Contests between Two Groups for a Group-Specific Public-Good/Bad Prize

By Kyung Hwan Baik*

Economics Letters, June 2023, 227: 111130.

Abstract

We study contests between groups for a group-specific public-good/bad prize in which the contest success function for a group can be represented by a continuous function in each group's effort level, where each group's effort level equals the sum of effort levels that the individual players in that group expend. The players expend their effort simultaneously and independently to win the prize or *not* to win it (or both). Obtaining the pure-strategy Nash equilibrium, we establish that, in each group, only the player with the highest valuation and the player with the lowest valuation may be active. We further establish that there are only two active players, either in the same group or in different groups, and the rest expend zero effort.

Keywords: Contest; Rent Seeking; Public-good prize; Public-bad prize; Free riding; Externalities; Sabotage activity; Outside allies

JEL classification: D72, H41, C72

*Department of Economics, Appalachian State University, Boone, NC 28608, USA, and Department of Economics, Sungkyunkwan University, Seoul 03063, South Korea (e-mail: khbaik@skku.edu). I am grateful to two anonymous referees and the Editor of the journal for their helpful comments and suggestions.

1. Introduction

Contests between groups for a group-specific public-good/bad prize are common. For example, consider a situation in which cities and towns compete to be selected as a location for housing development by a private developer or a local government. In this contest, some residents in the selected city or town will benefit from the housing development, and others in the selected city or town will be harmed by it, whereas the residents in the other (losing) cities and towns will neither benefit from, nor be harmed by, the housing development. This means that the housing development can be viewed as a group-specific public-good/bad prize. No doubt, residents in each city or town expend effort or make contributions to win the prize or to hinder their city or town from winning it (or both).

The purpose of this paper is to study such contests. Specifically, we study contests between groups for a group-specific public-good/bad prize in which the contest success function for a group can be represented by a *continuous* function in each group's effort level, where each group's effort level equals the *sum* of effort levels that the individual players in that group expend.

Formally, we consider contests in which two groups compete with each other to win a group-specific public-good/bad prize, every player's valuation for the prize is publicly known, and every player has a *constant* marginal cost of increasing effort. The individual players in each group expend their effort to win the prize or to hinder their group from winning it – or rather to help the other group win the prize – or both. All the players in both groups choose their effort levels simultaneously and independently. In particular, this paper focuses on examining who expends positive effort, how much effort each player expends, how severe the free-rider problem is, and what factors determine the players' effort levels.

Obtaining the pure-strategy Nash equilibrium, we establish that, in each group, only the player with the highest valuation and the player with the lowest valuation (that is negative) may be active – that is, they may expend positive effort. We further establish that there are only two active players, either in the same group or in different groups, and the rest expend zero effort.

There are four different cases: the case where only the highest-valuation player in each group is active, the two cases where only the highest-valuation player and the lowest-valuation player in the same group are active, and the case where only the lowest-valuation player in each group is active. These results basically come from the fact that a player with a higher [lower] valuation, when valuations are positive [negative], obtains a greater marginal gross payoff from increasing effort, while all the players incur the same marginal cost that is constant at unity.

Many papers study contests with a group-specific public-good prize: See, for example, Katz et al. (1990), Baik (1993, 2008), Baik and Shogren (1998), Baik et al. (2001), Epstein and Mealem (2009), Lee (2012), Kolmar and Rommeswinkel (2013), Chowdhury et al. (2013), Topolyan (2014), Barbieri et al. (2014), Chowdhury and Topolyan (2016), Barbieri and Malueg (2016), and Dasgupta and Neogi (2018). The most important difference of this paper from those papers is that, in this paper, the prize is a public good for some players and a public bad for others in the winning group, whereas, in those papers, the prize is a public good for all the players in the winning group.

2. The model

Consider a contest in which there are two groups, 1 and 2, that compete over a prize. Group 1 consists of m risk-neutral players, where $m \geq 1$, and group 2 consists of n risk-neutral players, where $n \geq 1$. The prize is a group-specific public-good/bad one – that is, it is a public good for some players and a public bad for others in the winning group, but it provides neither benefit nor harm to the players in the losing group. Players in each group expend effort or make contributions simultaneously and independently to win the prize or *not* to win it (or both).

