2^{\mathrm{o}}). \tag{8}$$

The equilibrium actions

Let the superscript * indicate the equilibrium actions of game I. First, using the leaders' reaction functions in (8) and the condition that $a_2^{\mathrm{o}} = a_2$, we obtain the equilibrium actions, a_1^* and a_2^*, of the leaders.[6] That is, the equilibrium actions of the leaders satisfy $a_1^* = a_1(a_2^*; a_2^*)$ and $a_2^* = a_2(a_1^*; a_2^*)$ simultaneously.

Then, substituting $a_2^{\mathrm{o}} = a_2^*$, $a_1 = a_1^*$, and $a_2 = a_2^*$ into follower 1's strategy (5) and follower 2's strategy (6), we obtain the equilibrium actions, x_1^* and x_2^*, of the followers: $x_1^* = x_1(a_1^*; a_2^*)$ and $x_2^* = x_2(a_1^*, a_2^*)$.

3.2.2. Game II

We view this game as the following two-stage game.[7] In the first stage, leader 1 chooses her action, and discloses it to the public. In the second stage, after observing leader 1's action, leader 2 chooses her action, but does not disclose it to the public; then followers 1 and 2 choose their actions simultaneously and independently.

To solve this two-stage game, or to obtain the equilibrium actions of this two-stage game, we take the following steps. First, we analyze the subgames which start at the second stage of the game. Then, we analyze the first stage in which leader 1 chooses her action. Finally, we obtain the equilibrium actions of the two-stage game – and thus those of game II – using the findings in the previous two steps.

Analyzing the subgames starting at the second stage

In the second stage, leader 2 and the followers observe leader 1's chosen action before they choose their actions. Analyzing the second stage, we obtain their equilibrium strategies, each of which shows her or his action given every possible action that leader 1 might choose.[8]

We begin by considering follower i's maximization problem, for $i = 1, 2$. After observing the action a_i chosen by leader i, follower i seeks to maximize his payoff over his action x_i, taking follower j's action x_j as given:

$$\max_{x_i \in X_i} \quad f_i(a_i, x_1, x_2).$$

From his maximization problem, we obtain follower i's reaction function:

$$x_i \ = \ x_i(x_j;\, a_i).$$

Next, our analysis of the followers' decisions goes further. Follower 1 knows his own reaction function $x_1 = x_1(x_2;\, a_1)$ for the action a_1 chosen by leader 1. Given his belief a_2^{o} about leader 2's chosen action, follower 1 also knows follower 2's reaction function $x_2 = x_2(x_1;\, a_2^{\mathrm{o}})$. Using these two reaction functions, we obtain follower 1's action $x_1(a_1;\, a_2^{\mathrm{o}})$ at their intersection. Then, given his belief a_2^{o}, follower 1's strategy is given by:

$$x_1 \ = \ x_1(a_1;\, a_2^{\mathrm{o}}). \tag{9}$$

By contrast, follower 2 knows both the action a_1 chosen by leader 1 and the action a_2 chosen by leader 2, and thus knows follower 1's reaction function $x_1 = x_1(x_2;\, a_1)$ and his own reaction function $x_2 = x_2(x_1;\, a_2)$. Using these two reaction functions, we obtain follower 2's action $x_2(a_1, a_2)$ at their intersection. Then, follower 2's strategy is given by:

$$x_2 \ = \ x_2(a_1, a_2). \tag{10}$$

Next, we consider leader 2's maximization problem. Knowing leader 1's chosen action a_1, leader 2 seeks to maximize her payoff over her action a_2, taking into account the followers' strategies:

$$\max_{a_2 \in A_2} \quad \pi_2(a_2, x_1(a_1;\, a_2^{\mathrm{o}}), x_2(a_1, a_2)).$$

From her maximization problem, we obtain leader 2's strategy:

$$a_2 \ = \ a_2(a_1;\, a_2^{\mathrm{o}}). \tag{11}$$

Now we obtain the equilibrium strategies of leader 2 and the followers. First, using leader 2's strategy (11) and the condition that $a_2^{o} = a_2$, we obtain leader 2's equilibrium strategy:

$$a_2 = a_2(a_1).$$

Then, substituting $a_2^{o} = a_2$ and $a_2 = a_2(a_1)$ into follower 1's strategy (9) and follower 2's strategy (10), we obtain the equilibrium strategies of the followers: $x_1 = x_1(a_1)$ and $x_2 = x_2(a_1)$.

Analyzing the first stage

Consider the first stage in which leader 1 chooses her action. Leader 1 seeks to maximize her payoff over her action a_1, taking into account the equilibrium strategies of leader 2 and the followers:

$$\max_{a_1 \in A_1} \quad \pi_1(a_1, x_1(a_1), x_2(a_1)). \tag{12}$$

We assume that maximization problem (12) has a unique interior solution, which is denoted by a_1^{**}. Note that a_1^{**} is the equilibrium action of leader 1.

The equilibrium actions

Substituting $a_1 = a_1^{**}$ into leader 2's equilibrium strategy $a_2 = a_2(a_1)$, we obtain the equilibrium action a_2^{**} of leader 2: $a_2^{**} = a_2(a_1^{**})$. Substituting $a_1 = a_1^{**}$ into follower 1's equilibrium strategy $x_1 = x_1(a_1)$ and follower 2's equilibrium strategy $x_2 = x_2(a_1)$, we obtain the equilibrium actions, x_1^{**} and x_2^{**}, of the followers: $x_1^{**} = x_1(a_1^{**})$ and $x_2^{**} = x_2(a_1^{**})$.

