Endogenous Timing in Contests with Delegation*

by

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Abstract

We study two-player contests in which each player hires a delegate, and the delegates decide endogenously when to expend their effort. First we look closely at the delegates' decisions on when to expend their effort, given contracts between the players and the delegates, and look at the players' decisions on their contracts. Then, we compare the outcomes of the endogenous-timing framework with those of the simultaneous-move framework. We show that the higher-valuation player offers her delegate greater contingent compensation than her opponent, the delegate of the higher-valuation player chooses his effort level after observing his counterpart's, the equilibrium expected payoff of the delegate of the higher-valuation player is greater than that of his counterpart, and economic rent for each delegate exists. We show that, in the endogenous-timing framework, each player offers her delegate better contingent compensation, each delegate's expected payoff is greater, and each player's expected payoff is smaller, as compared with the simultaneous-move framework.

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1. Introduction

Contests with delegation, where each player hires a delegate to expend effort or resources on the player's behalf to win a prize, are common.[1] An example is litigation between a plaintiff and a defendant in which each litigant hires an attorney who expends his effort to win the lawsuit on behalf of his client. Another example is a rent-seeking contest in which each firm hires a lobbyist who expends his effort to acquire a monopoly or a procurement contract from the government on behalf of his client. Yet another example is a patent contest in which each firm hires a group of independent researchers or university professors who conduct research to obtain a patent on behalf of their employer. Delegation is attractive to players because they each can use superior ability by hiring a delegate with more ability than herself, and achieve strategic commitments through delegation.

In such contests with delegation, we believe, delegates decide endogenously when to expend their effort after signing delegation contracts. Indeed, pursuing their self-interest, the delegates may well endogenize the order of their moves, and communicate it with each other before they expend their effort. We believe also that players take it into account, when designing their contracts for the delegates, that the delegates will endogenize the order of their moves. It is these ideas that motivate this paper.

Accordingly, the purpose of this paper is to systematically study contests with delegation in which delegates decide endogenously when to expend their effort after signing delegation contracts and before they expend their effort. We study two-player contests with delegation, focusing on not only the equilibrium delegation contracts between players and their delegates but also the equilibrium orders of the delegates' moves. Specifically, we set up and analyze the following three-stage game. In the first stage, two players each hire a delegate, and they simultaneously announce their contracts written with their delegates. Each player's contract specifies how much her delegate will be paid if he wins the contest and how much if he loses it. In the second stage, each delegate decides when to expend his effort, and the delegates announce

their decisions simultaneously. In the third stage, the delegates expend their effort according to their decisions in the second stage to win the prize on behalf of their employers.

We show that, in equilibrium, the delegate hired by the higher-valuation player chooses his effort level after observing his counterpart's, and his expected payoff is greater than that of his counterpart. This advantage of the delegate of the higher-valuation player can be explained by the fact that his employer, the higher-valuation player, offers better contingent compensation (as it will be defined in Section 2) to him than the lower-valuation player does to her delegate.

We compare the outcomes of the endogenous-timing framework – the three-stage game presented above – with those of the simultaneous-move framework, the game which is the same as the three-stage game with the exception that the delegates choose their effort levels simultaneously and this order of their moves is given exogenously. An interesting result is that each player offers her delegate a better contract in the endogenous-timing framework than in the simultaneous-move framework. Another interesting result is that, unless the valuation for the prize of the higher-valuation player is significantly greater than that of the lower-valuation player, then each delegate's expected payoff is greater in the endogenous-timing framework than in the simultaneous-move framework, whereas each player's expected payoff is less in the endogenous-timing framework than in the simultaneous-move framework.

In summary, pursuing their self-interest, the delegates decide endogenously when to expend their effort *given* contracts between the players and the delegates. This in turn leads to the players offering better contracts to their delegates. Consequently, the delegates are better off, but the players are worse off, as compared with the simultaneous-move framework. We are tempted to call the delegates' endogenous-timing behavior "betrayal" of their bosses by the delegates or "revenge" of the delegates against their bosses.