Let M_i, for $i = 1, 2$, represent the set of players in group i. Then we have that $M_1 \equiv \{1, . . . , m\}$ and $M_2 \equiv \{1, . . . , n\}$. Let v_{ik}, for $k \in M_i$, represent the valuation for the prize of player k in group i, where v_{ik} is positive or negative. Each player's valuation for the prize is publicly known at the start of the game.

Assumption 1. (*a*) *We assume that* $v_{i1} > v_{i2} \geq \ldots \geq v_{iz-1} > v_{iz}$ *for* $i = 1, 2$, *where* $z = m$ *for* $i = 1$ *and* $z = n$ *for* $i = 2$. (*b*) *We assume that* $v_{11}v_{1m} < 0$ *and* $v_{21}v_{2n} < 0$.

Part (*b*) of Assumption 1 indicates that the prize is a public good for some players and a public bad for others in each group. It eliminates from consideration less interesting cases, such as a case where all the players in either group have negative valuations for the prize.

Let x_{ik}, for $i = 1, 2$ and $k \in M_i$, represent the effort level expended by player k in group i to win the prize, where $x_{ik} \geq 0$. Let y_{ik} represent the effort level expended by player k in group i *not* to win the prize, where $y_{ik} \geq 0$. This is the effort level expended to hinder his own group from winning the prize and help the other group win the prize. It may include the effort expended in sabotage activities which harm his own group and benefit the other group (see, for example, Konrad, 2000; Chowdhury and Gürtler, 2015). Let $X_i \equiv \sum_{k=1}^{z} x_{ik}$ and $Y_i \equiv \sum_{k=1}^{z} y_{ik}$, where $z = m$ for $i = 1$ and $z = n$ for $i = 2$. The effort expended by each player is irreversible.

Let p_i, for $i = 1, 2$, represent the probability that group i wins the prize, where $0 \leq p_i \leq 1$ and $p_1 + p_2 = 1$. We assume the following contest success function for group i:

$$p_i = p_i(X_1 + \theta Y_2, X_2 + \theta Y_1),$$

where $\theta > 0$ and the function p_i has the properties specified in Assumption 2 below. The parameter θ reflects the relative effectiveness of effort that is expended by players in one group to help the other group win the prize. This contest success function says that group i's probability p_i of winning the prize depends on the effort levels expended by both groups *not* to win the prize as well as the effort levels expended by both groups to win the prize.

Assumption 2. (*a*) *We assume that* $\partial p_i/\partial X_i \geq 0$, $\partial p_i/\partial Y_j \geq 0$, $\partial^2 p_i/\partial X_i^2 \leq 0$, $\partial^2 p_i/\partial Y_j^2 \leq 0$, $\partial p_i/\partial X_j \leq 0$, $\partial p_i/\partial Y_i \leq 0$, $\partial^2 p_i/\partial X_j^2 \geq 0$, *and* $\partial^2 p_i/\partial Y_i^2 \geq 0$, *for* $i, j = 1, 2$ *with* $i \neq j$.

(b) *We further assume that* $\partial p_i/\partial X_i > 0$, $\partial p_i/\partial Y_j > 0$, $\partial^2 p_i/\partial X_i^2 < 0$, *and* $\partial^2 p_i/\partial Y_j^2 < 0$, *when* $X_j + Y_i > 0$; *and* $\partial p_i/\partial X_j < 0$, $\partial p_i/\partial Y_i < 0$, $\partial^2 p_i/\partial X_j^2 > 0$, *and* $\partial^2 p_i/\partial Y_i^2 > 0$, *when* $X_i + Y_j > 0$.

Assumption 2 says that, *ceteris paribus*, group i's probability of winning is increasing in its own effort level X_i at a decreasing rate; however, it is decreasing in the other group's effort level X_j at a decreasing rate, for $i, j = 1, 2$ with $i \neq j$. It says also that, *ceteris paribus*, group i's probability of winning is increasing in the other group's effort level Y_j at a decreasing rate; however, it is decreasing in its own effort level Y_i at a decreasing rate.

A specific form of the function p_i is: $p_i = (X_i + \theta Y_j)/S$ if $S > 0$, and $p_i = 1/2$ if $S = 0$, where $S \equiv X_1 + \theta Y_2 + X_2 + \theta Y_1$. This simplest logit-form contest success function is extensively used in the literature on the theory of contests.