4. Equilibrium contracts, output levels, and profits

In this section, we first obtain the equilibrium contracts, output levels, and profits of the firms in games A and B which we set up in Section 2. Then, we compare the outcomes of game A with those of game B.

4.1. Game A

Game A is an application of game I in Section 3. Hence, to obtain the equilibrium contracts, output levels, and profits of the firms in game A, we use the solution technique for game I which we proposed in Section 3.2.1. We relegate the analysis to Appendix A.

Lemma 1 summarizes the outcomes of game A.

Lemma 1. (a) *In the equilibrium of game A, owner* 1 *chooses* $\beta_1^* = (5c - a)/4c$, *owner* 2 *chooses* $\beta_2^* = 1$, *manager* 1 *chooses* $y_1^* = (a - c)/2b$, *and manager* 2 *chooses* $y_2^* = (a - c)/4b$. (b) *The equilibrium profits of firm* 1 *and those of firm* 2 *are* $\psi_1^* = (a - c)^2/8b$ *and* $\psi_2^* = (a - c)^2/16b$, *respectively.*

In equilibrium, owner 1 (or firm 1) chooses β_1^* that is *less than unity*, without observing firm 2's contract chosen by owner 2, and thereby makes manager 1 more aggressive, as compared with the case where $\beta_1 = 1$, in the output competition that follows.[9] By contrast, owner 2 (or firm 2) chooses $\beta_2^* = 1$, without observing firm 1's contract chosen by owner 1, and thereby makes manager 2 a *profit maximizer* in the subsequent output competition. Then, knowing β_1^* and forming his belief β_2^* about firm 2's contract chosen by owner 2 – thus, his belief is consistent with owner 2's equilibrium action – manager 1 chooses y_1^* in the output competition. Knowing both β_1^* and β_2^*, manager 2 chooses y_2^* in the output competition.

To understand Lemma 1 more deeply, consider first owner 2's decision on firm 2's contract. According to owner 2's reaction function (A7) in Appendix A, her decision on the value of β_2 is independent of β_1; however, it depends on manager 1's belief about firm 2's contract chosen by owner 2. Under the equilibrium condition that manager 1's belief be consistent with owner 2's chosen contract – or, equivalently, knowing that manager 1's belief will be correct – owner 2 chooses $\beta_2^* = 1$ without taking into account owner 1's decision on the value of β_1. Next, consider owner 1's decision on firm 1's contract. According to owner 1's reaction function (A5), her decision on the value of β_1 depends on β_2 as well as manager 1's belief about firm 2's contract chosen by owner 2. Knowing that manager 1's belief will be correct, owner 1 chooses β_1^*, taking $\beta_2^* = 1$ as given.

Intuitively, owner 1 chooses β_1^* that is less than unity because, due to its observability, such a strategic commitment can change in her favor the managers' behavior in the output competition, as compared with the case where $\beta_1 = 1$. By contrast, owner 2 chooses $\beta_2^* = 1$, which makes manager 2 a profit maximizer in the output competition, because she does not disclose her chosen contract and thus does not benefit from choosing a value of β_2 other than unity.

4.2. Game B

Game B is an application of game II in Section 3. Hence, to obtain the equilibrium contracts, output levels, and profits of the firms in game B, we use the solution technique for game II which we introduced in Section 3.2.2. We relegate the analysis to Appendix B.

Lemma 2 summarizes the outcomes of game B.

Lemma 2. (*a*) *In the equilibrium of game B, owner* 1 *chooses* $\beta_1^{**} = (5c - a)/4c$, *owner* 2 *chooses* $\beta_2^{**} = 1$, *manager* 1 *chooses* $y_1^{**} = (a - c)/2b$, *and manager* 2 *chooses* $y_2^{**} = (a - c)/4b$. (*b*) *The equilibrium profits of firm* 1 *and those of firm* 2 *are* $\psi_1^{**} = (a - c)^2/8b$ *and* $\psi_2^{**} = (a - c)^2/16b$, *respectively.*

In equilibrium, owner 1 chooses β_1^{**} that is less than unity, and thereby makes manager 1 more aggressive, as compared with the case where $\beta_1 = 1$, in the output competition. Next, after observing firm 1's contract chosen by owner 1, owner 2 chooses $\beta_2^{**} = 1$, and thereby makes manager 2 a profit maximizer in the output competition. Then, knowing β_1^{**} and forming his belief β_2^{**} about firm 2's contract chosen by owner 2, manager 1 chooses y_1^{**} in the output competition. Knowing both β_1^{**} and β_2^{**}, manager 2 chooses y_2^{**} in the output competition.

To understand Lemma 2 more clearly, consider first owner 2's decision on firm 2's contract. According to owner 2's strategy (B5) in Appendix B, her decision on the value of β_2 in the second stage is independent of β_1; however, it depends on manager 1's belief about firm 2's contract chosen by owner 2. Knowing that manager 1's belief will be correct, owner 2 chooses $\beta_2(\beta_1) = 1$ as her equilibrium strategy. This implies that, in equilibrium, owner 2 chooses $\beta_2^{**} = 1$ regardless of owner 1's decision on the value of β_1. Next, consider owner 1's decision on firm 1's contract in the first stage. Having perfect foresight about the managers' equilibrium strategies, $y_1(\beta_1)$ and $y_2(\beta_1)$, for any value of β_1, owner 1 chooses β_1^{**}.