This paper is related to Baik and Kim (1997), Wärneryd (2000), Schoonbeek (2002), Konrad et al. (2004), and Baik (2007, 2008), which study delegation in contests.[2] Baik and Kim (1997) study delegation in two-player contests in which delegation contracts are exogenous, and show that buying superior ability is an important motive of delegation. Considering two-player

contests with bilateral delegation, Wärneryd (2000) shows that compulsory delegation may be beneficial to the players in the case where the delegates' effort is unobservable. Schoonbeek (2002) considers unilateral delegation in a two-player contest, and shows that economic rent for the delegate may exist. Konrad et al. (2004) study delegation in a first-price all-pay auction with two buyers, and show that delegation is beneficial to the buyers. Baik (2007) considers two-player contests with bilateral delegation. Endogenizing delegation contracts, he finds the equilibrium contracts between the players and their delegates, and shows that economic rents for the delegates may exist. Baik (2008) studies litigation with bilateral delegation, and shows that the attorneys prefer the system with the nonnegative-fixed-fee constraint to the system with the contingent-fee cap, while the opposite holds for the litigants. A striking difference between this paper and the previous papers is that this paper allows endogenous timing of delegates' exerting effort, whereas the previous papers do not.

This paper is also related to Baik and Shogren (1992), Leininger (1993), Nitzan (1994b), Baik (1994), Morgan (2003), Fu (2006), and Konrad and Leininger (2007), which study endogenous timing, but not delegation, in contests. Baik and Shogren (1992) and Leininger (1993) show that the underdog always exerts effort before the favorite does.[3] Unlike the other papers, Konrad and Leininger (2007) use all-pay-auction contest success functions, and consider contests with more than two players in which each player's cost function is a general convex function of effort. In Baik and Shogren (1992), Leininger (1993), Nitzan (1994b), Baik (1994), and Konrad and Leininger (2007), each player's valuation for the prize is publicly known at the start of the game. However, in Morgan (2003) and Fu (2006), only the probability distribution of each player's valuation is publicly known at the start of the game; the players' valuations are realized after the players announce when they will expend their effort. The realized valuations are immediately revealed to both players in Morgan (2003), while they are immediately revealed to only one of the players in Fu (2006).

The paper proceeds as follows. Section 2 develops the model and sets up the three-stage game. In Section 3, we look at the delegates' decisions in the second and third stages of the

game. Specifically, we first look at the delegates' third-stage decisions on their effort levels, and then the delegates' second-stage decisions on when to expend their effort, given contracts between the players and the delegates. In Section 4, we look at the players' decisions on their contracts in the first stage of the game. In Section 5, to get more mileage, we modify the three-stage game by assuming simple logit-form contest success functions, and obtain its outcomes. In Section 6, we perform comparative statics of the outcomes of the modified game with respect to each player's valuation for the prize. In Section 7, we compare the outcomes of the endogenous-timing framework with those of the simultaneous-move framework. Finally, Section 8 offers our conclusions.

2. The model

Two players, 1 and 2, vie for a prize. Each player hires a delegate. The two delegates, 1 and 2, compete by expending their irreversible effort to win the prize on behalf of their employers. Each delegate bears the cost of expending his effort. There are two periods, the first and the second, between which each delegate chooses to expend his effort. Each delegate expends his effort in either of the two periods, but not in both periods. The probability that a delegate wins the prize is increasing in his own effort level, and decreasing in the rival delegate's effort level.

We formally model this situation as the following noncooperative game. In the first stage, the players hire delegates and simultaneously announce their contracts written with their delegates. In the second stage, after knowing both contracts, each delegate chooses independently whether he will expend his effort in the first period or in the second period. The delegates announce their choices simultaneously. In the third stage, after knowing the contracts and knowing when his rival expends effort, each delegate chooses his effort level in the period which he chose and announced in the second stage.[4] Once the winning player is determined at the end of this stage, each player (or only the winning player, as it will be clear shortly) pays compensation to her delegate according to her contract announced in the first stage.

The players and the delegates are risk-neutral. Player 1 values the prize at v_1, and player 2 at v_2, where $v_1 \geq v_2$.[5] We assume that v_1 and v_2 are positive, measured in monetary units, and publicly known. A contract that player i designs and offers delegate i in the first stage takes the following form: Compensation of $\alpha_i v_i$ is paid to delegate i if he wins the prize, and zero if he loses it, where $0 < \alpha_i < 1$.[6] We call compensation of $\alpha_i v_i$ delegate i's *contingent compensation*. Delegate i accepts player i's contract if it will yield him an expected payoff greater than or equal to his reservation wage, given his belief about the other player's contract. We assume that delegate i has a reservation wage of 0.[7]

