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Three-player contests with a potential inactive player: Endogenous timing of effort exertion

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Abstract

We study a contest where there are two active players in equilibrium when three players expend effort simultaneously to win a prize. We look at how endogenous timing of effort exertion affects the players' behavior. The players play the following game. First, they announce simultaneously whether they each will expend effort in period 1 or in period 2. Then, after knowing when they expend effort, each player expends effort in the period which he announced. We find interesting results, focusing on the players' decisions on when to expend effort, the identities of active players, and the effort levels in a subgame-perfect equilibrium.

KEYWORDS
active player, contest, endogenous timing, potential inactive player, rent seeking

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D72, D74, C72

1 | INTRODUCTION

A well-known result obtained from two-player asymmetric contests with endogenous timing of effort exertion is that, in equilibrium, the weak player—determined by the players' valuations for the prize and their relative abilities—expends effort before the strong player (see e.g., Baik & Shogren, 1992; Leininger, 1993).[1]

Recently, Baik et al. (2022) have studied a three-player contest with endogenous timing of effort exertion. In their model, there are two periods, 1 and 2, in which the players can expend their effort to win a prize. The players play the following game. First, the three players choose independently between *Period 1* and *Period 2*, and announce their choices simultaneously. Then, each player chooses his effort level in the period which he announced. The winner is determined at the end of the game. Restricting their analysis to cases where all three players are active in the equilibrium of every second-stage subgame, they show that each of the players announces *Period 1*, and thus they all choose their effort levels simultaneously in period 1. This result is in sharp contrast to the well-known result obtained from two-player asymmetric contests with endogenous timing of effort exertion. One may say that, given "moderate" asymmetries among three players, the presence of an additional player changes drastically the equilibrium timing of effort exertion.

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Now, an interesting question that arises is: What happens if asymmetries among three players are not moderate? In this case, does the presence of an additional player change the equilibrium timing of effort exertion, compared with two-player asymmetric contests with endogenous timing of effort exertion? Who are active players in the equilibrium? How much effort do the players expend?

To address these questions, we study a three-player asymmetric contest with endogenous timing of effort exertion in which asymmetries among three players are not moderate. More precisely, we study one in which there are only two active players—that is, only two players expend positive effort—in equilibrium when the three players choose their effort levels simultaneously.

One example of such a three-player asymmetric contest may be a United States presidential election because it has the following facts. First, there have often been three candidates: the nominee of the Democratic Party, the nominee of the Republican Party, and a third party or independent candidate. Second, asymmetries among the candidates have arisen because the candidates may have different valuations for winning the election, and may have different abilities to get peoples' votes. Third, throughout the history of presidential elections, even a popular (or strong) third party or independent candidate, such as Ross Perot in 1992, has been the weakest among the three candidates in the relevant election year and relatively inactive. Fourth, given two periods, the early and the late period, the three presidential candidates have first announced when they will run their major election campaigns, and then have actually run their election campaigns according to their announced campaign schedules.

Another example may come from competition in presidential primaries and caucuses to be selected as a party's nominee for president of the United States. It can be justified similarly to the above example.

Yet another example of the three-player asymmetric contest studied in the current paper may arise in competition among the United States vaccine makers to invent a COVID-19 vaccine at the beginning of the COVID-19 era because it had the following facts. First, there were three forerunners of COVID-19 vaccines: Pfizer, Moderna, and Johnson and Johnson. Second, asymmetries among the vaccine makers existed because the vaccine makers had different valuations for inventing a COVID-19 vaccine, and had different abilities to invent it. Third, there was the weakest one among the three vaccine makers, which was relatively behind the other two in terms of ability to invent a COVID-19 vaccine. Fourth, given two periods, the early and the late period, the three vaccine makers first announced (and committed to) their research and development plans for a COVID-19 vaccine, and then carried out their research and development activities according to their announced plans.

We consider the following game, which is similar to the one in Baik et al. (2022). In the first stage, the three players choose independently between *Period 1* and *Period 2*, and announce their choices simultaneously. In the second (or effort-expending) stage, after knowing when the players expend effort, each player independently chooses his effort level in the period which he announced. The winner is determined at the end of the game.

We first show that, given *not* significant asymmetries between the top two players and the weakest player, the game has no subgame-perfect equilibrium in pure strategies in which the weakest announces *Period 1* in the first stage. However, given significant asymmetries between them, the game has a subgame-perfect equilibrium in which the strongest player announces *Period 2* in the first stage and chooses his effort level in period 2, while the other players announce *Period 1* in the first stage and choose their effort levels in period 1.

We also show that, given very significant asymmetries between the top two players and the weakest player, the game has a subgame-perfect equilibrium in which the strongest player and the weakest player announce *Period 2* in the first stage, while the other player announces *Period 1* in the first stage.

Note that these results regarding the players' decisions on the timing of effort exertion is similar to the well-known result obtained from two-player asymmetric contests with endogenous timing of effort exertion.

Next, we show that, in the equilibrium in which the strongest player announces *Period 2* while the other players announce *Period 1*, the weakest player is active if asymmetry between the second strongest player and the weakest player is *not* very significant; however, he is inactive if asymmetry between them is very significant. We also show that, in the equilibrium in which the strongest player and the weakest player announce *Period 2* while the second strongest player announces *Period 1*, the weakest player is inactive.

Finally, we show that, in any of the subgame-perfect equilibria, the total effort level is lower—even when the weakest player is active—compared with the corresponding three-player simultaneous-move contest.[2]

Contests with endogenous timing of effort exertion have been studied by many other researchers. Nitzan (1994), Baik and Shogren (1994), Konrad and Leininger (2007), and Baik and Lee (2013) have studied contests in which players' valuations for a prize are exogenously fixed and publicly known. Baik and Shogren (1994) consider an environmental conflict between a citizens' group and a firm in which the citizens' group's legal expenses are reimbursed if the group

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wins, while the firm's expenses are not if the firm wins. They show that, if reimbursement is less than 50% of expenses, both the citizens' group and the firm exert their effort simultaneously. Konrad and Leininger (2007) consider an n-player contest with all-pay-auction contest success functions, and show that the player with the lowest cost of effort typically expends his effort late, while the rest expend their effort either early or late. Baik and Lee (2013) consider a two-player contest with delegation, and show that the delegate with low contingent compensation expends his effort before the delegate with high contingent compensation.

Morgan (2003) and Fu (2006) have studied contests in which each player's valuation for a prize is drawn from a probability distribution after the players announce when they will expend effort. Morgan (2003) considers a two-player contest with logit-form contest success functions in which the realized valuations are revealed to both players. He shows that the players expend effort sequentially. Fu (2006) considers a two-player contest with logit-form contest success functions in which the realized common valuation is revealed to only one player. He shows that the uninformed player expends effort before the informed player.

Hoffmann and Rota-Graziosi (2012) have considered a two-player contest with general contest success functions in which players' common valuation for a prize is endogenously determined, depending only on their effort levels. They show that in some cases the players expend effort sequentially, while in others they expend effort simultaneously. Hoffmann and Rota-Graziosi (2020) have studied two-player endogenous-timing games in which the payoff or the marginal payoff of a player becomes non-monotonic with respect to the strategy of the opponent. They propose a taxonomy of the subgame-perfect Nash equilibria.

The remainder of this paper proceeds as follows. In Section 2, we present a model of a three-player asymmetric contest with endogenous timing of effort exertion, and set up a noncooperative game. In Section 3, we examine whether one of the weakest player's first-stage actions strictly or weakly dominates the other of his first-stage actions. In Sections 4 and 5, we first obtain a subgame-perfect equilibrium of the game in the case where the weakest player announces *Period 1* in the first stage. Then, we examine the identities of active players and the individual and total effort levels in the subgame-perfect equilibrium of the game. In Section 6, we obtain a subgame-perfect equilibrium of the game in the case where the weakest player announces *Period 2* in the first stage. Finally, Section 7 offers our conclusions.

2 | THE MODEL

Consider a contest in which there are three players, 1 through 3, who compete to win a prize, and there are two periods, 1 and 2, in which the players can expend their effort. The players can each expend their effort either in period 1 or in period 2, but not in both periods. The structure of strategic interactions between the players is as follows: The players first decide independently and announce simultaneously whether they each will expend their effort in period 1 or in period 2, and then each player independently chooses his effort level in the period which he announced.

Let v_i, for $i = 1, 2, 3$, represent player i's valuation for the prize, where $v_i > 0$. The players' valuations for the prize are publicly known at the start of the game. Without loss of generality, let $v_i = \beta_i v_3$, where $\beta_i > 0$ and $\beta_3 = 1$.

Let x_i, for $i = 1, 2, 3$, represent player i's effort level, where $x_i \geq 0$. Let $p_i(x_1, x_2, x_3)$ represent the probability that player i wins the prize. We assume the following contest success function for player i:

$$p_i(x_1, x_2, x_3) = \begin{cases} \sigma_i x_i / X & \text{for } X > 0 \\ 1/3 & \text{for } X = 0, \end{cases} \tag{1}$$

where $X = \sigma_1 x_1 + \sigma_2 x_2 + \sigma_3 x_3$, $\sigma_i > 0$, and $\sigma_3 = 1$.[3] The parameter σ_i reflects player i's relative ability to convert effort into probability of winning. For example, given that $x_j = x_k > 0$, for $j, k = 1, 2, 3$ with $j \neq k$, *ceteris paribus*, $\sigma_j > \sigma_k$ implies that player j's probability of winning is greater than player k's. In this case, we may say that player j has more ability than player k. We assume that the parameter σ_i is publicly known at the start of the game.

Let $s_i \equiv \beta_i \sigma_i$ for $i = 1, 2, 3$. Then s_i reflects player i's relative "composite strength" in the contest. We assume in Assumption 1 that player 1 is the strongest, and player 3 is the weakest, in terms of composite strength.