Let π_{ik}, for $i = 1, 2$ and $k \in M_i$, represent the expected payoff for player k in group i. Then the payoff function for player k in group i is

$$\pi_{ik} \;=\; v_{ik}\, p_i(X_1 + \theta Y_2, X_2 + \theta Y_1) \;-\; x_{ik} \;-\; y_{ik}. \tag{1}$$

We formally consider the following simultaneous-move game. At the beginning of the game, every player knows the sizes of the two groups and the players' valuations for the prize. Next, all the players in both groups choose their effort levels simultaneously and independently – that is, when player k in group i, for $i = 1, 2$ and $k \in M_i$, chooses his effort levels, x_{ik} and y_{ik}, he does not know the other players' effort levels. Finally, the winning group is determined at the end of the game.

We assume that all of the above is common knowledge among the players. We employ Nash equilibrium as the solution concept.

3. Active players and free riders in equilibrium

We begin by showing that, in equilibrium, each player with a positive valuation expends zero effort to help the other group win the prize, while each player with a negative valuation expends zero effort to help his own group win it. Let M_i^+, for $i = 1, 2$, represent the set of players in group i who have positive valuations. Let M_i^-, for $i = 1, 2$, represent the set of players in group i who have negative valuations. By Assumption 1, neither set is empty. Let the superscript * represent the equilibrium values of effort levels.

Lemma 1. *In equilibrium, we have for* $i = 1, 2$*:* (*a*) $y_{ik}^* = 0$ *for* $k \in M_i^+$, *and* (*b*) $x_{ih}^* = 0$ *for* $h \in M_i^-$.

The proof of Lemma 1 is straightforward, and therefore omitted. Lemma 1 is intuitively natural. Each player in M_i^+ has a positive valuation for the prize. This implies that, if his own group wins the prize, he gets a positive benefit from it; otherwise, he gets nothing. Accordingly, he wants his group to win the prize, and thus he does not help the other group win the prize. By contrast, each player in M_i^- has a negative valuation for the prize. This implies that, if his own group wins the prize, he suffers a harm from it; otherwise, he suffers no harm. Accordingly, he wants the other group to win the prize, and thus he does not help his own group win the prize.

Based on Lemma 1, in order to obtain the pure-strategy Nash equilibrium of the game, we will henceforth restrict attention only to the strategy profiles at which, for $i = 1, 2$, $y_{ik} = 0$ for $k \in M_i^+$ and $x_{ih} = 0$ for $h \in M_i^-$.

3.1. Four possible active players in equilibrium

Consider player k in group 1, for $k \in M_1^+$, whose valuation for the prize is positive, and player h in group 2, for $h \in M_2^-$, whose valuation for the prize is negative. These players want group 1 to win the prize. We show that, in equilibrium, only player 1 in M_1^+ and player n in M_2^- may be active.

Player k, for $k \in M_1^+$, seeks to maximize his expected payoff

$$\pi_{1k} \;=\; v_{1k}\, p_1(X_1 + \theta Y_2, X_2 + \theta Y_1) \;-\; x_{1k} \tag{2}$$

over his effort level, $x_{1k} \geq 0$, given effort levels of all the other players. Note that, based on Lemma 1, we set $y_{1k} = 0$ in expression (2). Let x_{1k}^b represent the best response of player k in M_1^+ to a list of the other players' effort levels. Then, it satisfies the first-order condition:

$$\begin{aligned} &\partial\pi_{1k}/\partial x_{1k} = v_{1k}(\partial p_1/\partial x_{1k}) - 1 = v_{1k}\{\partial p_1/\partial(X_1 + \theta Y_2)\} - 1 = 0 \quad \text{for } x_{1k}^b > 0 \\ &\text{or} \\ &\partial\pi_{1k}/\partial x_{1k} = v_{1k}(\partial p_1/\partial x_{1k}) - 1 = v_{1k}\{\partial p_1/\partial(X_1 + \theta Y_2)\} - 1 \leq 0 \quad \text{for } x_{1k}^b = 0. \end{aligned} \tag{3}$$

Under Assumption 2, the payoff function, π_{1k}, is strictly concave in the effort level, x_{1k}, which implies that the second-order condition for maximizing π_{1k} is satisfied and x_{1k}^b is unique.