Let x_i denote the effort level that delegate i expends in the third stage. Each delegate's effort level is positive, measured in monetary units, and may or may not be observable to his employer.[8] Let $p_i(x_1, x_2)$ represent the probability that delegate i wins the prize when delegates 1 and 2 expend x_1 and x_2, respectively, where $0 \leq p_i(x_1, x_2) \leq 1$ and $p_1(x_1, x_2) + p_2(x_1, x_2) = 1$. We assume that the contest success function for delegate i is

$$p_i(x_1, x_2) = h(x_i)/\{h(x_1) + h(x_2)\}, \tag{1}$$

where the function h has the properties specified in Assumption 1 below.[9] Using function (1), we obtain $p_1(y, z) = p_2(z, y)$ for every pair, y and z, of effort levels which indicates that the delegates have equal ability for the contest.

Assumption 1. *We assume that* $h(0) \geq 0$, *h is twice differentiable,* $h'(x_i) > 0$, *and* $h''(x_i) \leq 0$, *for all* x_i *in* R_+, *where* h' *and* h'' *denote, respectively, the first and second derivatives of the function h, and* R_+ *denotes the set of all positive real numbers.*

Assumption 1, together with function (1), implies that $\partial p_i/\partial x_i > 0$ for $x_j > 0$ and $\partial p_i/\partial x_j < 0$ for $x_i > 0$, and that $\partial^2 p_i/\partial x_i^2 < 0$ for $x_j > 0$ and $\partial^2 p_i/\partial x_j^2 > 0$ for $x_i > 0$, where j is the other delegate. Thus Assumption 1 together with function (1) implies that, given the rival's effort level, each delegate's probability of winning is increasing in his own effort level at a

decreasing rate; it is decreasing in his rival's effort level at a decreasing rate, given that his own effort level remains constant.

Let G_i denote the expected payoff for player i. Then the payoff function for player i is

$$G_i = (1 - \alpha_i)v_i p_i(x_1, x_2). \tag{2}$$

Let π_i denote the expected payoff for delegate i. Then the payoff function for delegate i is

$$\pi_i = \alpha_i v_i p_i(x_1, x_2) - x_i. \tag{3}$$

Note that the players and the delegates compute these expected payoffs at the start of the game, believing that players 1 and 2 will announce, respectively, their contracts α_1 and α_2 in the first stage, and delegates 1 and 2 will expend, respectively, their effort levels x_1 and x_2 in the third stage. They do so because they need to know their payoff functions, at the beginning of the game, to choose their optimal strategies.

All of the above is common knowledge among the players and delegates. We employ subgame-perfect equilibrium as the solution concept.

3. Delegates' decisions in the second and third stages

To obtain a subgame-perfect equilibrium of the game, we work backward. In this section, we analyze first the subgames which start at the third stage, and then consider the delegates' decisions, in the second stage, on when to expend their effort.[10] Note that, in these stages, the delegates know both contracts – or, equivalently, the values of α_1 and α_2 – chosen in the first stage.

There are three *distinct* subgames which start at the third stage: the simultaneous-move subgame, the 1L sequential-move subgame, and the 2L sequential-move subgame.[11] The simultaneous-move subgame arises when both delegates announce that they will expend their effort in the same period, either the first period or the second one. If delegate 1 announces that he will expend his effort in the first period but delegate 2 announces the second period, then the

$1L$ sequential-move subgame arises. Finally, the $2L$ sequential-move subgame arises when delegate 1 announces the second period but delegate 2 announces the first period.

3.1. The simultaneous-move subgame

In this subgame, the delegates choose their effort levels simultaneously. To characterize a Nash equilibrium of the subgame, we begin by looking at the delegates' reaction functions. Let $x_1 = r_1(x_2)$ denote delegate 1's reaction function, and let $x_2 = r_2(x_1)$ denote delegate 2's reaction function. In Appendix A, we find the shapes of the delegates' reaction functions.