Assumption 1 We assume that $s_1 > s_2 > s_3 = 1$.

We assume that the players are risk-neutral. Let π_i, for $i = 1, 2, 3$, represent the expected payoff for player i. Then the payoff function for player i is

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$$\pi_i = v_i p_i(x_1, x_2, x_3) - x_i. \tag{2}$$

We formally consider the following game. In the first stage, the three players each choose independently between *Period 1* and *Period 2*. They announce (and commit to) their choices simultaneously. In the second (or effort-expending) stage, after knowing when the players choose their effort levels, each player independently chooses his effort level in the period which he announced in the first stage. We assume that a player, if any, who chooses his effort level in period 2 observes the effort levels, if any, chosen in period 1 before he does. The winner is determined at the end of the game.

We assume that all of the above is common knowledge among the players. We employ subgame-perfect equilibrium as the solution concept.

3 | PLAYER 3'S STRICTLY OR WEAKLY DOMINANT ACTION IN THE FIRST STAGE

We begin our analysis by examining whether one of player 3's first-stage actions strictly or weakly dominates the other of his first-stage actions. To examine it, we first solve the proper subgames which start at the second (or effort-expending) stage, and obtain player 3's equilibrium expected payoffs in these subgames. Then, we identify player 3's strictly or weakly dominant action in the first stage, if any, by comparing player 3's equilibrium expected payoffs in the subgames.

We have eight proper subgames starting at the second stage. If the three players all announce *Period 1* in the first stage, or if they all announce *Period 2* in the first stage, then we have a simultaneous-move subgame in which the three players choose their effort levels simultaneously. If player i, for $i = 1, 2, 3$, announces *Period 1* but the other players announce *Period 2*, then we have the *iL* sequential-move subgame—the subgame with player i as the leader—in which player i chooses his effort level in period 1, and then, after observing player i's effort level, the other two players choose their effort levels simultaneously in period 2. If players j and k, for $j = 1, 2$ and $k = 2, 3$ with $j \neq k$, announce *Period 1* but the remaining player announces *Period 2*, then we have the *jkL* sequential-move subgame—the subgame with players j and k as the leaders—in which players j and k choose their effort levels simultaneously in period 1, and then, after observing their effort levels, the remaining player chooses his effort level in period 2.

In Appendixes A–G, we solve these subgames, and obtain player 3's equilibrium expected payoffs in these subgames. In Appendix H, we compare player 3's equilibrium expected payoffs in the subgames. We obtain the following lemma, which immediately follows from Lemma H2.

Lemma 1 *Under Assumption 1, we have the following. (a) If $s_1 < 2$, then player 3's first-stage action* Period 1 *strictly dominates his first-stage action* Period 2*. (b) If $2 \leq s_1 < s_2/(s_2 - 1)$, then player 3's first-stage action* Period 1 *weakly dominates his first-stage action* Period 2*. (c) If $s_1 \geq s_2/(s_2 - 1)$ and $s_2 < 2$, then player 3's first-stage action* Period 1 *weakly dominates his first-stage action* Period 2*. (d) If $s_2 \geq 2$, then player 3's first-stage action* Period 1 *yields the same expected payoff as his first-stage action* Period 2 *for every list of the other players' first-stage actions.*

Note that Lemma 1 covers all the relevant values of s_1 and s_2 under Assumption 1, which can be verified by Figure 1. In Figure 1, curve I represents the equation $s_1 = s_2/(s_2 - 1)$.

Lemma 1 says that, for any values of s_1 and s_2 satisfying Assumption 1, player 3's first-stage action *Period 1* yields at least as much an expected payoff as his action *Period 2*, no matter what the other players announce in the first stage. The intuition behind this is clear. Competing against the top two players, if player 3, the weakest player, announces *Period 2* (rather than *Period 1*) in the first stage, then he cannot enjoy a first-mover advantage and can suffer from the second-mover disadvantage in the effort-expending stage.

Based on Lemma 1, we mainly focus on the case where player 3, the weakest player, announces *Period 1* in the first stage (see Sections 4 and 5). However, we also look at the case where player 3 announces *Period 2* in the first stage (see Section 6).

4 | ANALYSIS OF THE PROPER SUBGAMES WITH PLAYER 3 ANNOUNCING *PERIOD 1*

To obtain a subgame-perfect equilibrium of the full game in the case where player 3 announces *Period 1* in the first stage, we first analyze the following proper subgames which start at the second (or effort-expending) stage: the

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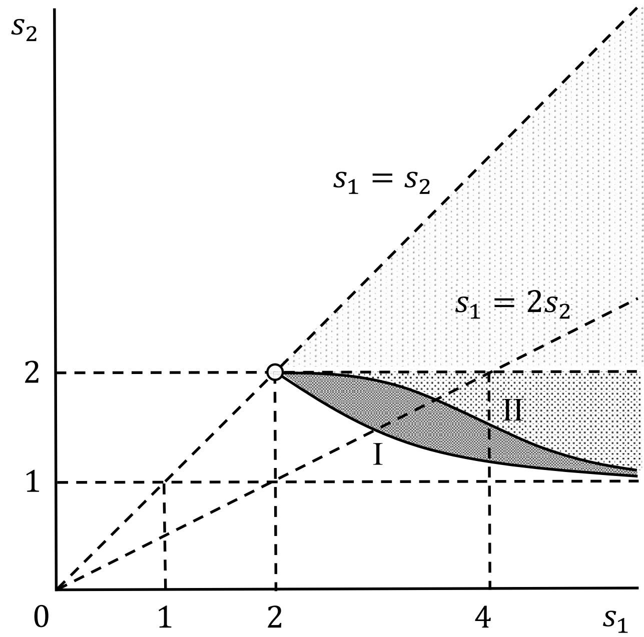

FIGURE 1 The values of the parameters at which player 2 expends effort before player 1 in equilibrium.

simultaneous-move subgame and three sequential-move subgames. The simultaneous-move subgame arises when players 1 and 2 both choose and announce *Period 1* in the first stage. In this subgame, the three players choose their effort levels simultaneously in period 1. If players 1 and 2 both choose and announce *Period 2*, then the $3L$ sequential-move subgame arises in which player 3 chooses his effort level in period 1, and then, after observing player 3's effort level, players 1 and 2 choose their effort levels simultaneously in period 2. If player 1 announces *Period 1* but player 2 announces *Period 2*, then the $13L$ sequential-move subgame arise in which players 1 and 3 choose their effort levels simultaneously in period 1, and then, after observing their effort levels, player 2 chooses his effort level in period 2. Finally, the $23L$ sequential-move subgame arises when player 2 announces *Period 1* but player 1 announces *Period 2*. In this subgame, players 2 and 3 choose their effort levels simultaneously in period 1, and then, after observing their effort levels, player 1 chooses his effort level in period 2.

4.1 | The simultaneous-move subgame

In this subgame, the three players choose their effort levels simultaneously in period 1. Each player seeks to maximize his expected payoff over his effort level, given his belief about the other players' effort levels.

We obtain a unique Nash equilibrium of the simultaneous-move subgame in Appendix A. Let x_i^S, for $i = 1, 2, 3$, represent player i's equilibrium effort level, and let π_i^S represent player i's expected payoff at the Nash equilibrium. We report them in Lemmas A1 and A2 in Appendix A.

Note that players 1 and 2 are always active at the Nash equilibrium. However, player 3, the weakest player, is active at the Nash equilibrium if $s_1 + s_2 > s_1 s_2$ or, equivalently, $s_1 < s_2/(s_2 - 1)$, and he is inactive if $s_1 + s_2 \leq s_1 s_2$ or, equivalently, $s_1 \geq s_2/(s_2 - 1)$.

4.2 | The $3L$ sequential-move subgame

In this subgame, player 3 first chooses his effort level in period 1. Then, after observing player 3's effort level, players 1 and 2 choose their effort levels simultaneously in period 2.

Let x_i^{3L}, for $i = 1, 2, 3$, represent player i's effort level specified in a subgame-perfect equilibrium of the $3L$ sequential-move subgame. Let π_i^{3L} represent player i's expected payoff in the subgame-perfect equilibrium. We obtain them in Appendix B, and report them in Lemmas B1 and B2.

Note that players 1 and 2 are always active in the subgame-perfect equilibrium. However, player 3, the weakest player, is active in the subgame-perfect equilibrium if $s_1 + s_2 > s_1 s_2$ or, equivalently, $s_1 < s_2/(s_2 - 1)$, and he is inactive if $s_1 + s_2 \leq s_1 s_2$ or, equivalently, $s_1 \geq s_2/(s_2 - 1)$.

4.3 | The 13L sequential-move subgame

In this subgame, players 1 and 3 first choose their effort levels simultaneously in period 1. Then, after observing their effort levels, player 2 chooses his effort level in period 2.

Let x_i^{13L}, for $i = 1, 2, 3$, represent player i's effort level specified in a subgame-perfect equilibrium of the 13L sequential-move subgame. Let π_i^{13L} represent player i's expected payoff in the subgame-perfect equilibrium. We obtain them in Appendix C, and report them in Lemmas C1, C2, and C3.

Note that player 1 is always active in the subgame-perfect equilibrium. However, player 2 is active in the subgame-perfect equilibrium if $s_1 < 2s_2$, and he is inactive if $s_1 \geq 2s_2$. Player 3, the weakest player, is active in the subgame-perfect equilibrium if $s_1 < 2$, and he is inactive if $s_1 \geq 2$.

4.4 | The 23L sequential-move subgame

In this subgame, players 2 and 3 first choose their effort levels simultaneously in period 1. Then, after observing their effort levels, player 1 chooses his effort level in period 2.