Player h, for $h \in M_2^-$, seeks to maximize his expected payoff

$$\begin{aligned} \pi_{2h} \;&=\; v_{2h}\, p_2(X_1 + \theta Y_2, X_2 + \theta Y_1) \;-\; y_{2h} \\ &=\; v_{2h}\{1 - p_1(X_1 + \theta Y_2, X_2 + \theta Y_1)\} \;-\; y_{2h} \end{aligned}$$

over his effort level, $y_{2h} \geq 0$, given effort levels of all the other players. Let y_{2h}^b represent the best response of player h in M_2^- to a list of the other players' effort levels. Then, it satisfies the first-order condition:

$$\begin{aligned} &\partial\pi_{2h}/\partial y_{2h} = v_{2h}(\partial p_2/\partial y_{2h}) - 1 = -\theta v_{2h}\{\partial p_1/\partial(X_1 + \theta Y_2)\} - 1 = 0 \quad \text{for } y_{2h}^b > 0 \\ &\text{or} \\ &\partial\pi_{2h}/\partial y_{2h} = v_{2h}(\partial p_2/\partial y_{2h}) - 1 = -\theta v_{2h}\{\partial p_1/\partial(X_1 + \theta Y_2)\} - 1 \leq 0 \quad \text{for } y_{2h}^b = 0. \end{aligned} \tag{4}$$

Under Assumption 2, the payoff function, π_{2h}, is strictly concave in the effort level, y_{2h}, which implies that the second-order condition for maximizing π_{2h} is satisfied and y_{2h}^b is unique.

Lemma 2, together with Lemma 1, shows that all players in $M_1^+ \setminus \{1\}$ and all players in $M_2^- \setminus \{n\}$ expend zero effort in equilibrium.

Lemma 2. *In equilibrium, we have:* (*a*) $x^*_{1k} = 0$ *for all* $k \in M_1^+ \setminus \{1\}$, *and* (*b*) $y^*_{2h} = 0$ *for all* $h \in M_2^- \setminus \{n\}$.

Proof. (*a*) Consider a Nash equilibrium, $(x^*_{11}, y^*_{11}, \ldots, x^*_{1m}, y^*_{1m}, x^*_{21}, y^*_{21}, \ldots, x^*_{2n}, y^*_{2n})$. Suppose on the contrary that $x^*_{1k} > 0$ for some $k \in M_1^+ \setminus \{1\}$. Then, from expression (3), we have that $v_{1k}\{\partial p_1/\partial(X_1 + \theta Y_2)\} - 1 = 0$. This, together with $v_{11} > v_{1k} > 0$, yields that $\partial\pi_{11}/\partial x_{11} = v_{11}\{\partial p_1/\partial(X_1 + \theta Y_2)\} - 1 > 0$, which implies that player 1 in group 1 has an incentive to increase his effort level x_{11} in the Nash equilibrium. This contradicts the assumption that x^*_{11} is his equilibrium effort level.
(*b*) Consider a Nash equilibrium, $(x^*_{11}, y^*_{11}, \ldots, x^*_{1m}, y^*_{1m}, x^*_{21}, y^*_{21}, \ldots, x^*_{2n}, y^*_{2n})$. Suppose on the contrary that $y^*_{2h} > 0$ for some $h \in M_2^- \setminus \{n\}$. Then, from expression (4), we have that $-\theta v_{2h}\{\partial p_1/\partial(X_1 + \theta Y_2)\} - 1 = 0$. This, together with $0 > v_{2h} > v_{2n}$, yields that $\partial\pi_{2n}/\partial y_{2n} = -\theta v_{2n}\{\partial p_1/\partial(X_1 + \theta Y_2)\} - 1 > 0$, which implies that player n in group 2 has an incentive to increase his effort level y_{2n} in the Nash equilibrium. This contradicts the assumption that y^*_{2n} is his equilibrium effort level. □

In part (*a*) of Lemma 2, every player in $M_1^+ \setminus \{1\}$ expends zero effort for the prize in equilibrium because his marginal gross payoff is less than his marginal cost at the equilibrium total effort level – in other words, because he expects the equilibrium total effort level to be large enough from his perspective. Similarly, in part (*b*) of Lemma 2, every player in $M_2^- \setminus \{n\}$ expends zero effort for the prize in equilibrium because his marginal gross payoff is less than his marginal cost at the equilibrium total effort level.