The simultaneous-move subgame has a unique Nash equilibrium.[12] Let (x_1^N, x_2^N) denote the Nash equilibrium. If $\alpha_1 v_1 > \alpha_2 v_2$, then we have $x_1^N > x_2^N$. If $\alpha_1 v_1 < \alpha_2 v_2$, then we have $x_1^N < x_2^N$. If $\alpha_1 v_1 = \alpha_2 v_2$, then we have $x_1^N = x_2^N$. Let the favorite [the underdog] be the delegate who has a probability of winning greater [less] than $1/2$ at the Nash equilibrium of the simultaneous-move subgame. Then, when $\alpha_1 v_1 > \alpha_2 v_2$, delegate 1 is the favorite and delegate 2 the underdog. When $\alpha_1 v_1 < \alpha_2 v_2$, delegate 1 is the underdog and delegate 2 the favorite. When $\alpha_1 v_1 = \alpha_2 v_2$, each delegate's probability of winning equals $1/2$ at the Nash equilibrium.

3.2. The sequential-move subgames

In the $1L$ sequential-move subgame, delegate 1 chooses his effort level, and then after observing delegate 1's effort level, delegate 2 chooses his effort level. Let (x_1^{1L}, x_2^{1L}) denote the pair of the delegates' effort levels specified in the subgame-perfect equilibrium of the $1L$ sequential-move subgame. If $\alpha_1 v_1 > \alpha_2 v_2$, then we have $x_1^{1L} > x_1^N$ and $x_2^{1L} < x_2^N$. If $\alpha_1 v_1 < \alpha_2 v_2$, then we have $x_1^{1L} < x_1^N$ and $x_2^{1L} < x_2^N$. If $\alpha_1 v_1 = \alpha_2 v_2$, then we have $x_i^{1L} = x_i^N$ for $i = 1, 2$. Next, consider the $2L$ sequential-move subgame in which delegate 2 chooses his effort level, and then after observing delegate 2's effort level, delegate 1 chooses his effort level. Let (x_1^{2L}, x_2^{2L}) denote the pair of the delegates' effort levels specified in the subgame-perfect equilibrium of the $2L$ sequential-move subgame. If $\alpha_1 v_1 > \alpha_2 v_2$, then we have $x_1^{2L} < x_1^N$ and

$x_2^{2L} < x_2^N$. If $\alpha_1 v_1 < \alpha_2 v_2$, then we have $x_1^{2L} < x_1^N$ and $x_2^{2L} > x_2^N$. If $\alpha_1 v_1 = \alpha_2 v_2$, then we have $x_i^{2L} = x_i^N$ for $i = 1, 2$.

3.3. Timing of delegates' exerting effort

We now consider the second stage of the full game in which the delegates choose and announce when to expend their effort. We begin by comparing each delegate's equilibrium expected payoffs in the three subgames. Denote by π_i^N delegate *i*'s expected payoff at the Nash equilibrium of the simultaneous-move subgame. Denote by π_i^{1L} delegate *i*'s expected payoff in the subgame-perfect equilibrium of the 1*L* sequential-move subgame. Denote by π_i^{2L} delegate *i*'s expected payoff in the subgame-perfect equilibrium of the 2*L* sequential-move subgame. Then we obtain the following results. If $\alpha_1 v_1 > \alpha_2 v_2$, then we have $\pi_1^N < \pi_1^{2L}$ and $\pi_2^{1L} < \pi_2^N < \pi_2^{2L}$. If $\alpha_1 v_1 < \alpha_2 v_2$, then we have $\pi_1^{2L} < \pi_1^N < \pi_1^{1L}$ and $\pi_2^N < \pi_2^{1L}$. If $\alpha_1 v_1 = \alpha_2 v_2$, then we have $\pi_i^{1L} = \pi_i^N = \pi_i^{2L}$ for $i = 1, 2$.

Using these results, we obtain Lemma 1.

Lemma 1. (*a*) *If* $\alpha_1 v_1 > \alpha_2 v_2$, *then delegate* 1 *announces the second period while delegate* 2 *announces the first period.* (*b*) *If* $\alpha_1 v_1 < \alpha_2 v_2$, *then delegate* 1 *announces the first period while delegate* 2 *announces the second period.* (*c*) *If* $\alpha_1 v_1 = \alpha_2 v_2$, *then each delegate announces either the first period or the second period.*

Lemma 1 per se is interesting. It says that, in the case where $\alpha_1 v_1 \neq \alpha_2 v_2$, the underdog announces the first period while the favorite announces the second period.[13] The intuition behind this result is obvious. The underdog, or the delegate with less contingent compensation, has a strong incentive to be the leader in the effort-expending stage because he, as the leader, can signal (to his formidable rival) his intention to avoid a big fight. On the other hand, the favorite has an incentive to be the follower. Indeed, he benefits from yielding the leadership role because

he can ease up and respond efficiently to his small rival's challenge. The two delegates' incentives are not conflicting.