Let x_i^{23L}, for $i = 1, 2, 3$, represent player i's effort level specified in a subgame-perfect equilibrium of the 23L sequential-move subgame. Let π_i^{23L} represent player i's expected payoff in the subgame-perfect equilibrium. We obtain them in Appendix D, and report them in Lemmas D1 and D2.

Note that players 1 and 2 are always active in the subgame-perfect equilibrium. However, player 3, the weakest player, is active in the subgame-perfect equilibrium if $s_2 < 2$, and he is inactive if $s_2 \geq 2$.

5 | FIRST-STAGE DECISIONS ON THE TIMING OF EFFORT EXERTION WHEN PLAYER 3 CHOOSES *PERIOD 1*

We now look at the players' first-stage decisions (or announcements) on when to expend effort. Recall that we consider here the case where player 3 announces *Period 1* in the first stage.

In the first stage, each player has perfect foresight about the equilibrium of every (relevant) proper subgame, and thus about the players' equilibrium expected payoffs reported in Appendixes. For example, player 1 knows that, given that players 2 and 3 will announce *Period 1*, if he announces *Period 1*, then the players will end up with the payoffs, (π_1^S, π_2^S, π_3^S), reported in Lemma A1 or A2.

Figure 2 illustrates the strategic interaction between players 1 and 2 in the first stage in the case where player 3 announces *Period 1*. Since players 1 and 2 each announce either *Period 1* or *Period 2*, there are four possible combinations

Player 2

Player 1		*Period 1*	*Period 2*
	Period 1	π_1^S, π_2^S	π_1^{13L}, π_2^{13L}
	Period 2	π_1^{23L}, π_2^{23L}	π_1^{3L}, π_2^{3L}

FIGURE 2 The strategic interaction between players 1 and 2 in the first stage in the case where player 3 announces *Period 1*.

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of actions resulting from their announcements: (*Period 1*, *Period 1*), (*Period 1*, *Period 2*), (*Period 2*, *Period 1*), and (*Period 2*, *Period 2*).

The objective of this paper is to look at how endogenous timing of effort exertion affects players' behavior in a case where there are only two active players in equilibrium when three players choose their effort levels simultaneously to win a prize. Accordingly, we restrict our analysis to the case where only players 1 and 2 are active at the Nash equilibrium of the corresponding three-player simultaneous-move contest. It follows from Lemma A2 that this case occurs when Assumption 2 holds.[4]

Assumption 2 We assume that $s_1 + s_2 \le s_1 s_2$ or, equivalently, $s_1 \ge s_2/(s_2 - 1)$.

Figure 1 illustrates the values of s_1 and s_2 which satisfy both Assumption 1 and Assumption 2. In the figure, curve I represents the equation $s_1 + s_2 = s_1 s_2$ or, equivalently, $s_1 = s_2/(s_2 - 1)$. At the values of s_1 and s_2 located in the darkly, medium, or lightly shaded area of Figure 1, only players 1 and 2 are active at the Nash equilibrium of the three-player simultaneous-move contest (see Lemma A2).

Under Assumptions 1 and 2, which period does each player choose and announce in a subgame-perfect equilibrium of the full game? To answer this question, we first compare the equilibrium expected payoffs for player i, for $i = 1, 2$, in the four (relevant) proper subgames analyzed in Appendixes A–D. Using the lemmas therein, it is straightforward to obtain Lemma 2.

Lemma 2 *Under Assumptions 1 and 2, we have: (a)* $\pi_1^S < \pi_1^{23L}$, *(b)* $\pi_1^{13L} > \pi_1^{3L}$, *(c)* $\pi_2^S > \pi_2^{13L}$, *and (d)* $\pi_2^{23L} < \pi_2^{3L}$ *if* $3(2s_2 - 1)^2(s_1 + s_2)^2 < 4s_1s_2(1 + s_2)^3$, *but* $\pi_2^{23L} \ge \pi_2^{3L}$ *otherwise.*

Part (*d*) is stated in more detail as follows: $\pi_2^{23L} < \pi_2^{3L}$ if $3(2s_2 - 1)^2(s_1 + s_2)^2 < 4s_1s_2(1 + s_2)^3$, but $\pi_2^{23L} \ge \pi_2^{3L}$ if $3(2s_2 - 1)^2(s_1 + s_2)^2 \ge 4s_1s_2(1 + s_2)^3$ and $s_2 < 2$ hold or if $s_2 \ge 2$ holds.

In Figure 1, curve II represents the equation $3(2s_2 - 1)^2(s_1 + s_2)^2 = 4s_1s_2(1 + s_2)^3$, and it intersects curve I at (6.082, 1.197). The strict inequality $3(2s_2 - 1)^2(s_1 + s_2)^2 < 4s_1s_2(1 + s_2)^3$ holds at the values of s_1 and s_2 in the area enclosed by curves I and II. Thus, at the values of s_1 and s_2 located in the darkly shaded area, $\pi_2^{23L} < \pi_2^{3L}$ holds. At the values of s_1 and s_2 located in the medium or lightly shaded area, $\pi_2^{23L} \ge \pi_2^{3L}$ holds.

Next, we compare the equilibrium expected payoffs for player 3 in the (relevant) proper subgames. We have the comparison result, which is more than we need, in Lemma 1 (see also Lemma H2): For any values of s_1 and s_2 satisfying Assumption 1, player 3's first-stage action *Period 1* yields at least as much an expected payoff as his action *Period 2*, no matter what the other players announce in the first stage. Note that this implies immediately that, under Assumption 1, player 3 has no incentive to deviate from *Period 1*.

Now, using Lemma 2 and the comparison result in the preceding paragraph, we obtain the following proposition.

Proposition 1 *Under Assumptions 1 and 2, (a) if* $3(2s_2 - 1)^2(s_1 + s_2)^2 < 4s_1s_2(1 + s_2)^3$, then the game has no subgame-perfect equilibrium in pure strategies (in which player 3 announces *Period 1* in the first stage); (b) otherwise, the game has a subgame-perfect equilibrium in which player 1 announces *Period 2* in the first stage while players 2 and 3 announce *Period 1* in the first stage.

The proof of Proposition 1 is straightforward. First recall that player 3 has no incentive to deviate from *Period 1*, no matter what the other players announce in the first stage. Next, consider the combination (*Period 1*, *Period 1*) at which players 1 and 2 each announce *Period 1* in the first stage (see Figure 2). It is immediate from part (*a*) of Lemma 2 that player 1 has an incentive to deviate to *Period 2*. Next, consider the combination (*Period 2*, *Period 2*). It follows immediately from part (*b*) of Lemma 2 that player 1 has an incentive to deviate to *Period 1*. Next, consider the combination (*Period 1*, *Period 2*). Part (*c*) of Lemma 2 implies that player 2 has an incentive to deviate to *Period 1*. Finally, consider the combination (*Period 2*, *Period 1*). If $3(2s_2 - 1)^2(s_1 + s_2)^2 < 4s_1s_2(1 + s_2)^3$, then it follows from part (*d*) of Lemma 2 that player 2 has an incentive to deviate to *Period 2*, which, together with the above, leads to part (*a*) of Proposition 1. However, if $3(2s_2 - 1)^2(s_1 + s_2)^2 \ge 4s_1s_2(1 + s_2)^3$, then it follows from part (*d*) of Lemma 2 that player 2 has no incentive to deviate from *Period 1*, which, together with the fact that player 1 has no incentive to deviate from *Period 2*, leads to part (*b*) of Proposition 1.

In Figure 1, the result of part (*a*) of Proposition 1 holds at the values of s_1 and s_2 located in the darkly shaded area, and the result of part (*b*) holds at the values of s_1 and s_2 located in the medium or lightly shaded area. Note that we may

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say that asymmetries in composite strength between the top two players and player 3 are not significant in the darkly shaded area.

In Proposition 1, the existence of a subgame-perfect equilibrium in pure strategies depends on player 2's (first-stage) best response to player 1's first-stage action *Period 2* (as well as player 3's first-stage action *Period 1*).

Consider part (a) of Proposition 1. At the values of s_1 and s_2 located in the darkly shaded area, if player 2 announces *Period 1* in the first stage, then player 3 expends positive effort in the effort-expending stage (see Lemma D1). This happens because, in the effort-expending stage, player 2 (as one of the two weak leaders) chooses a restrained effort level to trigger a softened response from player 1, the strong follower, which makes room for player 3 (as another weak leader) to be active (since asymmetry in composite strength between player 2 and player 3 is not significant). However, if player 2 announces *Period 2* in the first stage, then player 3 expends zero effort in the effort-expending stage (see Lemma B2). This happens because, in the effort-expending stage, player 3 (as the weak leader) expects that there will be a big fight between the two strong followers (since asymmetry in composite strength between player 1 and player 2 is not significant). Consequently, it turns out that player 2 is better off by announcing *Period 2* rather than *Period 1*. This then leads to the nonexistence of a subgame-perfect equilibrium in pure strategies.

Next, consider part (b) of Proposition 1. At the values of s_1 and s_2 located in the medium or lightly shaded area, player 2 is better off by announcing *Period 1* rather than *Period 2*, and thus the game has a subgame-perfect equilibrium in which player 1 announces *Period 2* while players 2 and 3 announce *Period 1*. This happens because, given (rather) significant asymmetries in composite strength between the top two players and player 3, player 2 can ignore the presence of player 3, the weakest player.

Indeed, part (b) of Proposition 1 follows from the following facts. The first is that player 2 announces *Period 1* in the first stage, no matter what player 1 announces. This fact, which is immediate from parts (c) and (d) of Lemma 2, can be explained as follows.