Lemma 2 implies that only player 1 in M_1^+ and player n in M_2^- may be active in equilibrium.

Next, consider player k in group 2, for $k \in M_2^+$, whose valuation for the prize is positive, and player h in group 1, for $h \in M_1^-$, whose valuation for the prize is negative. These players want group 2 to win the prize. We show that, in equilibrium, only player 1 in M_2^+ and player m in M_1^- may be active.

Player k, for $k \in M_2^+$, seeks to maximize his expected payoff

$$\pi_{2k} \;=\; v_{2k}\, p_2(X_1 + \theta Y_2, X_2 + \theta Y_1) \;-\; x_{2k}$$

over his effort level, $x_{2k} \geq 0$, given effort levels of all the other players. Let x_{2k}^b represent the best response of player k in M_2^+ to a list of the other players' effort levels. Then, it satisfies the first-order condition:

$$\partial\pi_{2k}/\partial x_{2k} = v_{2k}(\partial p_2/\partial x_{2k}) - 1 = v_{2k}\{\partial p_2/\partial(X_2 + \theta Y_1)\} - 1 = 0 \quad \text{for } x_{2k}^b > 0$$

or (5)

$$\partial\pi_{2k}/\partial x_{2k} = v_{2k}(\partial p_2/\partial x_{2k}) - 1 = v_{2k}\{\partial p_2/\partial(X_2 + \theta Y_1)\} - 1 \leq 0 \quad \text{for } x_{2k}^b = 0.$$

Under Assumption 2, the payoff function, π_{2k}, is strictly concave in the effort level, x_{2k}, which implies that the second-order condition for maximizing π_{2k} is satisfied and x_{2k}^b is unique.

Player h, for $h \in M_1^-$, seeks to maximize his expected payoff

$$\begin{aligned}\pi_{1h} \;&=\; v_{1h}\, p_1(X_1 + \theta Y_2, X_2 + \theta Y_1) \;-\; y_{1h} \\ &=\; v_{1h}\{1 - p_2(X_1 + \theta Y_2, X_2 + \theta Y_1)\} \;-\; y_{1h}\end{aligned}$$

over his effort level, $y_{1h} \geq 0$, given effort levels of all the other players. Let y_{1h}^b represent the best response of player h in M_1^- to a list of the other players' effort levels. Then, it satisfies the first-order condition:

$$\partial\pi_{1h}/\partial y_{1h} = v_{1h}(\partial p_1/\partial y_{1h}) - 1 = \; - \theta v_{1h}\{\partial p_2/\partial(X_2 + \theta Y_1)\} - 1 = 0 \quad \text{for } y_{1h}^b > 0$$

or (6)

$$\partial\pi_{1h}/\partial y_{1h} = v_{1h}(\partial p_1/\partial y_{1h}) - 1 = \; - \theta v_{1h}\{\partial p_2/\partial(X_2 + \theta Y_1)\} - 1 \leq 0 \quad \text{for } y_{1h}^b = 0.$$

Under Assumption 2, the payoff function, π_{1h}, is strictly concave in the effort level, y_{1h}, which implies that the second-order condition for maximizing π_{1h} is satisfied and y_{1h}^b is unique.

Lemma 3, together with Lemma 1, shows that all players in $M_2^+ \setminus \{1\}$ and all players in $M_1^- \setminus \{m\}$ expend zero effort in equilibrium.

Lemma 3. *In equilibrium, we have:* (*a*) $x_{2k}^* = 0$ *for* $k \in M_2^+ \setminus \{1\}$, *and* (*b*) $y_{1h}^* = 0$ *for* $h \in M_1^- \setminus \{m\}$.

The proof and explanation of Lemma 3 are similar to those of Lemma 2, and therefore omitted.

Based on Lemmas 1, 2, and 3, we conclude that, in equilibrium, only players 1 and m in group 1 and players 1 and n in group 2 may expend positive effort and the rest expend zero effort.