4. Players' decisions in the first stage

In the first stage, player *i* has perfect foresight about the subgame-perfect equilibria of each subgame which starts at the second stage of the full game. Taking player *j*'s contract α_j as given, player *i* seeks to maximize her expected payoff over her contract α_i, where *j* is the other player. Using payoff function (2) and Lemma 1, we obtain player *i*'s expected payoffs, each depending on the value of α_i that player 1 chooses, that are computed in the first stage but take into account the equilibria of the subgames starting at the second stage. Given a value of α_j, if player *i* chooses a value of α_i such that $\alpha_1 v_1 > \alpha_2 v_2$, then in the second stage, delegate 1 announces the second period while delegate 2 announces the first period, which leads to the 2*L* sequential-move subgame analyzed in Section 3.2. In this case, player *i*'s expected payoff is $G_i^{2L} = (1 - \alpha_i) v_i p_i(x_1^{2L}, x_2^{2L})$. Similarly, if player *i* chooses a value of α_i such that $\alpha_1 v_1 < \alpha_2 v_2$, then player *i*'s expected payoff is $G_i^{1L} = (1 - \alpha_i) v_i p_i(x_1^{1L}, x_2^{1L})$. If player *i* chooses a value of α_i such that $\alpha_1 v_1 = \alpha_2 v_2$, then player *i*'s expected payoff is $G_i^{2L} = G_i^{1L} = G_i^N$, where $G_i^N = (1 - \alpha_i) v_i p_i(x_1^N, x_2^N)$.[14] Thus player *i* faces the following maximization problem: Given a value of α_j,

$$\underset{\alpha_i}{\text{Max}} \; G_i, \tag{4}$$

where $G_i = G_i^{iL}$ for $0 < \alpha_i \leq \alpha_j v_j / v_i$ and $G_i = G_i^{jL}$ for $\alpha_j v_j / v_i \leq \alpha_i < 1$.

Let $b_i(\alpha_j)$ denote player *i*'s best response to α_j which solves (4). Using the players' reaction functions, $\alpha_1 = b_1(\alpha_2)$ and $\alpha_2 = b_2(\alpha_1)$, we obtain the equilibrium contracts, α_1^* and α_2^*, of the players.

5. Outcomes of the three-stage game with a specific form of the function *h*

To get more mileage, we assume henceforth that $h(x_i) = x_i$.[15] With this specific form of the function h, we obtain first the equilibrium contracts of the players, and then other outcomes of the three-stage game.

Taking the steps explained above, we find player i's reaction function:[16]

$$\alpha_i = b_i(\alpha_j) = \begin{cases} \sqrt{\alpha_j v_j / 2v_i} & \text{for } 0 < \alpha_j \leq v_i/2v_j \\ 1/2 & \text{for } v_i/2v_j < \alpha_j < 1, \end{cases} \tag{5}$$

for $i, j = 1, 2$ with $i \neq j$. Figure 1 illustrates the players' reaction functions resulting when $v_1 > v_2$. In Figure 1, the straight dotted line emanating from the origin represents the locus of points which satisfy $\alpha_1 v_1 = \alpha_2 v_2$. Thus $\alpha_1 v_1 < \alpha_2 v_2$ holds at any point above the line, and $\alpha_1 v_1 > \alpha_2 v_2$ holds at any point below the line. This, together with Lemma 1, implies that any point above the line yields player 1's expected payoff of G_1^{1L} and player 2's expected payoff of G_2^{1L}; any point below the line yields player 1's expected payoff of G_1^{2L} and player 2's expected payoff of G_2^{2L}.[17] This in turn implies, together with the locations of the players' reaction functions in Figure 1, the following. For $0 < \alpha_j \leq v_i/2v_j$, player i's best response $b_i(\alpha_j)$ to α_j is a value of α_i which maximizes her expected payoff G_i^{jL}, and for $v_i/2v_j < \alpha_j < 1$, it is a value of α_i which maximizes her expected payoff G_i^{iL}.[18]

Now, we find the equilibrium contracts of the players, which satisfy simultaneously the players' reaction functions represented by equation (5). Solving the pair of simultaneous equations, we obtain: $\alpha_1^* = \sqrt{v_2/4v_1}$ and $\alpha_2^* = 1/2$. Point S in Figure 1 represents the equilibrium contracts of the players resulting when $v_1 > v_2$.