Consider first the case where player 1 announces *Period 1*. Player 2 has two options: either to announce *Period 1* or to announce *Period 2*. If he announces *Period 1*, then he will compete with the other players on equal footing in the effort-expending stage. On the other hand, if player 2 announces *Period 2*, then he will suffer seriously from the second-mover disadvantage in the effort-expending stage because he will face the aggressive leaders including the strongest player. Accordingly, given that player 1 announces *Period 1*, player 2 also announces *Period 1*.

Next, consider the case where player 1 announces *Period 2*. In this case, if player 2 announces *Period 1*, then he will be one of the two weak leaders in the effort-expending stage, and can signal (to player 1, the strongest player) his intention to avoid a big fight by choosing a restrained effort level. However, if player 2 announces *Period 2*, then he will be a follower together with player 1 in the effort-expending stage, and cannot avoid a big fight against player 1, the strongest player. It turns out that player 2 is better off by announcing *Period 1* rather than *Period 2*.

The second fact is that, given that player 2 announces *Period 1*, player 1 announces *Period 2*. This fact, which is immediate from part (a) of Lemma 2, can be explained as follows. If player 1 announces *Period 1*, then he will compete with the other players on equal footing in the effort-expending stage: The three players will choose their effort levels simultaneously and independently in period 1. On the other hand, if player 1 announces *Period 2*, then he will become the only follower in the effort-expending stage, and he, the strongest player, can compete efficiently against the two weak players by easing up and responding with an appropriate level of effort to restrained effort levels of the two intimidated weak leaders. Consequently, player 1 is better off by announcing *Period 2* rather than *Period 1*.

A well-known result obtained from two-player asymmetric contests with endogenous timing of effort exertion is that, in equilibrium, the strong player announces *Period 2* while the weak player announces *Period 1* (see e.g., Baik & Shogren, 1992; Leininger, 1993). Part (a) of Proposition 1 implies that this well-known result (or a similar one) does not hold with the presence of an additional player if asymmetries between the top two players and the additional player are not significant—that is, the additional player is unignorable. However, part (b) of Proposition 1 implies that the well-known result (or a similar one) holds with the presence of an additional player if asymmetries between the top two players and the additional player are significant—that is, the additional player is ignorable.

5.1 | Active players in equilibrium

We henceforth focus on the subgame-perfect equilibrium indicated in part (b) of Proposition 1. In the equilibrium, player 1 announces *Period 2* in the first stage, while players 2 and 3 announce *Period 1*, and thus the 23L sequential-move subgame arises in the effort-expending stage. An interesting question that arises is: Who are active players in the

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equilibrium? It follows from Lemmas D1 and D2 that, under the relevant restrictions on the parameters, players 1 and 2 are always active in the equilibrium, but player 3 is not always active. Specifically, we obtain the following proposition regarding player 3's activeness in the equilibrium.

Proposition 2 *Under Assumptions 1 and 2, (a) if* $3(2s_2 - 1)^2(s_1 + s_2)^2 \geq 4s_1s_2(1 + s_2)^3$ and $s_2 < 2$, then player 3 expends positive effort in the subgame-perfect equilibrium; (b) if $s_2 \geq 2$, then player 3 expends zero effort in the subgame-perfect equilibrium.

In Figure 1, the result of part (*a*) holds at the values of s_1 and s_2 located in the medium shaded area, and the result of part (*b*) holds at the values of s_1 and s_2 located in the lightly shaded area.

Under Assumptions 1 and 2, if the three players are *exogenously* assumed to choose their effort levels simultaneously, player 3 is inactive at the Nash equilibrium of the three-player simultaneous-move contest. However, Proposition 2 says that, if the players decide *endogenously* on when to expend effort, then player 3 can be active in equilibrium. Specifically, it says that player 3 is active in equilibrium if asymmetry in composite strength between player 2 and player 3 is not very significant; however, he is inactive in equilibrium if asymmetry between them is very significant.

The intuition behind part (*a*) of Proposition 2 is as follows. Facing an ignorable additional player, players 1 and 2 announce *Period 2* and *Period 1*, respectively, in the first stage, as in two-player asymmetric contests with endogenous timing of effort exertion. Then, in the effort-expending stage, player 2 (as one of the two weak leaders) chooses a restrained effort level to trigger a softened response from player 1, the strong follower.[5] This makes room for player 3 (as another weak leader) to be active when asymmetry between player 2 and player 3 is not very significant.

5.2 | Effort levels in equilibrium

It is of interest to compare the equilibrium effort levels obtained in the current model with those in the corresponding three-player simultaneous-move contest. Let the superscripts $*$ and SM indicate the equilibrium effort levels obtained in the current model and those of the three-player simultaneous-move contest, respectively. Let $T*$ represent the equilibrium total effort level obtained in the current model, so that $T* \equiv x_1^* + x_2^* + x_3^*$. Let T^{SM} represent the equilibrium total effort level in the three-player simultaneous-move contest, so that $T^{SM} \equiv x_1^{SM} + x_2^{SM} + x_3^{SM}$.

The equilibrium effort levels obtained in the current model are equal to those in the 23L sequential-move subgame. The equilibrium effort levels in the three-player simultaneous-move contest are equal to those in the simultaneous-move subgame in the current model. Hence, we compare the equilibrium effort levels in the 23L sequential-move subgame with those in the simultaneous-move subgame. Using Lemmas A2, D1, and D2, we obtain the following proposition which summarizes the comparison results.

Proposition 3 *Under Assumptions 1 and 2, (a) if* $3(2s_2 - 1)^2(s_1 + s_2)^2 \geq 4s_1s_2(1 + s_2)^3$ and $s_2 < 2$, then we have that $x_1^* < x_1^{SM}$, $x_2^* < x_2^{SM}$, $x_3^* > x_3^{SM} = 0$, and $T^* < T^{SM}$; (b) if $s_2 \geq 2$, then we have that $x_1^* < x_1^{SM}$, $x_2^* < x_2^{SM}$, $x_3^* = x_3^{SM} = 0$, and $T^* < T^{SM}$.

Note that, when comparing the equilibrium total effort levels in part (*a*), we assume, for tractability, that $\sigma_1 = \sigma_2 = 1$.

In Figure 1, the result of part (*a*) holds at the values of s_1 and s_2 located in the medium shaded area, and the result of part (*b*) holds at the values of s_1 and s_2 located in the lightly shaded area.

Proposition 3 says that the equilibrium total effort level is lower in the current model than in the corresponding three-player simultaneous-move contest.[6] This result is interesting because it holds even when player 3 is active in the current model.

The intuition behind the result is as follows. In the subgame-perfect equilibrium of the current model, the two weak players, players 2 and 3, first choose their effort levels in period 1, and then the strongest player, player 1, chooses his effort level in period 2. Given this order of choosing effort levels, the two intimidated weak leaders choose restrained effort levels in order to avoid a big fight against (or to trigger a softened response from) the strongest player. In response to the weak leaders' restrained behavior, the strongest player eases up—compared with the corresponding three-player simultaneous-move contest—and chooses an appropriate level of effort. Consequently, the equilibrium total effort level is lower in the current model than in the corresponding three-player simultaneous-move contest.

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6 | FIRST-STAGE DECISIONS ON THE TIMING OF EFFORT EXERTION WHEN PLAYER 3 CHOOSES *PERIOD 2*

To obtain a subgame-perfect equilibrium of the full game in the case where player 3 announces *Period 2* in the first stage, we first analyze the following proper subgames: the simultaneous-move subgame, the $1L$ sequential-move subgame, the $2L$ sequential-move subgame, and the $12L$ sequential-move subgame. Appendixes A, E, F, and G report the outcomes of these subgames.

Then, under Assumption 2 (in addition to Assumption 1), we look at the players' first-stage decisions (or announcements) on when to expend effort. Note that, in the first stage, each player has perfect foresight about the players' equilibrium expected payoffs in every (relevant) proper subgame.

Figure 3 illustrates the strategic interaction between players 1 and 2 in the first stage in the case where player 3 announces *Period 2*. Using Lemmas A2, E3, E4, F3, and G3, it is straightforward to obtain Lemma 3, which compares the equilibrium expected payoffs for player i, for $i = 1, 2$, in the four (relevant) proper subgames.

Lemma 3 *Under Assumptions 1 and 2, we have: (a)* $\pi_1^{12L} < \pi_1^{2L}$, *(b)* $\pi_1^{1L} > \pi_1^{S}$, *(c)* $\pi_2^{12L} > \pi_2^{1L}$, *and (d)* $\pi_2^{2L} > \pi_2^{S}$.

In Figure 1, Lemma 3 holds at the values of s_1 and s_2 located in the darkly, medium, or lightly shaded area.

Next, we compare the equilibrium expected payoffs for player 3 in the relevant proper subgames. Lemma 3 implies that, in any subgame-perfect equilibrium of the full game, player 1 announces *Period 2* in the first stage while player 2 announces *Period 1* in the first stage. Due to this fact, we compare the equilibrium expected payoffs for player 3 only in the case where player 1 announces *Period 2* in the first stage while player 2 announces *Period 1* in the first stage. From parts (c) and (d) of Lemma H2, we have under Assumption 1: $\pi_3^{23L} > \pi_3^{2L}$ if $s_1 \geq s_2/(s_2 - 1)$ and $s_2 < 2$, and $\pi_3^{23L} = \pi_3^{2L}$ if $s_2 \geq 2$. Put differently, $\pi_3^{23L} > \pi_3^{2L}$ holds at the values of s_1 and s_2 located in the darkly or medium shaded area of Figure 1, and $\pi_3^{23L} = \pi_3^{2L}$ holds at the values of s_1 and s_2 located in the lightly shaded area. Note that this implies immediately that, given that player 1 announces *Period 2* and player 2 announces *Period 1*, player 3 has an incentive to deviate from *Period 2* if $s_1 \geq s_2/(s_2 - 1)$ and $s_2 < 2$; but he has no incentive to do so if $s_2 \geq 2$.