3.2. Two active players in equilibrium

We assume, for simplicity, that $v_{11} \neq \theta|v_{2n}|$ and $v_{21} \neq \theta|v_{1m}|$. In this case, we first show that two among the four possible active players are actually inactive in equilibrium.

Lemma 4. *In equilibrium, we have:* (*a*) $y_{2n}^* = y_{1m}^* = 0$ if $v_{11} > \theta|v_{2n}|$ *and* $v_{21} > \theta|v_{1m}|$, (*b*) $y_{2n}^* = x_{21}^* = 0$ if $v_{11} > \theta|v_{2n}|$ *and* $v_{21} < \theta|v_{1m}|$, (*c*) $x_{11}^* = y_{1m}^* = 0$ if $v_{11} < \theta|v_{2n}|$ *and* $v_{21} > \theta|v_{1m}|$, *and* (*d*) $x_{11}^* = x_{21}^* = 0$ if $v_{11} < \theta|v_{2n}|$ *and* $v_{21} < \theta|v_{1m}|$.

Proof. (*a*) Consider a Nash equilibrium, $(x_{11}^*, y_{11}^*, \ldots, x_{1m}^*, y_{1m}^*, x_{21}^*, y_{21}^*, \ldots, x_{2n}^*, y_{2n}^*)$. Suppose on the contrary that $y_{2n}^* > 0$. Then, from expression (4), we have that $-\theta v_{2n}\{\partial p_1/\partial(X_1 + \theta Y_2)\} - 1 = 0$. This, together with $v_{11} > \theta|v_{2n}|$, yields that $\partial\pi_{11}/\partial x_{11} = v_{11}\{\partial p_1/\partial(X_1 + \theta Y_2)\} - 1 > 0$ (see expression (3)), which implies that player 1 in group 1 has an incentive to increase his effort level x_{11} in the Nash equilibrium. This contradicts the assumption that x_{11}^* is his equilibrium effort level.

Next, consider a Nash equilibrium, $(x^*_{11}, y^*_{11}, \ldots, x^*_{1m}, y^*_{1m}, x^*_{21}, y^*_{21}, \ldots, x^*_{2n}, y^*_{2n})$. Suppose on the contrary that $y^*_{1m} > 0$. Then, from expression (6), we have that $-\theta v_{1m}\{\partial p_2/\partial(X_2 + \theta Y_1)\} - 1 = 0$. This, together with $v_{21} > \theta|v_{1m}|$, yields that $\partial \pi_{21}/\partial x_{21} = v_{21}\{\partial p_2/\partial(X_2 + \theta Y_1)\} - 1 > 0$ (see expression (5)), which implies that player 1 in group 2 has an incentive to increase his effort level x_{21} in the Nash equilibrium. This contradicts the assumption that x^*_{21} is his equilibrium effort level.

The proof of parts (b), (c), and (d) is similar to that of part (a), and therefore omitted. □

Based on Lemmas 1 through 4, we conclude that, in equilibrium, there are only two active players and the rest free ride.

3.3. The pure-strategy Nash equilibrium of the game

Now we obtain the pure-strategy Nash equilibrium of the game. At the Nash equilibrium, denoted by $(x^*_{11}, y^*_{11}, \ldots, x^*_{1m}, y^*_{1m}, x^*_{21}, y^*_{21}, \ldots, x^*_{2n}, y^*_{2n})$, each player's pair of effort levels, one expended to win the prize and the other expended *not* to win the prize, is the best response to the other players' pairs of effort levels. Using Lemmas 1 through 4, we obtain the following proposition.

Proposition 1. *There are four different cases to consider, depending on the values of* v_{11}, v_{1m}, v_{21}, v_{2n}, *and the parameter* θ. *The following strategy profile in each case constitutes the Nash equilibrium of the game.*

(a) *If* $v_{11} > \theta|v_{2n}|$ *and* $v_{21} > \theta|v_{1m}|$, *the players play the following strategies:* $x^*_{11} = x^b_{11}$, $x^*_{21} = x^b_{21}$, $x^*_{ij} = 0$ *for* $i = 1, 2$ *and* $j \in M_i \setminus \{1\}$, *and* $y^*_{ij} = 0$ *for* $i = 1, 2$ *and* $j \in M_i$. *In this case, we have:* $X^*_1 = x^b_{11}$, $X^*_2 = x^b_{21}$, *and* $Y^*_1 = Y^*_2 = 0$.