Let x_i^* represent the effort level of delegate i that is specified in the subgame-perfect equilibria. Let $p_1(x_1^*, x_2^*)$ be the probability that delegate 1 and player 1 win the prize in the subgame-perfect equilibria. Let π_i^* and G_i^* represent the expected payoff for delegate i and that for player i, respectively, in the subgame-perfect equilibria. Then, using Lemma 1 and expressions (1) through (3), we obtain Proposition 1.[19]

Proposition 1. (*a*) $\alpha_1^* = \sqrt{v_2/4v_1}$ *and* $\alpha_2^* = 1/2$, *so that* $\alpha_1^* \leq \alpha_2^*$ *and* $\alpha_1^* v_1 \geq \alpha_2^* v_2$. (*b*) *If* $v_1 > v_2$, *then delegate* 1 *announces the second period while delegate* 2 *announces the first period. If* $v_1 = v_2$, *then each delegate announces either the first period or the second period.* (*c*) $x_1^* = v_2(2\sqrt{v_1} - \sqrt{v_2})/8\sqrt{v_1}$ *and* $x_2^* = v_2\sqrt{v_2}/8\sqrt{v_1}$, *so that* $x_1^* \geq x_2^*$. (*d*) $p_1(x_1^*, x_2^*) = 1 - \sqrt{v_2}/2\sqrt{v_1} \geq 1/2$. (*e*) $\pi_1^* = \sqrt{v_2}(2\sqrt{v_1} - \sqrt{v_2})^2/8\sqrt{v_1}$ *and* $\pi_2^* = v_2\sqrt{v_2}/8\sqrt{v_1}$, *so that* $\pi_1^* \geq \pi_2^* > 0$. (*f*) $G_1^* = (2\sqrt{v_1} - \sqrt{v_2})^2/4$ *and* $G_2^* = v_2\sqrt{v_2}/4\sqrt{v_1}$, *so that* $G_1^* \geq G_2^* > 0$.

Note that the lower-valuation player pays her delegate half her valuation for the prize if he wins the prize. Note also that, if $v_1 > v_2$, then we obtain: $\alpha_1^* < \alpha_2^*$, $\alpha_1^* v_1 > \alpha_2^* v_2$, $x_1^* > x_2^*$, $p_1(x_1^*, x_2^*) > 1/2$, $\pi_1^* > \pi_2^*$, and $G_1^* > G_2^*$. Parts (*a*) and (*b*) of Proposition 1 say that, trying to overcome her relative "weakness" in the valuations for the prize, the lower-valuation player (player 2) offers her delegate (delegate 2) the "maximum" value of α_2 that she is willing to offer, but fails to make him the favorite, so that her delegate chooses to be the leader in the effort-expending stage. Part (*e*) of Proposition 1 says that the equilibrium expected payoff for the delegate hired by the higher-valuation player is greater than that for his counterpart. The *identical* delegates – before signing up for their employers – end up having different expected payoffs because they are offered different contracts: The higher-valuation player strategically offers her delegate greater contingent compensation and motivates him more strongly than her opponent does. Part (*e*) says also that economic rent for each delegate exists – that is, each delegate's equilibrium expected payoff is greater than his reservation wage.[20]

6. Comparative statics

In this section, we examine how those outcomes of the game obtained in Section 5 respond when the asymmetry between the players changes – that is, we perform comparative statics of those outcomes of the game with respect to each player's valuation for the prize.

Using Proposition 1, we examine first the effects of *increasing* player 1's valuation v_1 for the prize on the outcomes of the game. Proposition 2 summarizes the comparative statics results.[21]

Proposition 2. *As* v_1 *increases from* v_2, (*a*) α_1^* *decreases but* $\alpha_1^* v_1$ *increases, and thus the gap between* $\alpha_1^* v_1$ *and* $\alpha_2^* v_2$ *widens,* (*b*) *delegate* 1 *expends more effort while delegate* 2 *expends less,* (*c*) *total effort level remains unchanged,* (*d*) *delegate* 1*'s probability of winning increases while delegate* 2*'s decreases, and* (*e*) *delegate* 1*'s and player* 1*'s expected payoffs each increase while delegate 2's and player* 2*'s expected payoffs each decrease*.

Parts (*b*) and (*c*) are stated in more detail as follows. