

STRATEGIC GROUPS AND RENT DISSIPATION

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Abstract

We consider a rent-seeking contest in which players can form strategic groups before expending their outlays. We examine the profitability of endogenous group formation and the effect of such group formation on rent dissipation. We show the following. When just one strategic group is formed in equilibrium, group formation is beneficial both to the group members and to the nonmembers, and rent dissipation is smaller than with usual individual rent seeking. However, when more than two strategic groups are formed in equilibrium, group formation is never profitable to any players and rent dissipation is greater than with individual rent seeking. (JEL D72, C72)

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I. INTRODUCTION

A rent-seeking contest is a situation in which players compete with one another by expending outlays to win a rent. Examples abound. When a positive monopoly rent is secured under government protection, firms lobby to win the monopoly. When governmental decisions to establish tariffs or other trade barriers create rents, firms compete to capture these rents. Firms compete to acquire a rent generated by rights of ownership to an import quota. Firms compete to obtain a rent generated by a government procurement contract.

Beginning with the seminal work of Tullock (1967), the early literature on rent seeking has concentrated on individual rent seeking – that is, contests in which players compete individually to win the rent. This includes, for example, Krueger (1974), Posner (1975), Tullock (1980), Hillman and Katz (1984), Appelbaum and Katz (1987), Hillman and Riley (1989), Hirshleifer (1989), Leininger (1993), and Hurley and Shogren (1998).¹

Recently, several economists have studied collective rent seeking – that is, contests in which competition for the rent arises among groups of players and the rent is awarded to a group. For example, Nitzan (1991a, 1991b), Baik and Shogren (1995), Lee (1995), Hausken (1995), and Baik and Lee (1997) study collective rent seeking with a private-good rent. Katz et al. (1990), Ursprung (1990), Baik (1993), Riaz et al. (1995), Katz and Tokatlidu (1996), and Baik et al. (2001) study collective rent seeking with a group-specific public-good rent. A salient feature of the rent-seeking contests in these articles is that the players expend their outlays noncooperatively.

This article studies individual rent seeking with a private-good rent. But unlike the previous work, we consider a rent-seeking contest with endogenous group formation in which players can form groups before they expend their outlays.² What type of groups do we consider? As we will explain it in section II, if a group is formed and then a player in that group wins the rent, the winner "shares" it with the other players in that group. Examples of such groups may include coalitions among firms, political parties, or interest

groups, and R&D joint ventures among firms. This article focuses on the profitability of endogenous group formation and the effect of such group formation on rent dissipation. Specifically, we consider a three-stage game in which the players first decide whether to form groups, then the players in each group decide how to share the rent if "they" win it, and finally all the players in the contest expend their outlays independently to win the rent. The rent is awarded to a *single* player.

Why do players form such groups? An intuitive explanation is that each player wants to insure against his failure in winning the rent – in other words, he wants to share the risk of his failure with the other players in his group. Or, as it will be clear in section II, each player wants to make his prize bigger in case he becomes the winner. Another explanation is that they can benefit by achieving strategic commitments through such group formation: They can change their opponents' behavior in their favor by using their sharing rule and treating members differently from nonmembers. To highlight the idea of strategic group formation, we call groups formed herein *strategic groups*.³ We define a strategic group as follows: (a) the players in each group first choose their sharing rule – winner's fractional share – and then expend their outlays noncooperatively; and (b) if a player in a group wins the rent, the winner "shares" the rent with the other players in that group according to the predetermined sharing rule. The players in a strategic group do not need to know how much outlays their group members expended, when they share the rent.

This article endogenizes the number of strategic groups, their sizes, and their sharing rules. Defining an equilibrium as a situation in which no payoff-increasing *individual* move-in or move-out is possible, we obtain the number of strategic groups and their sizes in equilibrium. Given the number of players, the equilibrium number of strategic groups is not unique. When just one strategic group is formed in equilibrium, there may be players who do not belong to the single strategic group and, for more than five players, the equilibrium group size equals the smallest of integers greater than half the

number of players. When more than one strategic group is formed in equilibrium, every player belongs to one of the groups and the difference in equilibrium group size between any two groups is at most one. Equilibrium winner's fractional share is less than one when just one strategic group is formed, and is greater than one – the winner takes all the rent and further receives "bounties" from the other players in his group – for all the strategic groups when more than two strategic groups are formed.

Examining the extent of rent dissipation is one of the main issues in the literature on rent seeking. It is important because the opportunity costs of resources expended on rent-seeking activities are social costs and thus create economic inefficiency. Examining the profitability of endogenous group formation and the effect of such group formation on rent dissipation, we show the following. When just one strategic group is formed in equilibrium, group formation is beneficial both to the group members and to the nonmembers, and rent dissipation (or total outlay) is smaller than with usual individual rent seeking – that is, the social cost associated with rent seeking decreases by such group formation. However, when more than two strategic groups are formed in equilibrium, group formation is never profitable to any players and rent dissipation is greater than with individual rent seeking. We also show that total rent-seeking outlay is less than the rent, regardless of the number of strategic groups formed.

This article is related to the literature on the noncooperative theory of endogenous formation of coalitions – the literature which deals with situations in which players first form coalitions and then, given the coalition structure determined, engage in noncooperative competition. Papers in this literature include Bloch (1995, 1996), Yi (1996, 1997, 1998), Konishi et al. (1997), Belleflamme (2000), Yi and Shin (2000), and Morasch (2000). They concentrate on examining the equilibrium (or stable) structures of coalitions – the equilibrium numbers and the sizes of coalitions – in models with various applications. Bloch (1995), Yi (1998), Belleflamme (2000), and Yi and Shin (2000) examine the equilibrium structures of associations (such as R&D joint ventures) in

oligopolies. Yi (1996) examines the equilibrium structure of customs unions in a situation in which countries can form customs unions freely. Morasch (2000) obtains the equilibrium structures of strategic alliances of firms and, using them, examines the loose competition policy toward strategic alliances as an alternative to the strategic trade policy. One main finding of these articles is that the grand coalition is typically not an equilibrium coalition structure, with the exception that, in the case where coalition formation among symmetric players creates negative externalities for nonmembers, the grand coalition is an equilibrium coalition structure under the open membership rule – the rule that stipulates that a coalition admit new members on a nondiscriminatory basis. We obtain a similar result in this article: The grand coalition never occurs if the number of players exceeds five.