Now, using Lemma 3 and the comparison results in the preceding paragraph, we obtain the following proposition.

Proposition 4 *Under Assumptions 1 and 2, (a) if* $s_2 < 2$, then the game has no subgame-perfect equilibrium in pure strategies (in which player 3 announces *Period 2* in the first stage); (b) if $s_2 \geq 2$, the game has a subgame-perfect equilibrium in which players 1 and 3 announce *Period 2* in the first stage while player 2 announces *Period 1* in the first stage.

In Figure 1, the result of part (a) holds at the values of s_1 and s_2 located in the darkly or medium shaded area, and the result of part (b) holds at the values of s_1 and s_2 located in the lightly shaded area.

The explanation for Proposition 4 can be made similarly to that for Proposition 1, and therefore is omitted. We may well say that part (b) arises because, if $s_2 \geq 2$, player 3 is indifferent between choosing *Period 1* or *Period 2* in the first stage (see part (d) of Lemma 1).

In the subgame-perfect equilibrium indicated in part (b) of Proposition 4, we have the same outcome as in part (b) of Proposition 2 and part (b) of Proposition 3. Since we have provided there the intuition behind the outcome, we omit it here.

Player 2

Player 1		*Period 1*	*Period 2*
	Period 1	π_1^{12L}, π_2^{12L}	π_1^{1L}, π_2^{1L}
	Period 2	π_1^{2L}, π_2^{2L}	π_1^{S}, π_2^{S}

FIGURE 3 The strategic interaction between players 1 and 2 in the first stage in the case where player 3 announces *Period 2*.

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7 | CONCLUSIONS

We have studied a three-player contest in which there are only two active players in equilibrium when the three players choose their effort levels simultaneously to win a prize. Specifically, we have studied a three-player contest in which only players 1 and 2 are active at the Nash equilibrium of the corresponding three-player simultaneous-move contest.

We have looked at how endogenous timing of effort exertion affects the players' behavior. In the model, there are two periods, 1 and 2, in which the players can choose their effort levels. The players play the following game. First, the players decide independently and announce simultaneously whether they each will expend their effort in period 1 or in period 2. Then, after knowing when the players choose their effort levels, each player independently chooses his effort level in the period which he announced.

We have shown in Proposition 1 that, given *not* significant asymmetries between the top two players and player 3, the game has no subgame-perfect equilibrium in pure strategies in which player 3 announces *Period 1* in the first stage. However, given significant asymmetries between them, the game has a subgame-perfect equilibrium in which player 1 announces *Period 2* in the first stage and chooses his effort level in period 2, while players 2 and 3 announce *Period 1* in the first stage and choose their effort levels in period 1.

Next, we have shown in Proposition 4 that, given very significant asymmetries between the top two players and the weakest player, the game has a subgame-perfect equilibrium in which players 1 and 3 announce *Period 2* in the first stage, while player 2 announces *Period 1* in the first stage.

Next, we have shown in Proposition 2 that, in the subgame-perfect equilibrium indicated in part (*b*) of Proposition 1, player 3 is active if asymmetry between player 2 and player 3 is *not* very significant; however, he is inactive if asymmetry between them is very significant. We have also shown that, in the subgame-perfect equilibrium indicated in part (*b*) of Proposition 4, player 3 is inactive.

Finally, we have shown in Proposition 3 that, in any of the subgame-perfect equilibria, the total effort level is lower —even when player 3 is active—compared with the corresponding three-player simultaneous-move contest.

It would be interesting to experimentally investigate the theoretical predictions of the current model. It would also be interesting to compare the results of this experimental study with those of existing experimental studies of two-player asymmetric contests (see e.g., Baik et al., 1999). We leave them for future research.

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DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

ENDNOTES

1 In the theory of contests, a contest refers to a situation in which players compete by expending effort to win a prize. Examples include elections, rent-seeking contests, patent-seeking contests, environmental conflicts, all-pay auctions, and sporting contests. See for example Tullock (1980), Dixit (1987), Hillman and Riley (1989), Baye et al. (1993), Corchón (2007), Epstein and Nitzan (2007), Congleton et al. (2008), Konrad (2009), Chowdhury and Gürtler (2015), and Vojnović (2015).

2 A three-player simultaneous-move contest refers to a contest in which three players compete by choosing their effort levels simultaneously to win a prize.

3 This logit-form contest success function is extensively used in the theory of contests. See for example Tullock (1980), Hillman and Riley (1989), Leininger (1993), Baik and Shogren (1994), Morgan (2003), Baik (2004), Epstein and Nitzan (2007), Konrad (2009), Baik and Lee (2013), Vojnović (2015), Baik et al. (2022), and Barbieri and Serena (2022).

4 Assumption 2 can be rewritten as $s_3 \leq s_1 s_2/(s_1 + s_2)$ because we assume that $s_3 = 1$. Interestingly, Lemma A2 shows that we have $x_1^S/v_1 = x_2^S/v_2$ or, equivalently, $x_1^S/x_2^S = v_1/v_2$ at the Nash equilibrium at which only the top two players are active (see Baik, 2004).

5 Using Lemmas A2 and D1, it is straightforward to check that, under Assumptions 1 and 2, we have $x_2^{23L} < x_2^S$ if $3(2s_2 - 1)^2(s_1 + s_2)^2 \geq 4s_1 s_2(1 + s_2)^3$ and $s_2 < 2$. Note that player 2's effort level x_2^S at the Nash equilibrium of the simultaneous-move subgame is equal to his effort level at the Nash equilibrium of the three-player simultaneous-move contest.

6 Baik and Shogren (1992) and Leininger (1993) study two-player asymmetric contests with endogenous timing of effort exertion, and show that the equilibrium total effort level is lower than in the corresponding two-player simultaneous-move contest.

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APPENDIX A

ANALYSIS OF THE SIMULTANEOUS-MOVE SUBGAME

Player 1 seeks to maximize his expected payoff π_1 in Equation (2) over his effort level x_1, given his belief about the other players' effort levels. From the first-order condition for maximizing π_1, we obtain player 1's reaction function:

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$$x_1 = \begin{cases} \left\{\sqrt{s_1 v_3(\sigma_2 x_2 + x_3)} - (\sigma_2 x_2 + x_3)\right\}/\sigma_1 & \text{for } 0 < \sigma_2 x_2 + x_3 \le s_1 v_3 \\ 0 & \text{for } \sigma_2 x_2 + x_3 > s_1 v_3. \end{cases}$$

It is straightforward to check that the second-order condition for maximizing π_1 is satisfied. (Note that every maximization problem in this paper satisfies its second-order condition.)

Similarly, we obtain the reaction functions for players 2 and 3, respectively:

$$x_2 = \begin{cases} \left\{\sqrt{s_2 v_3(\sigma_1 x_1 + x_3)} - (\sigma_1 x_1 + x_3)\right\}/\sigma_2 & \text{for } 0 < \sigma_1 x_1 + x_3 \le s_2 v_3 \\ 0 & \text{for } \sigma_1 x_1 + x_3 > s_2 v_3 \end{cases}$$

and

$$x_3 = \begin{cases} \sqrt{v_3(\sigma_1 x_1 + \sigma_2 x_2)} - (\sigma_1 x_1 + \sigma_2 x_2) & \text{for } 0 < \sigma_1 x_1 + \sigma_2 x_2 \le v_3 \\ 0 & \text{for } \sigma_1 x_1 + \sigma_2 x_2 > v_3. \end{cases}$$

Next, we obtain the Nash equilibrium, (x_1^S, x_2^S, x_3^S), by solving the system of three simultaneous equations which consists of the three reaction functions above. Finally, we obtain player i's expected payoff π_i^S at the Nash equilibrium by substituting the players' equilibrium effort levels into Equation (2).

Lemma A1 *In the case where $s_1 + s_2 > s_1 s_2$ or, equivalently, $s_1 < s_2/(s_2 - 1)$, we obtain the following at the Nash equilibrium of the simultaneous-move subgame. (a) The players' effort levels are: $x_1^S = 2\beta_1 s_2(s_1 s_2 + s_1 - s_2)v_3/(s_1 s_2 + s_1 + s_2)^2$, $x_2^S = 2\beta_2 s_1(s_1 s_2 + s_2 - s_1)v_3/(s_1 s_2 + s_1 + s_2)^2$, and $x_3^S = 2 s_1 s_2(s_1 + s_2 - s_1 s_2)v_3/(s_1 s_2 + s_1 + s_2)^2$. (b) The players' expected payoffs are: $\pi_1^S = \beta_1(s_1 s_2 + s_1 - s_2)^2 v_3/(s_1 s_2 + s_1 + s_2)^2$, $\pi_2^S = \beta_2(s_1 s_2 + s_2 - s_1)^2 v_3/(s_1 s_2 + s_1 + s_2)^2$, and $\pi_3^S = (s_1 + s_2 - s_1 s_2)^2 v_3/(s_1 s_2 + s_1 + s_2)^2$.*

Lemma A2 *In the case where $s_1 + s_2 \le s_1 s_2$ or, equivalently, $s_1 \ge s_2/(s_2 - 1)$, we obtain the following at the Nash equilibrium of the simultaneous-move subgame. (a) The players' effort levels are: $x_1^S = \beta_1 s_1 s_2 v_3/(s_1 + s_2)^2$, $x_2^S = \beta_2 s_1 s_2 v_3/(s_1 + s_2)^2$, and $x_3^S = 0$. (b) The players' expected payoffs are: $\pi_1^S = \beta_1 s_1^2 v_3/(s_1 + s_2)^2$, $\pi_2^S = \beta_2 s_2^2 v_3/(s_1 + s_2)^2$, and $\pi_3^S = 0$.*

Note that, in Lemma A1, the three players are all active at the Nash equilibrium, whereas, in Lemma A2, only players 1 and 2 are active at the Nash equilibrium. It is immediate from these lemmas that player 3, the weakest player, is active at the Nash equilibrium if $s_1 + s_2 > s_1 s_2$, but he is inactive if $s_1 + s_2 \le s_1 s_2$.