Intuitively, owner 1 enjoys a first-mover advantage by disclosing firm 1's chosen contract before owner 2 chooses firm 2's contract. She chooses β_1^{**} that is less than unity, which changes in her favor the managers' behavior in the output competition, as compared with the case where $\beta_1 = 1$. By contrast, owner 2 chooses firm 2's contract after observing firm 1's chosen contract, and does not disclose her chosen contract. In this case, yielding strategic leadership to owner 1, owner 2 chooses $\beta_2^{*} = 1$, and thereby makes manager 2 a profit maximizer and avoids stiff output competition.

4.3. Comparison of the outcomes of games A and B

Now, using Lemmas 1 and 2, we compare the outcomes of game A with those of game B. Proposition 1 reports the comparison results. Recall that the superscripts * and ** in Proposition 1 indicate the outcomes of game A and those of game B, respectively.

Proposition 1. *For the quantity-setting firms, we obtain:* (*i*) $\beta_1^{*} = \beta_1^{**} < 1$ *and* $\beta_2^{*} = \beta_2^{**} = 1$, (*ii*) $y_1^{*} = y_1^{**}$ *and* $y_2^{*} = y_2^{**}$, *and* (*iii*) $\psi_1^{*} = \psi_1^{**}$ *and* $\psi_2^{*} = \psi_2^{**}$.

Proposition 1 says that each outcome in game A is the same as the corresponding outcome in game B. We first explain the result in part (*ii*): $y_i^{*} = y_i^{**}$ for $i = 1, 2$. In games A and B, each manager has the same information structure regarding the firms' chosen contracts when choosing his firm's output level. Accordingly, in both games, each manager plays the same strategy (see (A2) and (B2) for manager 1 and (A3) and (B3) for manager 2). Further, in equilibrium, since the owners each choose the same contract in both games, each manager chooses the same output level in both games.

Next, we explain the result that $\beta_2^* = \beta_2^{**} = 1$. In both games, owner 2's decision on the value of β_2 is independent of β_1 (see her reaction function (A7) and her strategy (B5)); furthermore, (A7) and (B5) are the same function of manager 1's belief β_2^o about firm 2's chosen contract. This, together with the equilibrium condition that that $\beta_2^o = \beta_2$, leads to the result that $\beta_2^* = \beta_2^{**} = 1$. Intuitively, in both games, owner 2 does not benefit from a strategic commitment to her firm's contract because she does not disclose her chosen contract. Accordingly, in both games, owner 2 chooses a value of β_2 which is equal to unity, which makes manager 2 a profit maximizer in the output competition.

Finally, we explain the result that $\beta_1^* = \beta_1^{**}$. Recall that, in both games, owner 2's decision on the value of β_2 is independent of β_1, and that $\beta_2^* = \beta_2^{**} = 1$. Given this, the best response β_1^* of owner 1 to β_2^* in game A and the committed contract β_1^{**} of owner 1 as the leader in game B are the same.

5. Price-setting duopoly with delegation: one disclosed and one hidden contract

We consider a duopoly in which firms 1 and 2 sell differentiated products and compete in prices. Each firm, consisting of an owner and a manager, has two sequential moves: The first move is the owner's decision on the firm's contract, and the second the manager's decision on the firm's price.

We assume that firm 1 discloses to the public the information about the chosen contract between owner 1 and manager 1 before the managers of both firms choose their firms' prices. However, firm 2's contract is observed only by owner 2 and manager 2; it is hidden from owner 1 and manager 1. We assume that the managers choose their firms' prices simultaneously and independently.

The market demand function facing firm i, for $i = 1, 2$, is given by

$$q_i = d - p_i + kp_j,$$

where d and k are positive constants, p_i denotes firm i's price, p_j denotes firm j's price, and q_i denotes the quantity of firm i's product demanded. We assume that $0 < k < 1$. The cost function of firm i is given by $c(q_i) = cq_i$ for all $q_i \in R_+$, where q_i denotes firm i's output level and c is a positive constant. We assume that $0 < c < d/(1-k)$.

As in Section 2, manager i, for $i = 1, 2$, is given an incentive to maximize

$$H_i = \beta_i \psi_i + (1 - \beta_i) S_i. \tag{13}$$

We assume that owner i's objective is to maximize firm i's profits net of manager i's compensation. Because any delegation contract provides manager i with equilibrium compensation exactly equal to his reservation wage (see footnote 4), this assumption is mathematically equivalent to assuming that owner i seeks to maximize firm i's profits

$$\psi_i = (p_i - c)\,(d - p_i + kp_j).$$

By contrast, given the incentive structure (13), manager i seeks to maximize

$$H_i = p_i(d - p_i + kp_j) - \beta_i c(d - p_i + kp_j). \tag{14}$$

As in Section 2, we set up two games. The first game, called game C, has the following structure and timing. First, the owner of each firm writes a contract with her manager – concisely, owner i (or firm i) chooses a value of β_i – without observing the contract chosen by the other firm. Next, owner 1 (or firm 1) discloses the chosen contract between owner 1 and manager 1 to the public, but owner 2 (or firm 2) does not disclose to the public the chosen contract between owner 2 and manager 2. As a result, manager 1 knows only the value of β_1, whereas manager 2 knows the values of both parameters, β_1 and β_2. Finally, the managers choose their firms' prices simultaneously and independently.