The remainder of the article is organized as follows. In section II, we develop the model and set up the three-stage game. In section III, we solve a subgame that starts at the second stage of the three-stage game. In this subgame, given the number of groups and their sizes, the players in groups choose their winner's fractional shares and then all the players in the contest expend their outlays. In section IV, we analyze the first stage of our three-stage game and thereby obtain the equilibrium numbers of strategic groups and equilibrium group sizes. In section V, we compute the equilibrium winner's fractional shares of the groups and examine the profitability of endogenous group formation and the effect of such group formation on rent dissipation. Finally, section VI offers our conclusions.

II. THE MODEL

There are n risk-neutral players who want to win a fixed rent, where $n > 1$. The rent is worth V and will be awarded to one of the players. The probability that a player wins the rent is equal to his outlay divided by the players' total outlay if the total outlay is positive, and it is equal to $1/n$ if all the players expend zero.⁴

The players can form groups before they compete by expending outlays. Groups formed are called strategic groups. We define a strategic group as follows. Suppose that a group consists of m players, where m is an integer and $1 < m \leq n$. The m players write an agreement. The agreement specifies how much of the rent the winner and the losers each will take, when a player in the group wins the rent. Then all the players in the contest expend their outlays noncooperatively. If a player in that group wins the rent, the winner "shares" it with the other players in that group according to the previously written agreement. We assume that the winning player takes σV and each losing player "takes" $(1 - \sigma)V/(m - 1)$. The winner's fractional share, σ , is assumed to be greater than or equal to $1/m$. If $\sigma = 1/m$ holds, the players in that group share the rent equally when a player in that group wins it. In the case where $1/m \leq \sigma < 1$, the winner helps the other players in that group, and thus the prize to the winner is less than the rent. When the winner's fractional share is equal to unity, the winner takes all the rent. In the case where $\sigma > 1$, the winner takes all the rent and further receives "bounties" from the other players in that group. Thus, in this case, the prize to the winner is greater than the rent. We assume that there is no transaction cost associated with organizing a strategic group, negotiating an agreement, and enforcing compliance.

We formally consider the following three-stage game. In the first stage, the players decide simultaneously and independently whether to form strategic groups.⁵ In the second stage, after knowing the number of groups and their sizes, the players in each group make a binding agreement that specifies how much of the rent the winner and the losers each will take when a player in the group wins the rent. In doing so, the players in each group set their winner's fractional share. Then all the groups announce their choices simultaneously. Note that, because the players in each group are identical, their decision on their winner's fractional share is unanimous. In the third stage, after knowing the number of groups, their sizes, and their winner's fractional shares, all the players in the contest choose their outlays simultaneously and independently. At the end of the third

stage, the winning player is chosen, and the winner "shares" the rent with the other players in his group according to the agreement written in the second stage. We assume that all of this is common knowledge. We employ subgame-perfect equilibrium as the solution concept.

III. EXPECTED PAYOFFS FOR THE PLAYERS GIVEN N STRATEGIC GROUPS

This section considers subgames which start at the second stage of our three-stage game. Suppose that N strategic groups (1 through N) are formed in the first stage, where $N \geq 1$. The players who do not belong to any of these N groups are treated as those in an imaginary strategic group (called group $N+1$) whose winner's fractional share is unity. Let m_i denote the number of players in group i . We have then $n = \sum_{k=1}^{N+1} m_k$. Without loss of generality, assume that $1 < m_N \leq \dots \leq m_2 \leq m_1$ and $m_{N+1} \geq 0$.

Specifically, we consider the following subgame of our three-stage game. In the second stage, given the number of groups, $N+1$, and their sizes, (m_1, \dots, m_{N+1}) , the players in groups 1 through N choose their winner's fractional shares simultaneously and independently. In the third stage, after learning the winner's fractional shares of the $N+1$ groups (including group $N+1$) and the size of each group, all the players in the contest choose their outlays simultaneously and independently. At the end of the third stage, the winner is chosen, and the winner "shares" the rent with the other players in his group according to the agreement written in the second stage. Note that we can view this subgame as a full game resulting when the number of groups and their sizes are exogenously given.

Lemma 1 describes the subgame-perfect equilibrium outcomes of the subgame in the case where $m_1 = n$ or, equivalently, $N = 1$ and $m_{N+1} = 0$.

LEMMA 1. *In the case where $m_1 = n$, the equilibrium winner's fractional share of group 1 is $1/n$ and each player's equilibrium outlay is zero.*

Proof. The brief proof of Lemma 1 follows. First recall that the winner's fractional share of a group is assumed to be greater than or equal to one divided by the number of players in the group. Hence, in the case where $m_1 = n$, the winner's fractional share is greater than or equal to $1/n$. One can easily check the following. If $\sigma = 1/n$, then each player expends zero outlays and his payoff is V/n . If $\sigma > 1/n$, then each player's outlay is positive and his expected payoff is less than V/n . Comparing these findings, we obtain Lemma 1. \square

Lemma 1 shows that, if all the players in the contest belong to the same group, they expend zero outlays and share the rent equally, whoever wins it. Thus, Lemma 1 implies that, if the grand coalition occurs, the inefficiency problem associated with rent seeking disappears.