APPENDIX B

ANALYSIS OF THE 3*L* SEQUENTIAL-MOVE SUBGAME

Player 3 first chooses his effort level in period 1. Then, after observing player 3's effort level, players 1 and 2 choose their effort levels simultaneously and independently in period 2.

To obtain a subgame perfect equilibrium *outcome* of this subgame, we work backward. In period 2, players 1 and 2 know player 3's effort level x_3. Player 1 seeks to maximize his expected payoff π_1 in Equation (2) over his effort level x_1, given his belief about player 2's effort level. From the first-order condition for maximizing π_1, we obtain player 1's reaction function:

$$x_1 = \begin{cases} \left\{\sqrt{s_1 v_3(\sigma_2 x_2 + x_3)} - (\sigma_2 x_2 + x_3)\right\}/\sigma_1 & \text{for } 0 < \sigma_2 x_2 + x_3 \le s_1 v_3 \\ 0 & \text{for } \sigma_2 x_2 + x_3 > s_1 v_3. \end{cases}$$

Similarly, we obtain the reaction function for player 2:

$$x_2 = \begin{cases} \left\{\sqrt{s_2 v_3(\sigma_1 x_1 + x_3)} - (\sigma_1 x_1 + x_3)\right\}/\sigma_2 & \text{for } 0 < \sigma_1 x_1 + x_3 \le s_2 v_3 \\ 0 & \text{for } \sigma_1 x_1 + x_3 > s_2 v_3. \end{cases}$$

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Using these two reaction functions, we obtain the Nash equilibrium in period 2:

$$x_1^N(x_3) = \left\{ s_1^2 s_2 v_3 - 2s_2(s_1+s_2)x_3 + s_1\sqrt{s_1^2 s_2^2 v_3^2 + 4s_1 s_2 v_3(s_1+s_2)x_3} \right\} / 2\sigma_1(s_1+s_2)^2$$

and

$$x_2^N(x_3) = \left\{ s_1 s_2^2 v_3 - 2s_1(s_1+s_2)x_3 + s_2\sqrt{s_1^2 s_2^2 v_3^2 + 4s_1 s_2 v_3(s_1+s_2)x_3} \right\} / 2\sigma_2(s_1+s_2)^2.$$

Next, consider period 1 in which player 3 chooses his effort level. Let $\pi_3(x_3)$ be player 3's expected payoff which takes into account the Nash equilibrium in period 2. Substituting $x_1^N(x_3)$ and $x_2^N(x_3)$ into Equation (2), we obtain

$$\pi_3(x_3) = 2(s_1+s_2)v_3 x_3 / \left\{ s_1 s_2 v_3 + \sqrt{s_1^2 s_2^2 v_3^2 + 4s_1 s_2 v_3(s_1+s_2)x_3} \right\} - x_3.$$

In period 1, player 3 has perfect foresight about $\pi_3(x_3)$ for any value of x_3. He chooses a value of x_3 which maximizes $\pi_3(x_3)$. From the first-order condition for maximizing $\pi_3(x_3)$, we obtain player 3's equilibrium effort level x_3^{3L}.

Substituting x_3^{3L} into $x_1^N(x_3)$ and $x_2^N(x_3)$ above, we obtain the equilibrium effort levels of players 1 and 2, x_1^{3L} and x_2^{3L}, respectively. Substituting into Equation (2) the players' equilibrium effort levels, we obtain player i's equilibrium expected payoff π_i^{3L}.

Lemma B1 *In the case where $s_1 + s_2 > s_1 s_2$ or, equivalently, $s_1 < s_2/(s_2 - 1)$, we obtain the following in the subgame-perfect equilibrium of the 3L sequential-move subgame. (a) The players' effort levels are: $x_1^{3L} = \{2s_1^3 s_2 + s_1^2 s_2^2 - (s_1+s_2)^2 + 2s_1^2 (s_1+s_2)\}v_3/4\sigma_1 s_1(s_1+s_2)^2$, $x_2^{3L} = \{2s_1 s_2^3 + s_1^2 s_2^2 - (s_1+s_2)^2 + 2s_2^2(s_1+s_2)\}v_3/4\sigma_2 s_2(s_1+s_2)^2$, and $x_3^{3L} = \{(s_1+s_2)^2 - \}$v3/4 $s_1 s_2(s_1+s_2)$. (b) The players' expected payoffs are: $\pi_1^{3L} = \{2s_1^2 + s_1 s_2 - (s_1+s_2)\}^2 v_3/4\sigma_1 s_1(s_1+s_2)^2$, $\pi_2^{3L} = \{2s_2^2 + s_1 s_2 - (s_1+s_2)\}^2 v_3/4\sigma_2 s_2(s_1+s_2)^2$, and $\pi_3^{3L} = (s_1+s_2-s_1 s_2)^2 v_3/4s_1 s_2(s_1+s_2)$.*

Lemma B2 *In the case where $s_1 + s_2 \le s_1 s_2$ or, equivalently, $s_1 \ge s_2/(s_2 - 1)$, we obtain the following in the subgame-perfect equilibrium of the 3L sequential-move subgame. (a) The players' effort levels are: $x_1^{3L} = \beta_1 s_1 s_2 v_3/(s_1+s_2)^2$, $x_2^{3L} = \beta_2 s_1 s_2 v_3/(s_1+s_2)^2$, and $x_3^{3L} = 0$. (b) The players' expected payoffs are: $\pi_1^{3L} = \beta_1 s_1^2 v_3/(s_1+s_2)^2$, $\pi_2^{3L} = \beta_2 s_2^2 v_3/(s_1+s_2)^2$, and $\pi_3^{3L} = 0$.*

Note that, in Lemma B1, the three players are all active in the subgame-perfect equilibrium, whereas, in Lemma B2, only players 1 and 2 are active in the subgame-perfect equilibrium. It is immediate from these lemmas that player 3, the weakest player, is active in the subgame-perfect equilibrium if $s_1 + s_2 > s_1 s_2$, but he is inactive if $s_1 + s_2 \le s_1 s_2$.

APPENDIX C

ANALYSIS OF THE 13*L* SEQUENTIAL-MOVE SUBGAME

Players 1 and 3 first choose their effort levels simultaneously and independently in period 1, and then, after observing their effort levels, player 2 chooses his effort level in period 2.

To obtain a subgame perfect equilibrium *outcome* of this subgame, we work backward. In period 2, player 2 knows player 1's effort level, x_1, and player 3's effort level, x_3. Player 2 seeks to maximize his expected payoff π_2 in Equation (2) over his effort level x_2. From the first-order condition for maximizing π_2, we obtain player 2's strategy in any subgame-perfect equilibrium:

$$x_2(x_1, x_3) = \begin{cases} \left\{ \sqrt{s_2 v_3(\sigma_1 x_1 + x_3)} - (\sigma_1 x_1 + x_3) \right\} / \sigma_2 & \textit{for } 0 < \sigma_1 x_1 + x_3 \le s_2 v_3 \\ 0 & \text{for } \sigma_1 x_1 + x_3 > s_2 v_3. \end{cases} \tag{C1}$$

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Next, consider period 1 in which players 1 and 3 choose their effort levels simultaneously and independently. Let $\pi_j(x_1, x_3)$, for j = 1, 3, be player j's expected payoff which takes into account player 2's equilibrium strategy in Equation (C1). Substituting $x_2(x_1, x_3)$ in Equation (C1) into Equation (2), we obtain

$$\pi_1(x_1, x_3) = s_1 v_3 x_1 / \sqrt{s_2 v_3(\sigma_1 x_1 + x_3)} - x_1 \quad \text{and} \quad \pi_3(x_1, x_3) = v_3 x_3 / \sqrt{s_2 v_3(\sigma_1 x_1 + x_3)} - x_3. \tag{C2}$$

In period 1, players 1 and 3 have perfect foresight about $\pi_1(x_1, x_3)$ and $\pi_3(x_1, x_3)$ for any values of x_1 and x_3.

We first obtain the reaction function for player 1. Player 1 seeks to maximize $\pi_1(x_1, x_3)$ in Equation (C2) over his effort level x_1, given his belief about player 3's effort level x_3. The first-order condition for maximizing $\pi_1(x_1, x_3)$ reduces to

$$4s_2(\sigma_1 x_1 + x_3)^3 - s_1^2 v_3(\sigma_1 x_1 + 2x_3)^2 = 0. \tag{C3}$$

Naturally, Equation (C3) is the implicit form of player 1's reaction function, $x_1 = x_1(x_3)$.

Similarly, the implicit form of player 3's reaction function, $x_3 = x_3(x_1)$, is

$$4s_2(\sigma_1 x_1 + x_3)^3 - v_3(2\sigma_1 x_1 + x_3)^2 = 0. \tag{C4}$$

Using Equations (C3) and (C4), we obtain the equilibrium effort levels of players 1 and 3, x_1^{13L} and x_3^{13L}, respectively.

Substituting x_1^{13L} and x_2^{13L} into $x_2(x_1, x_3)$ in Equation (C1), we obtain player 2's equilibrium effort level x_2^{13L}. Substituting into Equation (2) the players' equilibrium effort levels, we obtain player i's equilibrium expected payoff π_i^{13L}.