(b) *If* $v_{11} > \theta|v_{2n}|$ *and* $v_{21} < \theta|v_{1m}|$, *the players use the following strategies:* $x^*_{11} = x^b_{11}$, $y^*_{1m} = y^b_{1m}$, $x^*_{1j} = 0$ *for* $j \in M_1 \setminus \{1\}$, $y^*_{1j} = 0$ *for* $j \in M_1 \setminus \{m\}$, *and* $x^*_{2j} = y^*_{2j} = 0$ *for* $j \in M_2$. *In this case, we have:* $X^*_1 = x^b_{11}$, $Y^*_1 = y^b_{1m}$, *and* $X^*_2 = Y^*_2 = 0$.

(c) *If* $v_{11} < \theta|v_{2n}|$ *and* $v_{21} > \theta|v_{1m}|$, *the players use the following strategies:* $x^*_{21} = x^b_{21}$, $y^*_{2n} = y^b_{2n}$, $x^*_{1j} = y^*_{1j} = 0$ *for* $j \in M_1$, $x^*_{2j} = 0$ *for* $j \in M_2 \setminus \{1\}$, *and* $y^*_{2j} = 0$ *for* $j \in M_2 \setminus \{n\}$. *In this case, we have:* $X^*_2 = x^b_{21}$, $Y^*_2 = y^b_{2n}$, *and* $X^*_1 = Y^*_1 = 0$.

(d) *If* $v_{11} < \theta|v_{2n}|$ *and* $v_{21} < \theta|v_{1m}|$, *the players play the following strategies:* $y^*_{1m} = y^b_{1m}$, $y^*_{2n} = y^b_{2n}$, $x^*_{ij} = 0$ *for* $i = 1, 2$ *and* $j \in M_i$, *and* $y^*_{ij} = 0$ *for* $i = 1, 2$ *and* $j \in M_i \setminus \{z\}$, *where* $z = m$ *for* $i = 1$ *and* $z = n$ *for* $i = 2$. *In this case, we have:* $Y^*_1 = y^*_{1m}$, $Y^*_2 = y^*_{2n}$, *and* $X^*_1 = X^*_2 = 0$.

In part (a) of Proposition 1, only two players, player 1 in group 1 and player 1 in group 2, are active. Each of the two active players has the highest valuation for the prize in his own group, and expends his effort only to win the prize. His effort expended to hinder his own group from winning the prize is zero. Hence, the equilibrium effort level x^*_{11} of player 1 in group 1 is the best response to only the equilibrium effort level x^*_{21} of player 1 in group 2, and vice versa.

In part (b), only two players, players 1 and m in group 1, are active – accordingly, all players in group 2 are inactive. The first active player has the highest valuation for the prize in group 1, and expends his effort only to win the prize. The second active player has the lowest valuation for the prize in group 1, and expends his effort only to hinder group 1 from winning the prize – that is, only to help group 2 win the prize. Hence, the equilibrium effort level x^*_{11} of player 1 in group 1 is the best response to only the equilibrium effort level y^*_{1m} of player m in the same group, and vice versa.

In part (c), only two players, players 1 and n in group 2, are active – accordingly, all players in group 1 are inactive. The first active player has the highest valuation for the prize in group 2, and expends his effort only to win the prize. The second active player has the lowest valuation for the prize in group 2, and expends his effort only to hinder group 2 from winning the prize – that is, only to help group 1 win the prize. Hence, the equilibrium effort level x^*_{21} of player 1 in group 2 is the best response to only the equilibrium effort level y^*_{2n} of player n in the same group, and vice versa.

In part (*d*), only two players, player m in group 1 and player n in group 2, are active. Each of the two active players has the lowest valuation for the prize in his own group, and expends his effort only to hinder his own group from winning the prize – that is, only to help the other group win the prize. His effort expended to win the prize is zero. Hence, the equilibrium effort level y_{1m}^* of player m in group 1 is the best response to only the equilibrium effort level y_{2n}^* of player n in group 2, and vice versa.