To solve for the subgame-perfect equilibrium of the subgame in the case where $m_1 < n$, we work backward.⁶ Let σ_i represent the winner's fractional share of group i . Let x_{ij} represent the irreversible rent-seeking outlay of player j in group i and let X_i represent group i 's outlay. In the third stage, given (m_1, \dots, m_{N+1}) and an $(N+1)$ -tuple vector of winner's fractional shares, player j in group i seeks to maximize his expected payoff:

$$\begin{aligned} \pi_{ij} &= (\sigma_i V - x_{ij})(x_{ij}/S) + [(1 - \sigma_i)V/(m_i - 1) - x_{ij}][(X_i - x_{ij})/S] + (-x_{ij})[(S - X_i)/S] \\ &= \sigma_i V(x_{ij}/S) + [(1 - \sigma_i)V/(m_i - 1)][(X_i - x_{ij})/S] - x_{ij} \quad \text{for } i = 1, \dots, N, \end{aligned}$$

and

(1)

$$\pi_{ij} = V(x_{ij}/S) - x_{ij} \quad \text{for } i = N+1,$$

where $S = \sum_{k=1}^{N+1} X_k$.⁷ In the payoff functions, x_{ij}/S is the probability that player j in group i wins the rent, $(X_i - x_{ij})/S$ is the probability that any one of the other players in group i wins the rent, and $(S - X_i)/S$ is the probability that any one of the other groups' players wins the rent. Note that given $S > 0$, the probability that a player wins the rent depends only on his own outlay.

Given a positive outlay of the other players, the first-order condition for maximizing π_{ij} yields

$$\sigma_i V(S - x_{ij}) + (1 - \sigma_i) V(x_{ij} - X_i)/(m_i - 1) = S^2 \quad \text{for } i = 1, \dots, N,$$

and (2)

$$V(S - x_{ij}) = S^2 \quad \text{for } i = N+1.$$

The second-order condition is satisfied. At the Nash equilibrium of the third-stage subgame, the players in the same group expend the same outlay. Denote the third-stage equilibrium outlay of each player in group i by $x_i(\boldsymbol{\sigma}, \mathbf{m})$, where $\boldsymbol{\sigma} = (\sigma_1, \dots, \sigma_{N+1})$ and $\mathbf{m} = (m_1, \dots, m_{N+1})$. Then, at the third-stage Nash equilibrium, equation (2) reduces to (3):

$$\sigma_i V H - V x_i(\boldsymbol{\sigma}, \mathbf{m}) = H^2 \quad \text{for } i = 1, \dots, N+1, \quad (3)$$

where $H = \sum_{k=1}^{N+1} m_k x_k(\boldsymbol{\sigma}, \mathbf{m})$. Note that equation (3) holds for $i = N+1$ because $\sigma_{N+1} = 1$.

Multiplying (3) through by m_i , we have $\sigma_i m_i V H - m_i V x_i(\boldsymbol{\sigma}, \mathbf{m}) = m_i H^2$. Summing over all groups, we obtain $\sum_{k=1}^{N+1} \sigma_k m_k V H - \sum_{k=1}^{N+1} m_k V x_k(\boldsymbol{\sigma}, \mathbf{m}) = \sum_{k=1}^{N+1} m_k H^2$. This is simplified to

$VM - V = nH$, where $M = \sum_{k=1}^{N+1} \sigma_k m_k$. We have then

$$H = V(M - 1)/n. \quad (4)$$

Substituting equation (4) into (3), we obtain the outlay of each player in group i at the third-stage Nash equilibrium:

$$x_i(\boldsymbol{\sigma}, \mathbf{m}) = V(M - 1)(\sigma_i n - M + 1)/n^2. \quad (5)$$

Let $\pi_i(\boldsymbol{\sigma}, \mathbf{m})$ represent the expected payoff for each player in group i at the third-stage Nash equilibrium. Then, using (1), we obtain

$$\pi_i(\boldsymbol{\sigma}, \mathbf{m}) = (V - H)x_i(\boldsymbol{\sigma}, \mathbf{m})/H. \quad (6)$$

Substituting (4) and (5) into equation (6), we obtain

$$\pi_i(\boldsymbol{\sigma}, \mathbf{m}) = V(n - M + 1)(\sigma_i n - M + 1)/n^2. \quad (7)$$

Consider now the second stage in which the players in group i (in this paragraph, $i = 1, \dots, N$) choose their winner's fractional share. The N groups choose their winner's fractional shares simultaneously and independently. Because the players in group i are identical, the winner's fractional share that is "optimal" for a player is also optimal for the other players in the group. This implies that group i 's decision on its winner's fractional share is unanimous. In this stage, the players in group i know that $\sigma_{N+1} = 1$ and have perfect foresight about the third-stage competition. Given winner's fractional shares of the other groups, the best response of group i is the winner's fractional share that maximizes $\pi_i(\boldsymbol{\sigma}, \mathbf{m})$ subject to $\sigma_i \geq 1/m_i$. From the first-order condition for maximizing (7), we obtain

$$-m_i(\sigma_i n - M + 1) + (n - M + 1)(n - m_i) = 0.$$

It is easy to see that $\pi_i(\boldsymbol{\sigma}, \mathbf{m})$ is strictly concave in σ_i and thus the second-order condition for maximizing $\pi_i(\boldsymbol{\sigma}, \mathbf{m})$ is satisfied. Let $\sigma_i(\mathbf{m})$ represent group i 's winner's fractional share which is specified in the subgame-perfect equilibrium. From the above N equations and the fact that $\sigma_{N+1} = 1$, we obtain group i 's equilibrium winner's fractional share $\sigma_i(\mathbf{m})$.