The second game, called game D, has the following structure and timing. First, owner 1 writes a contract with manager 1 and discloses it to the public. Next, after observing the value of β_1, owner 2 writes a contract with manager 2, but does not disclose it to the public. As a result, manager 1 knows only the value of β_1, whereas manager 2 knows the values of both parameters, β_1 and β_2. Finally, the managers choose their firms' prices simultaneously and independently.

We assume that all of the above is common knowledge among the owners and managers.

5.1. Game C

Game C is an application of game I in Section 3. Hence, to obtain the equilibrium contracts, prices, and profits of the firms in game C, we use the solution technique for game I which we proposed in Section 3.2.1. We relegate the analysis to Appendix C.

Lemma 3 reports the outcomes of game C.

Lemma 3. (*a*) *In the equilibrium of game C, owner* 1 *chooses* $\beta_1^* = \{8c + 2(d - 3c)k^2 + (c + d)k^3 + ck^4\}/4c(2 - k^2)$, *owner* 2 *chooses* $\beta_2^* = 1$, *manager* 1 *chooses* $p_1^* = \{2(c + d) + (c + d)k - ck^2\}/2(2 - k^2)$, *and manager* 2 *chooses* $p_2^* = \{4(c + d) + 2(c + d)k - (c + d)k^2 - ck^3\}/4(2 - k^2)$. (*b*) *The equilibrium profits of firm* 1 *and those of firm* 2 *are* $\psi_1^* = (2 + k)^2 (d - c + ck)^2/8(2 - k^2)$ *and* $\psi_2^* = (4 + 2k - k^2)^2(d - c + ck)^2/16(2 - k^2)^2$, *respectively.*

In equilibrium, owner 1 (or firm 1) chooses β_1^* that is *greater than unity*, without observing firm 2's contract chosen by owner 2, and thereby makes manager 1 less aggressive, as compared with the case where $\beta_1 = 1$, in the price competition that follows.[10] By contrast, owner 2 (or firm 2) chooses $\beta_2^* = 1$, without observing firm 1's contract chosen by owner 1, and thereby makes manager 2 a *profit maximizer* in the subsequent price competition. Then, knowing β_1^* and forming his belief β_2^* about firm 2's contract chosen by owner 2, manager 1 chooses p_1^* in the price competition. Knowing both β_1^* and β_2^*, manager 2 chooses p_2^* in the price competition.

The further explanation and intuition for Lemma 3 can be made similarly to those for Lemma 1, and therefore are omitted.

5.2. Game D

Game D is an application of game II in Section 3. Hence, to obtain the equilibrium contracts, prices, and profits of the firms in game D, we use the solution technique for game II which we introduced in Section 3.2.2. We relegate the analysis to Appendix D.

Lemma 4 reports the outcomes of game D.

Lemma 4. (*a*) *In the equilibrium of game D, owner* 1 *chooses* $\beta_1^{**} = \{8c + 2(d - 3c)k^2 + (c + d)k^3 + ck^4\}/4c(2 - k^2)$, *owner* 2 *chooses* $\beta_2^{**} = 1$, *manager* 1 *chooses* $p_1^{**} = \{2(c + d) + (c + d)k - ck^2\}/2(2 - k^2)$, *and manager* 2 *chooses* $p_2^{**} = \{4(c + d) + 2(c + d)k - (c + d)k^2 - ck^3\}/4(2 - k^2)$. (*b*) *The equilibrium profits of firm* 1 *and those of firm* 2 *are* $\psi_1^{**} = (2 + k)^2 (d - c + ck)^2/8(2 - k^2)$ *and* $\psi_2^{**} = (4 + 2k - k^2)^2(d - c + ck)^2/16(2 - k^2)^2$, *respectively.*

In equilibrium, owner 1 chooses β_1^{**} that is greater than unity, and thereby makes manager 1 less aggressive, as compared with the case where $\beta_1 = 1$, in the price competition. Next, after observing firm 1's contract chosen by owner 1, owner 2 chooses $\beta_2^{**} = 1$, and thereby makes manager 2 a profit maximizer in the price competition. Then, knowing β_1^{**} and forming his belief β_2^{**} about firm 2's contract chosen by owner 2, manager 1 chooses p_1^{**} in the price competition. Knowing both β_1^{**} and β_2^{**}, manager 2 chooses p_2^{**} in the price competition.

The further explanation and intuition for Lemma 4 can be made similarly to those for Lemma 2, and therefore are omitted.

5.3. Comparison of the outcomes of games C and D

Now, using Lemmas 3 and 4, we compare the outcomes of game C with those of game D. Proposition 2 reports the comparison results. Recall that the superscripts * and ** in Proposition 2 indicate the outcomes of game C and those of game D, respectively.

Proposition 2. *For the price-setting firms, we obtain:* (*i*) $\beta_1^* = \beta_1^{**} > 1$ and $\beta_2^* = \beta_2^{**} = 1$, (*ii*) $p_1^* = p_1^{**}$ *and* $p_2^* = p_2^{**}$, *and* (*iii*) $\psi_1^* = \psi_1^{**}$ *and* $\psi_2^* = \psi_2^{**}$.

Proposition 2 says that each outcome in game C is the same as the corresponding outcome in game D. The explanations for Proposition 2 can be made similarly to those for Proposition 1, and therefore are omitted.

6. Conclusions

We have studied the quantity-setting [price-setting] duopoly with managerial delegation in which the owner of each firm first writes a delegation contract with the firm's manager, and then the managers compete in quantities [prices]. In both duopoly models, firm 1 discloses its chosen contract to the public before the managers compete, while firm 2 does not disclose its chosen contract. In each duopoly model, we have considered two distinct cases, cases I and II, and compared the outcomes from case I with those from case II. In case I, owner 2 writes a contract with manager 2 before observing firm 1's chosen contract. In case II, owner 2 writes a contract with manager 2 after observing firm 1's chosen contract.

We have shown in the quantity-setting [price-setting] duopoly model that the equilibrium contracts, output levels [prices], and profits of the firms in case I are the same, respectively, as those in case II. This interesting result comes from the fact that, in both cases, owner 2 makes manager 2 a profit maximizer in the output [price] competition by choosing a value of β_2 which is equal to unity. Owner 2 does so because she does not disclose her chosen contract and thus does not benefit from a strategic commitment to her firm's contract.

In Section 3, we have studied two general games (between two parties) with imperfect and asymmetric information which are applicable to the duopolies with managerial delegation studied in the current paper and other similar situations. We have set up games I and II, and developed the solution techniques for these games, one for each game. In both games, each party has two sequential moves. Party 1 discloses its chosen first action to the public before the parties simultaneously choose their second actions, while party 2 does not disclose its chosen first action. The two games differ in the timing of the parties' first moves. In game I, party 2 chooses its first action before observing party 1's chosen first action. In game II, party 2 chooses its first action after observing party 1's chosen first action.

We may apply these solution techniques to solve two distinct games which model a two-player contest with bilateral delegation but differ in the timing of the players' decisions on their delegation contracts (see, for example, Baik and Kim, 2014). In this case, it would be interesting to compare the outcomes of one game with those of the other game. We leave them for future research.

Footnotes

1. Duopolies with managerial delegation have been studied by many researchers. See, for example, Fershtman and Judd (1987), Sklivas (1987), Theilen (2007), and Baik and Lee (2020).

2. For the analysis of duopolies with managerial delegation in which both firms disclose their delegation contracts to the public or neither firm discloses its delegation contract, see Baik and Lee (2020).

3. For an example of similar situations, consider a duopoly in which two firms first make R&D investments to reduce their production costs, and then they produce their products at the resulting reduced costs, and compete in quantities [prices]. In particular, consider a situation in which only one firm discloses the amount of its R&D investment before the firms compete in quantities [prices] (see Baik and Kim, 2020).

For another example, consider a rent-seeking contest between two groups in which the players in each group first decide how to share the rent among themselves if they win it, then the players in both groups expend their effort simultaneously and independently to win the rent. In particular, consider a situation in which only one group discloses its sharing rule before the players expend effort (see Baik and Lee, 2012).

4. We assume that owner i, for $i = 1, 2$, uses a compensation scheme in which manager i is paid $F_i + \lambda_i H_i$, where F_i and λ_i are constants with $F_i \geq 0$ and $\lambda_i > 0$. Under this compensation scheme, any delegation contract designed by owner i provides manager i with equilibrium compensation exactly equal to his reservation wage. For detailed explanations of the compensation scheme, see, for example, Fershtman and Judd (1987) and Baik and Lee (2020).

5. Throughout the paper, when we use i and j at the same time, we mean that $i \neq j$.

6. We impose the equilibrium condition or requirement that follower 1's belief a_2^{o} about leader 2's chosen action be consistent with leader 2's chosen action a_2: $a_2^{o} = a_2$. Accordingly, in equilibrium, follower 1's belief about leader 2's chosen action is a_2^*. Note that follower 1's belief about leader 2's chosen action is a_2^* even when leader 2 deviates from a_2^*.

7. See Baik and Lee (2012) for an alternative solution technique for game II. The alternative solution technique yields the same equilibrium actions of game II.

8. Since the early part of this second-stage analysis is the same as the first part of the analysis in Section 3.2.1, its details are omitted.

9. Recall from Section 2 that $\beta_1 = 1$ indicates the case where manager 1 is given an incentive to maximize firm 1's profits.

10. Note that $\beta_1^* > 1$ implies that owner 1 gives a *negative* weight to the sales component S_1 in manager 1's performance measure H_1.

Appendix A: Analysis of game A

Now that we take exactly the same steps as in the solution technique for game I which we proposed in Section 3.2.1, we omit, for concise exposition, detailed derivations and explanations.

The managers' decisions

We begin by considering manager i's maximization problem, for $i = 1, 2$. After observing firm i's contract (or the value of β_i) chosen by owner i, manager i seeks to maximize H_i in (2) over his firm's output level y_i, taking firm j's output level y_j as given. From the first-order condition for maximizing H_i, we obtain manager i's reaction function:

$$y_i(y_j;\, \beta_i) \;=\; (a - \beta_i c)/2b \;-\; y_j/2. \tag{A1}$$

The second-order condition for maximizing H_i is satisfied. *Note that the second-order condition is satisfied for every maximization problem in Appendixes A through D.*

Next, our analysis of the managers' decisions goes further to obtain the managers' strategies. Manager 1 knows his own reaction function in (A1) for the value of β_1 chosen by owner 1. Given his belief β_2^{o} about the value of β_2 chosen by owner 2, manager 1 also knows manager 2's reaction function: $y_2(y_1;\, \beta_2^{\mathrm{o}}) = (a - \beta_2^{\mathrm{o}} c)/2b - y_1/2$. Using these two reaction functions, we obtain manager 1's output level at their intersection, and further obtain his strategy:

$$y_1(\beta_1;\, \beta_2^{\mathrm{o}}) \;=\; (a + \beta_2^{\mathrm{o}} c - 2\beta_1 c)/3b. \tag{A2}$$

By contrast, manager 2 knows both the value of β_1 chosen by owner 1 and the the value of β_2 chosen by owner 2, and thus knows manager 1's reaction function and his own reaction function in (A1). Using these two reaction functions, we obtain manager 2's output level at their intersection, and further obtain his equilibrium strategy:

$$y_2(\beta_1, \beta_2) \;=\; (a + \beta_1 c - 2\beta_2 c)/3b. \tag{A3}$$

The owners' decisions

We consider owner 1's maximization problem. Given firm 2's contract β_2, owner 1 seeks to maximize

$$\psi_1(\beta_1, y_1(\beta_1; \beta_2^{o}), y_2(\beta_1, \beta_2)) \;=\; (a - 3c + \beta_1 c + 2\beta_2 c - \beta_2^{o} c)\,(a + \beta_2^{o} c - 2\beta_1 c)/9b \tag{A4}$$

over firm 1's contract β_1. From the first-order condition for maximizing ψ_1 in (A4), we obtain owner 1's reaction function:

$$\beta_1(\beta_2; \beta_2^{o}) \;=\; (6c - a - 4\beta_2 c + 3\beta_2^{o} c)/4c. \tag{A5}$$

Next, we consider owner 2's maximization problem. Given firm 1's contract β_1, owner 2 seeks to maximize

$$\psi_2(\beta_2, y_1(\beta_1; \beta_2^{o}), y_2(\beta_1, \beta_2)) \;=\; (a - 3c + \beta_1 c + 2\beta_2 c - \beta_2^{o} c)\,(a + \beta_1 c - 2\beta_2 c)/9b \tag{A6}$$

over firm 2's contract β_2. From the first-order condition for maximizing ψ_2 in (A6), we obtain owner 2's reaction function:

$$\beta_2(\beta_1; \beta_2^{o}) \;=\; (3 + \beta_2^{o})/4. \tag{A7}$$

The equilibrium contracts and output levels

First, using the owners' reaction functions, (A5) and (A7), and the condition that $\beta_2^{o} = \beta_2$ (see footnote 6), we obtain the equilibrium contracts of the firms: $\beta_1^{*} = (5c - a)/4c$ and $\beta_2^{*} = 1$. Then, substituting $\beta_2^{o} = \beta_2^{*}$, $\beta_1 = \beta_1^{*}$, and $\beta_2 = \beta_2^{*}$ into manager 1's strategy (A2) and manager 2's strategy (A3), we obtain the equilibrium output levels of the firms: $y_1^{*} = (a - c)/2b$ and $y_2^{*} = (a - c)/4b$.

Appendix B: Analysis of game B

Because we take exactly the same steps as in the solution technique for game II which we introduced in Section 3.2.2, we omit, for concise exposition, detailed derivations and explanations.

Analyzing the subgames starting at the second stage

We begin by considering manager i's maximization problem, for $i = 1, 2$. After observing the value of β_i chosen by owner i, manager i seeks to maximize H_i in (2) over his firm's output level y_i, taking firm j's output level y_j as given. From the first-order condition for maximizing H_i, we obtain manager i's reaction function:

$$y_i(y_j;\, \beta_i) \;=\; (a - \beta_i c)/2b \;-\; y_j/2. \tag{B1}$$

Next, our analysis of the managers' decisions goes further. Manager 1 knows his own reaction function in (B1) for the value of β_1 chosen by owner 1. Given his belief β_2^{o} about the value of β_2 chosen by owner 2, manager 1 also knows manager 2's reaction function: $y_2(y_1;\, \beta_2^{\text{o}}) = (a - \beta_2^{\text{o}} c)/2b - y_1/2$. Using these two reaction functions, we obtain manager 1's output level at their intersection, and further obtain his strategy:

$$y_1(\beta_1;\, \beta_2^{\text{o}}) \;=\; (a + \beta_2^{\text{o}} c - 2\beta_1 c)/3b. \tag{B2}$$

By contrast, manager 2 knows both the value of β_1 chosen by owner 1 and the the value of β_2 chosen by owner 2, and thus knows manager 1's reaction function and his own reaction function in (B1). Using these two reaction functions, we obtain manager 2's output level at their intersection, and further obtain his strategy:

$$y_2(\beta_1, \beta_2) \;=\; (a + \beta_1 c - 2\beta_2 c)/3b. \tag{B3}$$

Next, we consider owner 2's maximization problem. Knowing the value of β_1 chosen by owner 1, owner 2 seeks to maximize

$$\psi_2(\beta_2, y_1(\beta_1;\beta_2^o), y_2(\beta_1,\beta_2)) \;=\; (a - 3c + \beta_1 c + 2\beta_2 c - \beta_2^o c)\,(a + \beta_1 c - 2\beta_2 c)/9b \qquad \text{(B4)}$$

over firm 2's contract β_2. From the first-order condition for maximizing ψ_2 in (B4), we obtain owner 2's strategy:

$$\beta_2(\beta_1;\beta_2^o) \;=\; (3 + \beta_2^o)/4. \qquad \text{(B5)}$$

Now we obtain the equilibrium strategies of owner 2 and the managers. First, using owner 2's strategy (B5) and the condition that $\beta_2^o = \beta_2$, we obtain owner 2's equilibrium strategy:

$$\beta_2(\beta_1) \;=\; 1. \qquad \text{(B6)}$$

Then, substituting $\beta_2^o = \beta_2$ and (B6) into manager 1's strategy (B2) and manager 2's strategy (B3), we obtain the equilibrium strategies of the managers: $y_1(\beta_1) = (a + c - 2\beta_1 c)\;/3b$ and $y_2(\beta_1) = (a - 2c + \beta_1 c)/3b$.

Analyzing the first stage

Consider the first stage in which owner 1 chooses a value of β_1. Having perfect foresight about $y_1(\beta_1)$ and $y_2(\beta_1)$ for any value of β_1, owner 1 seeks to maximize

$$\psi_1(\beta_1, y_1(\beta_1), y_2(\beta_1)) \;=\; (a - 2c + \beta_1 c)\,(a + c - 2\beta_1 c)/9b \qquad \text{(B7)}$$

over firm 1's contract β_1. From the first-order condition for maximizing ψ_1 in (B7) with respect to β_1, we obtain the equilibrium contract of firm 1: $\beta_1^{**} = (5c - a)/4c$.

The equilibrium contracts and output levels

Substituting $\beta_1 = \beta_1^{**}$ into owner 2's equilibrium strategy (B6), we obtain the equilibrium contract of firm 2: $\beta_2^{**} = 1$. Substituting $\beta_1 = \beta_1^{**}$ into manager 1's equilibrium strategy $y_1 = y_1(\beta_1)$ and manager 2's equilibrium strategy $y_2 = y_2(\beta_1)$, we obtain the equilibrium output levels of the firms: $y_1^{**} = (a - c)/2b$ and $y_2^{**} = (a - c)/4b$.

Appendix C: Analysis of game C

Since we take exactly the same steps as in the solution technique for game I which we proposed in Section 3.2.1, we omit, for concise exposition, detailed derivations and explanations.

The managers' decisions

We begin by considering manager i's maximization problem, for $i = 1, 2$. After observing firm i's contract (or the value of β_i) chosen by owner i, manager i seeks to maximize H_i in (14) over his firm's price p_i, taking firm j's price p_j as given. From the first-order condition for maximizing H_i, we obtain manager i's reaction function:

$$p_i(p_j;\, \beta_i) \;=\; (d + \beta_i c + kp_j)/2. \tag{C1}$$

Next, manager 1 knows his own reaction function in (C1) for the value of β_1 chosen by owner 1. Given his belief β_2^{o} about the value of β_2 chosen by owner 2, manager 1 also knows manager 2's reaction function: $p_2(p_1;\ \beta_2^{\mathrm{o}}) = (d + \beta_2^{\mathrm{o}} c + kp_1)/2$. Using these two reaction functions, we obtain manager 1's price at their intersection, and further obtain his strategy:

$$p_1(\beta_1;\, \beta_2^{\mathrm{o}}) \;=\; \{d(2+k) + c(\beta_2^{\mathrm{o}} k + 2\beta_1)\}/(4-k^2). \tag{C2}$$

By contrast, manager 2 knows both the value of β_1 chosen by owner 1 and the the value of β_2 chosen by owner 2, and thus knows manager 1's reaction function and his own reaction function in (C1). Using these two reaction functions, we obtain manager 2's price at their intersection, and further obtain his equilibrium strategy:

$$p_2(\beta_1, \beta_2) \;=\; \{d(2+k) + c(\beta_1 k + 2\beta_2)\}/(4-k^2). \tag{C3}$$

The owners' decisions

Given firm 2's contract β_2, owner 1 seeks to maximize

$$\begin{aligned}\psi_1(\beta_1, p_1(\beta_1;\, \beta_2^{\mathrm{o}}), p_2(\beta_1, \beta_2)) \;&=\; \{d(2+k) - c(4-k^2) + 2\beta_1 c + \beta_2^{\mathrm{o}} kc\} \\ &\quad \times \{d(2+k) - \beta_1 c(2-k^2) + 2\beta_2 kc - \beta_2^{\mathrm{o}} kc\}/(4-k^2)^2\end{aligned} \tag{C4}$$

over firm 1's contract β_1. From the first-order condition for maximizing ψ_1 in (C4), we obtain owner 1's reaction function:

$$\beta_1(\beta_2;\beta_2^{o}) \;=\; \{k^2d(2+k)+c(8-6k^2+k^4)+4\beta_2kc-\beta_2^{o}kc(4-k^2)\}/4c(2-k^2) \tag{C5}$$

Given firm 1's contract β_1, owner 2 seeks to maximize

$$\psi_2(\beta_2, p_1(\beta_1;\beta_2^{o}), p_2(\beta_1,\beta_2)) \;=\; \{d(2+k)-c(4-k^2)+\beta_1kc+2\beta_2c\} \times \{d(2+k)+\beta_1kc-2\beta_2c+\beta_2^{o}k^2c\}/(4-k^2)^2 \tag{C6}$$

over firm 2's contract β_2. From the first-order condition for maximizing ψ_2 in (C6), we obtain owner 2's reaction function:

$$\beta_2(\beta_1;\beta_2^{o}) \;=\; (4-k^2+\beta_2^{o}k^2)/4. \tag{C7}$$

The equilibrium contracts and prices

First, using the owners' reaction functions, (C5) and (C7), and the condition that $\beta_2^{o}=\beta_2$ (see footnote 6), we obtain the equilibrium contracts of the firms: $\beta_1^{*}=\{8c+2(d-3c)k^2+(c+d)k^3+ck^4\}\ /4c(2-k^2)$ and $\beta_2^{*}=1$. Then, substituting $\beta_2^{o}=\beta_2^{*}$, $\beta_1=\beta_1^{*}$, and $\beta_2=\beta_2^{*}$ into manager 1's strategy (C2) and manager 2's strategy (C3), we obtain the equilibrium prices of the firms: $p_1^{*}=\ \{2(c+d)+(c+d)k-ck^2\}/2(2-k^2)$ and $p_2^{*}=\{4(c+d)+2(c+d)k-(c+d)k^2-ck^3\}\ /4(2-k^2)$.