Consider the housing development example introduced in Section 1. Proposition 1 implies that there are only two people (or two "groups" of people) that expend effort or make contributions to win the group-specific public-good/bad prize, and the rest free ride. There are four different cases: the case where only the highest-valuation resident (or group of residents) in each city or town is active, the two cases where only the highest-valuation resident (or group of residents) and the lowest-valuation resident (or group of residents) in the same city or town are active, and the case where only the lowest-valuation resident (or group of residents) in each city or town is active.

4. Conclusions

We believe that common are contests between groups for a group-specific public-good/bad prize in which the contest success function for a group can be represented by a *continuous* function in each group's effort level, where each group's effort level equals the *sum* of effort levels that the individual players in that group expend. However, we also believe that there are contests between groups for a group-specific public-good/bad prize in which the contest success function for a group can be represented by the selection rule of all-pay auctions or a difference-form contest success function. In addition, we believe that there are contests between groups for a group-specific public-good/bad prize in which a group's effort level equals the minimum or maximum of effort levels that the individual players in that group expend. It would be interesting to study these contests.

References

Baik, Kyung Hwan. 1993. "Effort Levels in Contests: The Public-Good Prize Case." *Economics Letters* 41 (4): 363-67.

Baik, Kyung Hwan. 2008. "Contests with Group-Specific Public-Good Prizes." *Social Choice and Welfare* 30 (1): 103-17.

Baik, Kyung Hwan, In-Gyu Kim, and Sunghyun Na. 2001. "Bidding for a Group-Specific Public-Good Prize." *Journal of Public Economics* 82 (3): 415-29.

Baik, Kyung Hwan, and Jason F. Shogren. 1998. "A Behavioral Basis for Best-Shot Public-Good Contests." In *Advances in Applied Microeconomics: Contests*, Vol. 7, edited by Michael R. Baye, 169-78. Stamford, CT: JAI Press.

Barbieri, Stefano, and David A. Malueg. 2016. "Private-Information Group Contests: Best-Shot Competition." *Games and Economic Behavior* 98: 219-34.

Barbieri, Stefano, David A. Malueg, and Iryna Topolyan. 2014. "The Best-Shot All-Pay (Group) Auction with Complete Information." *Economic Theory* 57 (3): 603-40.

Chowdhury, Subhasish M., and Oliver Gürtler. 2015. "Sabotage in Contests: A Survey." *Public Choice* 164 (1-2): 135-55.

Chowdhury, Subhasish M., Dongryul Lee, and Roman M. Sheremeta. 2013. "Top Guns May Not Fire: Best-Shot Group Contests with Group-Specific Public Good Prizes." *Journal of Economic Behavior and Organization* 92: 94-103.

Chowdhury, Subhasish M., and Iryna Topolyan. 2016. "The Attack-and-Defense Group Contests: Best Shot versus Weakest Link." *Economic Inquiry* 54 (1): 548-57.

Dasgupta, Indraneel, and Ranajoy Guha Neogi. 2018. "Between-Group Contests over Group-Specific Public Goods with Within-Group Fragmentation." *Public Choice* 174 (3-4): 315-34.

Epstein, Gil S., and Yosef Mealem. 2009. "Group Specific Public Goods, Orchestration of Interest Groups with Free Riding." *Public Choice* 139 (3-4): 357-69.

Katz, Eliakim, Shmuel Nitzan, and Jacob Rosenberg. 1990. "Rent-Seeking for Pure Public Goods." *Public Choice* 65 (1): 49-60.

Kolmar, Martin, and Hendrik Rommeswinkel. 2013. "Contests with Group-Specific Public Goods and Complementarities in Efforts." *Journal of Economic Behavior and Organization* 89: 9-22.

Konrad, Kai A. 2000. "Sabotage in Rent-Seeking Contests." *Journal of Law, Economics, and Organization* 16 (1): 155-65.

Lee, Dongryul. 2012. "Weakest-Link Contests with Group-Specific Public Good Prizes." *European Journal of Political Economy* 28 (2): 238-48.

Topolyan, Iryna. 2014. "Rent-Seeking for a Public Good with Additive Contributions." *Social Choice and Welfare* 42 (2): 465-76.