LEMMA 2. *In the case where $m_1 < n$, group i 's equilibrium winner's fractional share is $\sigma_i(\mathbf{m}) = 1 + (n - 2m_i)/m_i[n(N - 1) + 2m_{N+1}]$ for $i = 1, \dots, N$.*

The winner's fractional share of group $N + 1$ is defined above as equal to unity.

Using Lemma 2, we find the following. First, group i 's equilibrium winner's fractional share is greater than $1/m_i$ (in this paragraph, $i = 1, \dots, N$). This can be explained as follows. Equal sharing brings a serious free-rider problem and fails to induce group i 's players to expend "optimal" outlays. Therefore, facing the rival groups, the players in group i adopt a "sharing" rule which allows the winner to take more than the equal share. Second, group i 's equilibrium winner's fractional share is less (greater) than unity if m_i is greater (less) than $n/2$. It is equal to unity if $m_i = n/2$ holds. Because group 1 is the largest of the N groups, we have the following. If $m_1 > n/2$ holds, then group 1's equilibrium winner's fractional share is less than unity and the equilibrium winner's fractional shares of groups 2 through N are greater than unity. Otherwise, all the equilibrium winner's fractional shares are greater than or equal to unity. Finally, two groups with the same number of players have the same equilibrium winner's fractional share, and a smaller group has a greater equilibrium winner's fractional share than a larger one: In terms of the symbols, $\sigma_1(\mathbf{m}) \leq \dots \leq \sigma_N(\mathbf{m})$ holds.

Let $x_i(\mathbf{m})$ represent the outlay of each player in group i in the subgame-perfect equilibrium, $X_i(\mathbf{m})$ group i 's equilibrium outlay, and $H(\mathbf{m})$ the equilibrium total outlay: $X_i(\mathbf{m}) = m_i x_i(\mathbf{m})$ and $H(\mathbf{m}) = \sum_{k=1}^{N+1} X_k(\mathbf{m}) = \sum_{k=1}^{N+1} m_k x_k(\mathbf{m})$. Using equation (5) and Lemma 2, we obtain Lemma 3.

LEMMA 3. *In the case where $m_1 < n$, the equilibrium outlays of the individual players, those of the groups, and the equilibrium total outlay are*

$$\begin{aligned} x_i(\mathbf{m}) &= V[n(N-1) + 2m_{N+1} - 1](n - m_i)/m_i[n(N-1) + 2m_{N+1}]^2 \quad \text{for } i = 1, \dots, N; \\ x_{N+1}(\mathbf{m}) &= V[n(N-1) + 2m_{N+1} - 1]/[n(N-1) + 2m_{N+1}]^2; \\ X_i(\mathbf{m}) &= V[n(N-1) + 2m_{N+1} - 1](n - m_i)/[n(N-1) + 2m_{N+1}]^2 \quad \text{for } i = 1, \dots, N; \\ X_{N+1}(\mathbf{m}) &= V[n(N-1) + 2m_{N+1} - 1]m_{N+1}/[n(N-1) + 2m_{N+1}]^2; \\ \text{and } H(\mathbf{m}) &= V\{1 - 1/[n(N-1) + 2m_{N+1}]\}. \end{aligned}$$

Using Lemma 3, we obtain the following. First, $x_1(\mathbf{m}) \leq \dots \leq x_N(\mathbf{m})$ and $X_1(\mathbf{m}) \leq \dots \leq X_N(\mathbf{m})$ hold. It means that the players in a smaller group expend more than those in a larger one. This is because a smaller group has a greater winner's fractional share than a larger one, and therefore the prize to the winner is greater in a smaller group than in a larger one – in other words, the players in a smaller group are motivated more than those in a larger one. Second, group i 's individual outlay $x_i(\mathbf{m})$, for $i = 1, \dots, N$, is less (greater) than $x_{N+1}(\mathbf{m})$ if m_i is greater (less) than $n/2$. It is equal to $x_{N+1}(\mathbf{m})$ if $m_i = n/2$ holds. Because group 1 is the largest of the N groups, if $m_1 > n/2$ holds, then group 1's individual outlay is less than $x_{N+1}(\mathbf{m})$ and the individual outlays of groups 2 through N are greater than $x_{N+1}(\mathbf{m})$. Otherwise, all the individual outlays are greater than or equal to $x_{N+1}(\mathbf{m})$. Third, group i 's outlay $X_i(\mathbf{m})$, for $i = 1, \dots, N$, is greater than $X_{N+1}(\mathbf{m})$ if m_i is less than $n - m_{N+1}$. It is equal to $X_{N+1}(\mathbf{m})$ if m_i is equal to $n - m_{N+1}$. In other words, group i 's outlay is greater than $X_{N+1}(\mathbf{m})$ if there is more than one strategic group excluding group

$N+1$ (i.e., $N > 1$). It is equal to $X_{N+1}(\mathbf{m})$ if there are just two groups, including group $N+1$. Finally, the equilibrium total outlay is less than the rent. Other things being equal, as the number of groups, the number of players in the contest, or m_{N+1} increases, the equilibrium total outlay increases.

Next, using equation (7), Lemmas 1 and 2, and the fact that $\sigma_{N+1} = 1$, we obtain Lemma 4, which describes each player's expected payoff in the subgame-perfect equilibrium.

LEMMA 4. *In the case where $m_1 = n$, the equilibrium expected payoff for each player is V/n . In the case where $m_1 < n$, the equilibrium expected payoff for each player in group i is $\pi_i(\mathbf{m}) = V(n - m_i)/m_i[n(N - 1) + 2m_{N+1}]^2$ for $i = 1, \dots, N$ and $\pi_{N+1}(\mathbf{m}) = V/[n(N - 1) + 2m_{N+1}]^2$.*

Lemma 4 shows that $\pi_1(\mathbf{m}) \leq \dots \leq \pi_N(\mathbf{m})$ holds. It means that the players in a smaller group have greater equilibrium expected payoffs than those in a larger one. Lemma 4 also shows that the equilibrium expected payoff for each player in group i , for $i = 1, \dots, N$, is less (greater) than that for each player in group $N+1$ if m_i is greater (less) than $n/2$. They are equal if $m_i = n/2$ holds. Because group 1 is the largest of the N groups, if $m_1 > n/2$ holds, then the equilibrium expected payoff for each player in group 1 is less than $\pi_{N+1}(\mathbf{m})$ and the equilibrium expected payoff for each player in groups 2 through N is greater than $\pi_{N+1}(\mathbf{m})$. Otherwise, the equilibrium expected payoff for each player in groups 1 through N is greater than or equal to $\pi_{N+1}(\mathbf{m})$.

IV. EQUILIBRIUM NUMBERS OF STRATEGIC GROUPS AND EQUILIBRIUM GROUP SIZES

We analyzed the second and the third stages of our three-stage game in section III. We now consider the first stage in which the players decide simultaneously and

independently whether to form strategic groups. By doing so, we obtain the equilibrium number of *strategic groups* and the equilibrium size of group i , denoted by N^* and m_i^* , respectively. Note that N^* and m_i^* are determined *at the same time* (see note 5). We obtain N^* and m_i^* , using Lemma 4 and the fact that in equilibrium no player has an incentive to deviate individually from his "position" – that is, no payoff-increasing individual move-in or move-out is possible. Given n , the equilibrium number of strategic groups is not unique. We obtain first the equilibrium group sizes when $N^* = 1$, which are summarized in Lemma 5 and Table 1.⁸

LEMMA 5. *When $N^* = 1$, the equilibrium group sizes are: (i) $m_1^* = n$ for $n = 2$ or 3 , (ii) $m_1^* = 3$ and 4 for $n = 4$ or 5 , (iii) $m_1^* = (n + 2)/2$ when n is even and $n \geq 6$, and (iv) $m_1^* = (n + 1)/2$ when n is odd and $n \geq 7$.*

Lemma 5 and Table 1 imply that when $N^* = 1$, there may be players who do not belong to the single strategic group. This occurs because, in the case where just one strategic group is formed, if the size of the single group is greater (less) than half the number of players, then the expected payoff for each member is less (greater) than that for each nonmember (see Lemma 4). Note that, for $n \geq 6$, the equilibrium group size equals the smallest of integers greater than half the number of players. This means that, as the number of players increases, the number of nonmembers weakly increases. Recall from Lemma 1 that the grand coalition eliminates the inefficiency problem associated with rent seeking. This and Lemma 5 tell us that, when $n \leq 4$, the inefficiency problem of rent seeking can be eliminated by endogenous strategic-group formation.

Next, we consider the case where $N^* \geq 2$.

LEMMA 6. (a) *When $N^* \geq 2$, every player is a strategic-group member: $n = m_1^* + \dots + m_{N^*}^*$.* (b) *When $N^* \geq 2$, the difference in equilibrium group size between*

group 1 and group N^* is either zero or one: In terms of the symbols, $m_1^* = m_{N^*}^*$ or $m_1^* = m_{N^*}^* + 1$. (c) Suppose that $n = m_1 + \dots + m_N$, and that $m_1 = m_N$ or $m_1 = m_N + 1$. Then, when $N \geq 5$, no player in group i with $m_i \geq 2$ has an incentive to move out of his strategic group. When $N \geq 2$, no player in group i with $m_i \geq 3$ has an incentive to move out of his strategic group.

Due to the assumption that $m_{N^*} \leq \dots \leq m_2 \leq m_1$, part (b) is equivalent to stating that the difference in equilibrium group size between *any* two groups is either zero or one. In other words, the N^* strategic groups are of the same size, or a largest strategic group has at most one more player when compared with a smallest group.

Lemma 6 and additional computation yield Lemma 7.

LEMMA 7. (a) When $N^* = 2$, the equilibrium group sizes are:

$$(m_1^*, m_2^*) = (n/2, n/2) \text{ for } n = 2k, \text{ and}$$

$$(m_1^*, m_2^*) = ((n+1)/2, (n-1)/2) \text{ for } n = 2k+1, \text{ where } k \text{ is an integer and } n \geq 6.$$

(b) When $N^* = 3$, the equilibrium group sizes are:

$$m_1^* = m_2^* = m_3^* = n/3 \text{ for } n = 3k,$$

$$m_1^* = (n+2)/3 \text{ and } m_2^* = m_3^* = (n-1)/3 \text{ for } n = 3k+1, \text{ and}$$

$$m_1^* = m_2^* = (n+1)/3 \text{ and } m_3^* = (n-2)/3 \text{ for } n = 3k+2,$$

where k is an integer and $n \geq 8$.

(c) When $N^* = 4$, the equilibrium group sizes are:

$$(i) \quad m_1^* = \dots = m_4^* = 2 \text{ for } n = 8, \text{ and}$$

$$(ii) \quad m_1^* = \dots = m_4^* = n/4 \text{ for } n = 4k,$$

$$m_1^* = (n+3)/4 \text{ and } m_2^* = m_3^* = m_4^* = (n-1)/4 \text{ for } n = 4k+1,$$

$$m_1^* = m_2^* = (n+2)/4 \text{ and } m_3^* = m_4^* = (n-2)/4 \text{ for } n = 4k+2, \text{ and}$$

$$m_1^* = m_2^* = m_3^* = (n+1)/4 \text{ and } m_4^* = (n-3)/4 \text{ for } n = 4k+3,$$

where k is an integer and $n \geq 11$.