Lemma C1 *In the case where $s_1 < 2$, we obtain the following in the subgame-perfect equilibrium of the 13L sequential-move subgame. (a) The players' effort levels are: $x_1^{13L} = 9s_1^2(2s_1 - 1)v_3/4\sigma_1 s_2(s_1 + 1)^3$, $x_2^{13L} = 3s_1(2s_1 s_2 + 2s_2 - 3s_1)v_3/4\sigma_2 s_2(s_1 + 1)^2$, and $x_3^{13L} = 9s_1^2(2-s_1)v_3/4s_2(s_1 + 1)^3$. (b) The players' expected payoffs are: $\pi_1^{13L} = 3s_1^2(2s_1 - 1)^2 v_3/4\sigma_1 s_2(s_1 + 1)^3$, $\pi_2^{13L} = (2s_1 s_2 + 2s_2 - 3s_1)^2 v_3/4\sigma_2 s_2(s_1 + 1)^2$, and $\pi_3^{13L} = 3s_1(2 - s_1)^2 v_3/4s_2(s_1 + 1)^3$.*

Lemma C2 *In the case where $2 \le s_1 < 2s_2$, we obtain the following in the subgame-perfect equilibrium of the 13L sequential-move subgame. (a) The players' effort levels are: $x_1^{13L} = s_1^2 v_3/4\sigma_1 s_2$, $x_2^{13L} = s_1(2s_2 - s_1)v_3/4\sigma_2 s_2$, and $x_3^{13L} = 0$. (b) The players' expected payoffs are: $\pi_1^{13L} = s_1^2 v_3/4\sigma_1 s_2$, $\pi_2^{13L} = (2s_2 - s_1)^2 v_3/4\sigma_2 s_2$, and $\pi_3^{13L} = 0$.*

Lemma C3 *In the case where $s_1 \ge 2s_2$, we obtain the following in the subgame-perfect equilibrium of the 13L sequential-move subgame. (a) The players' effort levels are: $x_1^{13L} = s_2 v_3/\sigma_1$, $x_2^{13L} = 0$, and $x_3^{13L} = 0$. (b) The players' expected payoffs are: $\pi_1^{13L} = (s_1 - s_2)v_3/\sigma_1$, $\pi_2^{13L} = 0$, and $\pi_3^{13L} = 0$.*

Note that, in Lemma C1, the three players are all active in the subgame-perfect equilibrium; in Lemma C2, only players 1 and 2 are active in the subgame-perfect equilibrium; and in Lemma C3, only player 1 is active in the subgame-perfect equilibrium. It is immediate from these lemmas that player 3, the weakest player, is active in the subgame-perfect equilibrium if $s_1 < 2$, but he is inactive if $s_1 \ge 2$. It is immediate from the lemmas that player 2 is active in the subgame-perfect equilibrium if $s_1 < 2s_2$, but he is inactive if $s_1 \ge 2s_2$. Lemma C3 shows that, if $s_1 \ge 2s_2$, then player 1 chooses the smallest of all effort levels to which player 2's best response is zero.

APPENDIX D

ANALYSIS OF THE 23*L* SEQUENTIAL-MOVE SUBGAME

Players 2 and 3 first choose their effort levels simultaneously and independently in period 1, and then, after observing their effort levels, player 1 chooses his effort level in period 2.

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Now that the analysis is similar to that for the 13*L* sequential-move subgame in Appendix C, we omit it.

Lemma D1 *In the case where $s_2 < 2$, we obtain the following in the subgame-perfect equilibrium of the 23L sequential-move subgame. (a) The players' effort levels are: $x_1^{23L} = 3s_2(2s_1s_2 + 2s_1 - 3s_2)v_3/4\sigma_1 s_1(s_2 + 1)^2$, $x_2^{23L} = 9s_2^2(2s_2 - 1)v_3/4\sigma_2 s_1(s_2 + 1)^3$, and $x_3^{23L} = 9s_2^2(2-s_2)v_3/4s_1(s_2 + 1)^3$. (b) The players' expected payoffs are: $\pi_1^{23L} = (2s_1s_2 + 2s_1 - 3s_2)^2 v_3/4\sigma_1 s_1(s_2 + 1)^2$, $\pi_2^{23L} = 3s_2^2(2s_2 - 1)^2 v_3/4\sigma_2 s_1(s_2 + 1)^3$, and $\pi_3^{23L} = 3s_2(2-s_2)^2 v_3/4s_1(s_2 + 1)^3$.*

Lemma D2 *In the case where $s_2 \geq 2$, we obtain the following in the subgame-perfect equilibrium of the 23L sequential-move subgame. (a) The players' effort levels are: $x_1^{23L} = s_2(2s_1 - s_2)v_3/4\sigma_1 s_1$, $x_2^{23L} = s_2^2 v_3/4\sigma_2 s_1$, and $x_3^{23L} = 0$. (b) The players' expected payoffs are: $\pi_1^{23L} = (2s_1 - s_2)^2 v_3/4\sigma_1 s_1$, $\pi_2^{23L} = s_2^2 v_3/4\sigma_2 s_1$, and $\pi_3^{23L} = 0$.*

Note that, in Lemma D1, the three players are all active in the subgame-perfect equilibrium, whereas, in Lemma D2, only players 1 and 2 are active in the subgame-perfect equilibrium. It is immediate from these lemmas that player 3, the weakest player, is active in the subgame-perfect equilibrium if $s_2 < 2$, but he is inactive if $s_2 \geq 2$.

APPENDIX E

ANALYSIS OF THE 1*L* SEQUENTIAL-MOVE SUBGAME

Player 1 first chooses his effort level in period 1. Then, after observing player 1's effort level, players 2 and 3 choose their effort levels simultaneously and independently in period 2.

Now that the analysis is similar to that for the 3*L* sequential-move subgame in Appendix B, we omit it.

Lemma E1 *In the case where $s_1 < (s_2 + 2)/(s_2 + 1)$, we obtain the following in the subgame-perfect equilibrium of the 1L sequential-move subgame. (a) The players' effort levels are: $x_1^{1L} = \{s_1^2(s_2 + 1)^2 - s_2^2\}v_3/4\sigma_1 s_2(s_2 + 1)$, $x_2^{1L} = \{2s_2^3 + s_2^2 - s_1^2(s_2 + 1)^2 + 2s_1 s_2^2(s_2 + 1)\}v_3/4\sigma_2 s_2(s_2 + 1)^2$, and $x_3^{1L} = \{s_2^2 + 2s_2 - s_1^2(s_2 + 1)^2 + 2s_1(s_2 + 1)\}v_3/4(s_2 + 1)^2$. (b) The players' expected payoffs are: $\pi_1^{1L} = \{s_1(s_2 + 1) - s_2\}^2 v_3/4\sigma_1 s_2(s_2 + 1)$, $\pi_2^{1L} = \{2s_2^2 + s_2 - s_1(s_2 + 1)\}^2 v_3/4\sigma_2 s_2 (s_2 + 1)^2$, and $\pi_3^{1L} = \{s_2 + 2 - s_1(s_2 + 1)\}^2 v_3/4(s_2 + 1)^2$.*

Lemma E2 *In the case where $s_1 \geq (s_2 + 2)/(s_2 + 1)$, we obtain the following in the subgame-perfect equilibrium of the 1L sequential-move subgame. (a) Player 3's effort level is zero: $x_3^{1L} = 0$. (b) Player 3's expected payoff is zero: $\pi_3^{1L} = 0$.*

It is immediate from Lemmas E1 and E2 that player 3, the weakest player, is active in the subgame-perfect equilibrium if $s_1 < (s_2 + 2)/(s_2 + 1)$, but he is inactive if $s_1 \geq (s_2 + 2)/(s_2 + 1)$.

We need Lemmas E3 and E4 to obtain the results in Section 6.

Lemma E3 *In the case where $s_2/(s_2 - 1) \leq s_1 < 2s_2$, we obtain the following in the subgame-perfect equilibrium of the 1L sequential-move subgame. (a) The players' effort levels are: $x_1^{1L} = s_1^2 v_3/4\sigma_1 s_2$, $x_2^{1L} = s_1(2s_2 - s_1)v_3/4\sigma_2 s_2$, and $x_3^{1L} = 0$. (b) The players' expected payoffs are: $\pi_1^{1L} = s_1^2 v_3/4\sigma_1 s_2$, $\pi_2^{1L} = (2s_2 - s_1)^2 v_3/4\sigma_2 s_2$, and $\pi_3^{1L} = 0$.*

Lemma E4 *In the case where $s_1 \geq s_2/(s_2 - 1)$ and $s_1 \geq 2s_2$, we obtain the following in the subgame-perfect equilibrium of the 1L sequential-move subgame. (a) The players' effort levels are: $x_1^{1L} = s_2 v_3/\sigma_1$, $x_2^{1L} = 0$, and $x_3^{1L} = 0$. (b) The players' expected payoffs are: $\pi_1^{1L} = (s_1 - s_2)v_3/\sigma_1$, $\pi_2^{1L} = 0$, and $\pi_3^{1L} = 0$.*

APPENDIX F

ANALYSIS OF THE 2*L* SEQUENTIAL-MOVE SUBGAME

Player 2 first chooses his effort level in period 1. Then, after observing player 2's effort level, players 1 and 3 choose their effort levels simultaneously and independently in period 2.

Now that the analysis is similar to that for the 3*L* sequential-move subgame in Appendix B, we omit it.

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Lemma F1 *In the case where $s_2 < (s_1 + 2)/(s_1 + 1)$ or, equivalently, $s_1 < (2 - s_2)/(s_2 - 1)$, we obtain the following in the subgame-perfect equilibrium of the 2L sequential-move subgame. (a) The players' effort levels are: $x_1^{2L} = \{2s_1^3 + s_1^2 - s_2^2(s_1 + 1)^2 + 2s_1^2 s_2(s_1 + 1)\}v_3/4\sigma_1 s_1(s_1 + 1)^2$, $x_2^{2L} = \{s_2^2(s_1 + 1)^2 - s_1^2\}v_3/4\sigma_2 s_1(s_1 + 1)$, and $x_3^{2L} = \{s_1^2 + 2s_1 - s_2^2(s_1 + 1)^2 + 2s_2(s_1 + 1)\}v_3/4(s_1 + 1)^2$. (b) The players' expected payoffs are: $\pi_1^{2L} = \{2s_1^2 + s_1 - s_2(s_1 + 1)\}^2 v_3/4\sigma_1 s_1(s_1 + 1)^2$, $\pi_2^{2L} = \{s_2(s_1 + 1) - s_1\}^2 v_3/4\sigma_2 s_1(s_1 + 1)$, and $\pi_3^{2L} = \{s_1 + 2 - s_2(s_1 + 1)\}^2 v_3/4(s_1 + 1)^2$.*

Lemma F2 *In the case where $s_2 \geq (s_1 + 2)/(s_1 + 1)$ or, equivalently, $s_1 \geq (2 - s_2)/(s_2 - 1)$, we obtain the following in the subgame-perfect equilibrium of the 2L sequential-move subgame. (a) Player 3's effort level is zero: $x_3^{2L} = 0$. (b) Player 3's expected payoff is zero: $\pi_3^{2L} = 0$.*

It is immediate from Lemmas F1 and F2 that player 3, the weakest player, is active in the subgame-perfect equilibrium if $s_2 < (s_1 + 2)/(s_1 + 1)$, but he is inactive if $s_2 \geq (s_1 + 2)/(s_1 + 1)$.

We need Lemma F3 to obtain the results in Section 6.

Lemma F3 *In the case where $s_1 \geq s_2/(s_2 - 1)$, we obtain the following in the subgame-perfect equilibrium of the 2L sequential-move subgame. (a) The players' effort levels are: $x_1^{2L} = s_2(2s_1 - s_2)v_3/4\sigma_1 s_1$, $x_2^{2L} = s_2^2 v_3/4\sigma_2 s_1$, and $x_3^{2L} = 0$. (b) The players' expected payoffs are: $\pi_1^{2L} = (2s_1 - s_2)^2 v_3/4\sigma_1 s_1$, $\pi_2^{2L} = s_2^2 v_3/4\sigma_2 s_1$, and $\pi_3^{2L} = 0$.*

APPENDIX G

ANALYSIS OF THE 12L SEQUENTIAL-MOVE SUBGAME

Players 1 and 2 first choose their effort levels simultaneously and independently in period 1, and then, after observing their effort levels, player 3 chooses his effort level in period 2.

Now that the analysis is similar to that for the 13L sequential-move subgame in Appendix C, we omit it.

Lemma G1 *In the case where $s_1 < 2s_2/(3s_2 - 2)$, we obtain the following in the subgame-perfect equilibrium of the 12L sequential-move subgame. (a) The players' effort levels are: $x_1^{12L} = 9s_1^2 s_2^2(2s_1 - s_2)v_3/4\sigma_1(s_1 + s_2)^3$, $x_2^{12L} = 9s_1^2 s_2^2(2s_2 - s_1)v_3/4\sigma_2(s_1 + s_2)^3$, and $x_3^{12L} = 3s_1 s_2(2s_1 + 2s_2 - 3s_1 s_2)v_3/4(s_1 + s_2)^2$. (b) The players' expected payoffs are: $\pi_1^{12L} = 3s_1^2 s_2(2s_1 - s_2)^2 v_3/4\sigma_1(s_1 + s_2)^3$, $\pi_2^{12L} = 3s_1 s_2^2(2s_2 - s_1)^2 v_3/4\sigma_2(s_1 + s_2)^3$, and $\pi_3^{12L} = (2s_1 + 2s_2 - 3s_1 s_2)^2 v_3/4(s_1 + s_2)^2$.*

Lemma G2 *In the case where $s_1 \geq 2s_2/(3s_2 - 2)$, we obtain the following in the subgame-perfect equilibrium of the 12L sequential-move subgame. (a) Player 3's effort level is zero: $x_3^{12L} = 0$. (b) Player 3's expected payoff is zero: $\pi_3^{12L} = 0$.*

It is immediate from Lemmas G1 and G2 that player 3, the weakest player, is active in the subgame-perfect equilibrium if $s_1 < 2s_2/(3s_2 - 2)$, but he is inactive if $s_1 \geq 2s_2/(3s_2 - 2)$.

We need Lemma G3 to obtain the results in Section 6.

Lemma G3 *In the case where $s_1 \geq s_2/(s_2 - 1)$, we obtain the following in the subgame-perfect equilibrium of the 12L sequential-move subgame. (a) The players' effort levels are: $x_1^{12L} = \beta_1 s_1 s_2 v_3/(s_1 + s_2)^2$, $x_2^{12L} = \beta_2 s_1 s_2 v_3/(s_1 + s_2)^2$, and $x_3^{12L} = 0$. (b) The players' expected payoffs are: $\pi_1^{12L} = \beta_1 s_1^2 v_3/(s_1 + s_2)^2$, $\pi_2^{12L} = \beta_2 s_2^2 v_3/(s_1 + s_2)^2$, and $\pi_3^{12L} = 0$.*

APPENDIX H

PLAYER 3'S STRICTLY OR WEAKLY DOMINANT ACTION IN THE FIRST STAGE

In order to examine whether one of player 3's first-stage actions strictly or weakly dominates the other of his first-stage actions, we compare player 3's equilibrium expected payoffs in the proper subgames which start at the second (or effort-expending) stage. Player 3's equilibrium expected payoffs in these proper subgames are obtained in Appendixes A–G. Using the lemmas therein, it is straightforward to obtain Lemma H1.

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Lemma H1 *(a)* $\pi_3^S > \pi_3^{12L}$ *if* $s_1 < s_2/(s_2 - 1)$*, but* $\pi_3^S = \pi_3^{12L} = 0$ *otherwise. (b)* $\pi_3^{13L} > \pi_3^{1L}$ *if* $s_1 < 2$*, but* $\pi_3^{13L} = \pi_3^{1L} = 0$ *otherwise. (c)* $\pi_3^{23L} > \pi_3^{2L}$ *if* $s_2 < 2$*, but* $\pi_3^{23L} = \pi_3^{2L} = 0$ *otherwise. (d)* $\pi_3^{3L} > \pi_3^S$ *if* $s_1 < s_2/(s_2 - 1)$*, but* $\pi_3^{3L} = \pi_3^S = 0$ *otherwise.*

Next, using Lemma H1, we obtain the following lemma.

Lemma H2 *(a) If* $s_1 < s_2/(s_2 - 1)$*,* $s_1 < 2$*, and* $s_2 < 2$*, then* $\pi_3^S > \pi_3^{12L}$*,* $\pi_3^{13L} > \pi_3^{1L}$*,* $\pi_3^{23L} > \pi_3^{2L}$*, and* $\pi_3^{3L} > \pi_3^S$*. (b) If* $s_1 < s_2/(s_2 - 1)$*,* $s_1 \geq 2$*, and* $s_2 < 2$*, then* $\pi_3^S > \pi_3^{12L}$*,* $\pi_3^{13L} = \pi_3^{1L} = 0$*,* $\pi_3^{23L} > \pi_3^{2L}$*, and* $\pi_3^{3L} > \pi_3^S$*. (c) If* $s_1 \geq s_2/(s_2 - 1)$*,* $s_1 > 2$*, and* $s_2 < 2$*, then* $\pi_3^S = \pi_3^{12L} = 0$*,* $\pi_3^{13L} = \pi_3^{1L} = 0$*,* $\pi_3^{23L} > \pi_3^{2L}$*, and* $\pi_3^{3L} = \pi_3^S = 0$*. (d) If* $s_1 > s_2/(s_2 - 1)$*,* $s_1 > 2$*, and* $s_2 \geq 2$*, then* $\pi_3^S = \pi_3^{12L} = 0$*,* $\pi_3^{13L} = \pi_3^{1L} = 0$*,* $\pi_3^{23L} = \pi_3^{2L} = 0$*, and* $\pi_3^{3L} = \pi_3^S = 0$*.*

Note that the values of s_1 and s_2 which satisfy each conditional statement in Lemma H2, together with Assumption 1, are located in a different area of Figure 1. For example, the values of s_1 and s_2 which satisfy the conditional statement in part (*d*) are located in the lightly shaded area.

Note also that each conditional statement in Lemma H2 can be simplified. For example, part (*a*) can be rewritten as: (*a*) If $s_1 < 2$, then $\pi_3^S > \pi_3^{12L}$, $\pi_3^{13L} > \pi_3^{1L}$, $\pi_3^{23L} > \pi_3^{2L}$, and $\pi_3^{3L} > \pi_3^S$. However, we choose not to simplify the conditional statements in Lemma H2 to help readers easily verify the lemma by using Lemma H1.

In part (*a*), player 3's first-stage action *Period 1* strictly dominates his first-stage action *Period 2* because his action *Period 1* yields a greater expected payoff than his action *Period 2* for every list of the other players' first-stage actions (see e.g., Osborne, 2004, pp. 45–47).

In parts (*b*) and (*c*), player 3's first-stage action *Period 1* weakly dominates his first-stage action *Period 2* because (i) his action *Period 1* yields at least as much an expected payoff as his action *Period 2* for every list of the other players' first-stage actions and (ii) his action *Period 1* yields a greater expected payoff than his action *Period 2* for some list of the other players' first-stage actions.

In part (*d*), player 3's first-stage action *Period 1* yields the same expected payoff as his first-stage action *Period 2* for every list of the other players' first-stage actions.

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