Appendix D: Analysis of game D

Since we take exactly the same steps as in the solution technique for game II which we introduced in Section 3.2.2, we omit, for concise exposition, detailed derivations and explanations.

Analyzing the subgames starting at the second stage

We begin by considering manager i's maximization problem, for $i = 1,\ 2$. After observing firm i's contract (or the value of β_i) chosen by owner i, manager i seeks to maximize H_i in (14) over his firm's price p_i, taking firm j's price p_j as given. From the first-order condition for maximizing H_i, we obtain manager i's reaction function:

$$p_i(p_j;\, \beta_i) \;=\; (d + \beta_i c + kp_j)/2. \tag{D1}$$

Next, manager 1 knows his own reaction function in (D1) for the value of β_1 chosen by owner 1. Given his belief β_2^{o} about the value of β_2 chosen by owner 2, manager 1 also knows manager 2's reaction function: $p_2(p_1;\ \beta_2^{\mathrm{o}}) = (d + \beta_2^{\mathrm{o}} c + kp_1)/2$. Using these two reaction functions, we obtain manager 1's price at their intersection, and further obtain his strategy:

$$p_1(\beta_1;\, \beta_2^{\mathrm{o}}) \;=\; \{d(2+k) + c(\beta_2^{\mathrm{o}} k + 2\beta_1)\}/(4 - k^2). \tag{D2}$$

By contrast, manager 2 knows both the value of β_1 chosen by owner 1 and the the value of β_2 chosen by owner 2, and thus knows manager 1's reaction function and his own reaction function in (D1). Using these two reaction functions, we obtain manager 2's price at their intersection, and further obtain his strategy:

$$p_2(\beta_1,\, \beta_2) \;=\; \{d(2+k) + c(\beta_1 k + 2\beta_2)\}/(4 - k^2). \tag{D3}$$

Next, we consider owner 2's maximization problem. Knowing the value of β_1 chosen by owner 1, owner 2 seeks to maximize

$$\begin{aligned}\psi_2(\beta_2, p_1(\beta_1;\, \beta_2^{\mathrm{o}}), p_2(\beta_1, \beta_2)) \;=\;& \{d(2+k) - c(4-k^2) + \beta_1 kc + 2\beta_2 c\} \\ &\times \{d(2+k) + \beta_1 kc - 2\beta_2 c + \beta_2^{\mathrm{o}} k^2 c\}/(4-k^2)^2\end{aligned} \tag{D4}$$

over firm 2's contract β_2. From the first-order condition for maximizing ψ_2 in (D4), we obtain owner 2's strategy:

$$\beta_2(\beta_1;\, \beta_2^{\mathrm{o}}) \;=\; (4 - k^2 + \beta_2^{\mathrm{o}} k^2)/4. \tag{D5}$$

Now we obtain the equilibrium strategies of owner 2 and the managers. First, using owner 2's strategy (D5) and the condition that $\beta_2^{\mathrm{o}} = \beta_2$, we obtain owner 2's equilibrium strategy:

$$\beta_2(\beta_1) \; = \; 1. \tag{D6}$$

Then, substituting $\beta_2^{\mathrm{o}} = \beta_2$ and (D6) into manager 1's strategy (D2) and manager 2's strategy (D3), we obtain the equilibrium strategies of the managers: $p_1(\beta_1) = \{d(2+k) + c(k + 2\beta_1)\}/(4 - k^2)$ and $p_2(\beta_1) = \{d(2+k) + c(\beta_1 k + 2)\}/(4 - k^2)$.

Analyzing the first stage

Having perfect foresight about $p_1(\beta_1)$ and $p_2(\beta_1)$ for any value of β_1, owner 1 seeks to maximize

$$\begin{aligned}\psi_1(\beta_1, p_1(\beta_1), p_2(\beta_1)) \; = \; &\{d(2+k) - c(4 - k - k^2) + 2\beta_1 c\} \\ &\times \{d(2+k) - \beta_1 c(2 - k^2) + kc\}/(4 - k^2)^2\end{aligned} \tag{D7}$$

over firm 1's contract β_1. From the first-order condition for maximizing ψ_1 in (D7), we obtain the equilibrium contract of firm 1: $\beta_1^{**} = \{8c + 2(d - 3c)k^2 + (c + d)k^3 + ck^4\}/4c(2 - k^2)$.

The equilibrium contracts and prices

Substituting $\beta_1 = \beta_1^{**}$ into owner 2's equilibrium strategy (D6), we obtain the equilibrium contract of firm 2: $\beta_2^{**} = 1$. Substituting $\beta_1 = \beta_1^{**}$ into manager 1's equilibrium strategy $p_1 = p_1(\beta_1)$ and manager 2's equilibrium strategy $p_2 = p_2(\beta_1)$, we obtain the equilibrium prices of the firms: $p_1^{**} = \{2(c + d) + (c + d)k - ck^2\}/2(2 - k^2)$ and $p_2^{**} = \{4(c + d) + 2(c + d)k - (c + d)k^2 - ck^3\}/4(2 - k^2)$.

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