(d) When $N^* = 5$, the equilibrium group sizes are:

$$m_1^* = \dots = m_5^* = n/5 \text{ for } n = 5k,$$

$$m_1^* = (n + 4)/5 \text{ and } m_2^* = \dots = m_5^* = (n - 1)/5 \text{ for } n = 5k + 1,$$

$$m_1^* = m_2^* = (n + 3)/5 \text{ and } m_3^* = m_4^* = m_5^* = (n - 2)/5 \text{ for } n = 5k + 2,$$

$$m_1^* = m_2^* = m_3^* = (n + 2)/5 \text{ and } m_4^* = m_5^* = (n - 3)/5 \text{ for } n = 5k + 3, \text{ and}$$

$$m_1^* = \dots = m_4^* = (n + 1)/5 \text{ and } m_5^* = (n - 4)/5 \text{ for } n = 5k + 4,$$

where k is an integer and $n \geq 10$.

Using Lemma 6, one can easily obtain the equilibrium group sizes when $N^* \geq 6$. It is easy to see that, given $N^* \geq 2$, the equilibrium size of group i is weakly increasing in the number of players. Table 2 shows the equilibrium group sizes for $n \leq 15$, when $2 \leq N^* \leq 5$.

In the literature on endogenous coalition formation, it is common to obtain the equilibrium coalition structures involving multiple coalitions (see, for example, Bloch, 1995; Yi, 1997, 1998; and Yi and Shin, 2000). Morasch (2000) considers a game of endogenous formation of strategic alliances. Examining the equilibrium structures of strategic alliances resulting when only national alliances are possible, he shows the following: All firms in the national industry join a single alliance if the industry consists of up to four firms, but alliance structures with outsiders and those with two or more alliances are quite likely if the number of firms in the industry exceeds four.

We end this section by highlighting the interesting results obtained from Lemmas 5, 6, and 7.

PROPOSITION 1. (a) *The equilibrium number of strategic groups is one for $n \leq 5$, but is not unique for $n \geq 6$.* (b) *The grand coalition never occurs for $n \geq 5$.* (c) *When more than one strategic group is formed, every player belongs to one of the groups and the*

difference in equilibrium group size between any two groups is at most one. (d) Given $N^ \geq 2$, the equilibrium size of group i is weakly increasing in the number of players.*

Parts (a) and (b) (and Lemmas 5, 6, and 7) say that, as the number of players increases, more players become nonmembers (in the case where just one strategic group is formed) or more strategic groups are formed. This implies that, as the number of players increases, cooperation among the players becomes weaker. Part (b) means that, for $n \geq 5$, if a player deviates individually from the grand coalition, his expected payoff increases. This happens because, as the number of players increases, the equal share in the grand coalition becomes smaller and the members in the group (formed without the deviating player) become less motivated. Indeed, they become less motivated because the winner's fractional share of the group – and thus the prize to the winning member – decreases as the number of players increases. Part (c) is explained by two facts derived from Lemma 4: (i) the expected payoff for each player in groups 2 through N is always greater than that for a player who does not belong to a strategic group, and (ii) the players in a smaller group have greater equilibrium expected payoffs than those in a larger one. The former tells us that a nonmember can increase his expected payoff by joining one of those groups. The latter fact tells us that, if the difference in group size between any two groups is greater than one, then a player in a larger group can increase his expected payoff by switching his membership to a smaller group. Part (d) follows immediately from part (c) and the assumption that $m_{N^*} \leq \dots \leq m_2 \leq m_1$.

V. WINNER'S FRACTIONAL SHARES, EXPECTED PAYOFFS, AND RENT DISSIPATION

We begin by computing the groups' equilibrium winner's fractional shares of our three-stage game. Using Lemmas 1, 2, 5, 6, and 7, we obtain Proposition 2.⁹

PROPOSITION 2. (a) *When just one strategic group is formed in equilibrium, the strategic group's equilibrium winner's fractional share is less than one.* (b) *When two strategic groups are formed in equilibrium, (i) each group's equilibrium winner's fractional share equals one when n is even, and (ii) the equilibrium winner's fractional share of the larger group is less than one and that of the smaller group is greater than one, when n is odd.* (c) *When more than two strategic groups are formed in equilibrium, the groups' equilibrium winner's fractional shares are greater than one.*

In the case where just one strategic group is formed in equilibrium, if a player in the group wins the rent, the winner helps the losers in the group and thus the prize to the winner is less than the rent. This confirms our explanation in section I that one motive of group formation is that players want to insure against their failure in winning the rent. Part (b) says that when two strategic groups are formed and n is even, the winner takes all the rent. Part (c) says that when more than two strategic groups are formed in equilibrium, the winner takes all the rent and further receives "bounties" from the other players in his group. Thus, in this case, the prize to the winner is greater than the rent.

Why do group members write an agreement that if a player in their group wins the rent, the other members in the group pay "bounties" to the winner? One reason is that they can achieve strategic commitments: They can change their opponents' behavior in their favor by using such a "sharing" rule. Another and more convincing reason is that they each – as the potential beneficiary – can be motivated to exert more effort and thus can be more aggressive in the outlays-expending stage because the "sharing" rule makes the winner's prize bigger. In short, each group member agrees to pay "bounties" to increase his own expected payoff. Indeed, each group member supports himself by promising to support the winning member.

An explanation for part (c) is then: competing against the members in the other groups who are more aggressive than individual players, the members in each group make themselves more aggressive by setting their winner's fractional share greater than one.

Next, using Lemmas 4 through 7, we compute the equilibrium expected payoffs for the players, and compare them with the players' expected payoffs resulting from usual individual rent seeking. Note that each player's expected payoff resulting from individual rent seeking, denoted by $\pi(IR)$, is V/n^2 (see note 4). We obtain Proposition 3.

PROPOSITION 3. *(a) When just one strategic group is formed in equilibrium, each player's equilibrium expected payoff is greater than $\pi(IR)$. (b) When two strategic groups are formed in equilibrium, (i) each player's equilibrium expected payoff equals $\pi(IR)$ when n is even, and (ii) the equilibrium expected payoff for each player in a larger group is less than $\pi(IR)$ and that for each player in a smaller group is greater than $\pi(IR)$, when n is odd. (c) When more than two strategic groups are formed in equilibrium, each player's equilibrium expected payoff is less than $\pi(IR)$.*

When just one strategic group is formed in equilibrium, group formation is beneficial both to the group members and to the nonmembers. This means that group formation creates a positive externality for the nonmembers.¹⁰ However, when more than two strategic groups are formed in equilibrium, group formation is never profitable to any players. The intuitive explanations for these follow. When one strategic group is formed, only the group members move before the outlays-expending stage by announcing their sharing rule. According to the sharing rule, if a player in the group wins the rent, the winner must help the losers in the group. The exclusive move and rent sharing mitigate the competition among the players in the following outlays-expending stage, and thus enables the group members to increase their expected payoffs, as compared with those

earned in the case of individual rent seeking. Interestingly, the nonmembers also benefit by that exclusive move.¹¹ By contrast, when more than two strategic groups are formed, the groups announce their sharing rules simultaneously. And, according to each sharing rule, a winner takes all the rent and further receives "bounties" from the other players in his group. These simultaneous moves and the prize bigger than the rent lead to an intense competition, and thus the players' expected payoffs decrease as compared with individual rent seeking.

Yi (1996) examines the welfare effects of endogenous formation of customs unions on member and nonmember countries. He shows that formation of customs unions improves the aggregate welfare of member countries but reduces the welfare of nonmember countries. He also shows that, in any customs-union structure, a member of a large union has a higher level of welfare than a member of a small union. Morasch (2000) shows in the linear Cournot model that, in the case where only national alliances are possible, the members of a smaller alliance earn higher profits because they are more aggressive in the output market. To compare these results with ours, recall the result in Lemma 4 that the players in a smaller group have greater equilibrium expected payoffs than those in a larger one.

Finally, using Lemmas 1, 3, 5, 6, and 7, we compute the equilibrium outlays of the individual players and the equilibrium total outlay, and compare them with the individual outlays and total outlay resulting from usual individual rent seeking. Note that each player's outlay and the total outlay resulting from individual rent seeking, denoted by $x(IR)$ and $H(IR)$, are $V(n-1)/n^2$ and $V(n-1)/n$, respectively (see note 4). We obtain Proposition 4.

PROPOSITION 4. (a) *When just one strategic group is formed in equilibrium, each group member's equilibrium outlay is less than $x(IR)$, each nonmember's equilibrium outlay is greater than $x(IR)$, and the equilibrium total outlay is less than $H(IR)$.* (b) *When two*

strategic groups are formed in equilibrium, (i) each player's equilibrium outlay and the equilibrium total outlay equal $x(IR)$ and $H(IR)$, respectively, when n is even; (ii) the equilibrium outlay of each player in a larger group is less than $x(IR)$ and that of each player in a smaller group is greater than $x(IR)$, when n is odd; and (iii) the equilibrium total outlay equals $H(IR)$. (c) When more than two strategic groups are formed in equilibrium, (i) the equilibrium outlays of some players may be less than $x(IR)$, and (ii) the equilibrium total outlay is greater than $H(IR)$ but is less than the rent.

How much of the rent is dissipated by rent-seeking activities? Examining the extent of rent dissipation is one of the main issues in the literature on rent seeking. It is important because the opportunity costs of resources expended on rent-seeking activities are social costs and thus create economic inefficiency.

Proposition 4 says that, in the case where just one strategic group is formed in equilibrium, rent dissipation (or total outlay) is smaller than with individual rent seeking – that is, the social cost associated with rent seeking decreases by such group formation. However, in the case where more than two strategic groups are formed in equilibrium, rent dissipation is greater than with individual rent seeking. This result that group formation increases rent dissipation compared with individual rent seeking, is new in the literature on rent seeking. Baik (1994) shows in a model similar to ours that group formation always decreases rent dissipation compared with individual rent seeking. The main reason why we obtain the different result is that the winner's fractional share can be greater than unity in this article but cannot be so in Baik (1994). Proposition 4 also implies that the equilibrium total outlay is less than the rent, V , regardless of the number of strategic groups formed. This establishes that less than complete dissipation of the contested rent occurs when the players endogenously form strategic groups.¹²

Using Lemmas 3 and 6, we obtain that, when $N^* \geq 2$, the equilibrium total outlay is $V[1 - 1/n(N^* - 1)]$. It follows from this that, given the number of players, n , rent

dissipation increases as the equilibrium number of strategic groups, N^* , increases. It also follows that, given $N^* \geq 2$, rent dissipation increases as the number of players, n , increases. Another interesting result is that, in contrast with individual rent seeking, rent dissipation may decrease as the number of players increases.¹³ This occurs when an increase in the number of players is accompanied by a decrease in the equilibrium number of strategic groups.

VI. CONCLUSION

We have considered a rent-seeking contest in which players can form strategic groups before they expend their outlays. After obtaining the equilibrium numbers of strategic groups and equilibrium group sizes, we have examined the profitability of endogenous group formation and the effect of such group formation on rent dissipation. We have found the following. When just one strategic group is formed in equilibrium, group formation is beneficial both to the group members and to the nonmembers, and rent dissipation is smaller than with usual individual rent seeking. However, when more than two strategic groups are formed in equilibrium, group formation is never profitable to any players and rent dissipation is greater than with individual rent seeking. Finally, total rent-seeking outlay is less than the rent, regardless of the number of strategic groups formed.

We have shown in section IV that, when the number of players does not exceed five, the grand coalition occurs and, therefore, the inefficiency problem associated with rent seeking disappears.

In section IV, to obtain the equilibrium numbers of strategic groups and equilibrium group sizes, we have checked only whether a player has an incentive to deviate individually from his position. This means that our notion of equilibrium does not rule out the possibility of profitable coalitional-deviations from an equilibrium. That is, in an equilibrium, no player has an incentive to deviate individually, but coalitions of players may have incentives to deviate collectively. Hence, if we use an equilibrium concept that

takes into account coalitional deviations also, we may obtain fewer equilibria than we have in section IV. Indeed, we can refine the "Nash equilibrium set" obtained in section IV – we can obtain sharper predictions about the equilibrium numbers of strategic groups and equilibrium group sizes – by using the concept of coalition-proof Nash equilibrium introduced by Bernheim et al. (1987).¹⁴ However, our main results in section V remain unchanged since the coalition-proof Nash equilibrium set is merely a subset of the Nash equilibrium set.

FOOTNOTES

1. Nitzan (1994) provides an excellent survey of the literature on rent seeking.
2. Baik (1994) and Baik and Shogren (1995) formally study group formation. But the number of groups formed is exogenously restricted to unity. By contrast, we endogenize the number of groups formed.
3. To highlight the first motive to form such groups, one may wish to call them something different, say, *support groups*.
4. This simplest logit-form probability-of-winning function (also called contest success function) is extensively used in the literature on rent seeking. Examples include Tullock (1980), Appelbaum and Katz (1987), Hillman and Riley (1989), Hirshleifer (1989), Katz et al. (1990), Ursprung (1990), Nitzan (1991a, 1991b), Leininger (1993), Baik (1994), Baik and Shogren (1995), Lee (1995), and Che and Gale (1997). For other forms of probability of winning functions, see Baik (1998) and Baik et al. (2001).

Consider the case in which the players compete individually to win the rent. Assume that the players choose their outlays simultaneously. Then, at the Nash equilibrium, each player's expected payoff is V/n^2 , each player's outlay is $V(n-1)/n^2$, and the players' total outlay is $V(n-1)/n$ (see Baik, 1994). We will use these values as the comparative benchmarks when we examine the effects of endogenous group formation on each player's expected payoff, each player's outlay, and the players' total outlay.

5. Imagine the following procedure. Each player throws his name tag into one of n jars. The players who do not want to form a strategic group should throw their names in one particular jar, say, the n th jar. The players who throw their names in the same jar (other than the n th one) will form a strategic group. If the j th jar contains only one name, then the player forms a strategic group with, say, the k th jar players. In order not to have a trivial equilibrium in which no strategic group is formed, we assume that one player throws his name in a jar (other than the n th one) before the other players choose their jars.

6. In the rest of this section, if you consider the case where $m_{N+1} = 0$, then ignore the discussions and analyses for the players in group $N + 1$ and substitute m_{N+1} with zero in all the mathematical results for the players in groups 1 through N .
7. These payoff functions hold for $S > 0$. If all the players expend zero, that is, $S = 0$, then $\pi_{ij} = V/n$ for $i = 1, \dots, N+1$.
8. The proofs of Lemmas 5, 6, and 7 are straightforward and therefore omitted.
9. The proofs of Propositions 2, 3, and 4 are straightforward and therefore omitted.
10. Yi (1997) states that research-coalition formation in oligopolies and customs-union formation in international markets each create negative externalities for nonmembers, whereas formation of output cartels in oligopolies and formation of coalitions to provide public goods each create positive externalities.
11. It follows immediately from Lemmas 4 and 5 that each group member's expected payoff is less than each nonmember's.
12. Many economists obtain the underdissipation-of-rents result. Examples include Tullock (1980), Hillman and Katz (1984), Hillman and Riley (1989), Katz et al. (1990), Ursprung (1990), Guttman et al. (1992), Baik (1994), and Baik and Shogren (1995).
13. Nitzan (1991b) also obtains this result.
14. The coalition-proof Nash equilibrium set may be empty. Yi and Shin (2000) obtain stable structures of research joint ventures, using the concept of coalition-proof Nash equilibrium.

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TABLE 1The Equilibrium Group Sizes When $N^* = 1$

n	m_1^*
2	2
3	3
4	3 and 4
5	3 and 4
6	4
7	4
8	5
9	5
\vdots	
even	$(n + 2)/2$
odd	$(n + 1)/2$

TABLE 2The Equilibrium Group Sizes When $2 \leq N^* \leq 5$

n	(m_1^*, m_2^*)	(m_1^*, m_2^*, m_3^*)	$(m_1^*, m_2^*, m_3^*, m_4^*)$	$(m_1^*, m_2^*, m_3^*, m_4^*, m_5^*)$
6	(3, 3)			
7	(4, 3)			
8	(4, 4)	(3, 3, 2)	(2, 2, 2, 2)	
9	(5, 4)	(3, 3, 3)		
10	(5, 5)	(4, 3, 3)		(2, 2, 2, 2, 2)
11	(6, 5)	(4, 4, 3)	(3, 3, 3, 2)	(3, 2, 2, 2, 2)
12	(6, 6)	(4, 4, 4)	(3, 3, 3, 3)	(3, 3, 2, 2, 2)
13	(7, 6)	(5, 4, 4)	(4, 3, 3, 3)	(3, 3, 3, 2, 2)
14	(7, 7)	(5, 5, 4)	(4, 4, 3, 3)	(3, 3, 3, 3, 2)
15	(8, 7)	(5, 5, 5)	(4, 4, 4, 3)	(3, 3, 3, 3, 3)
⋮				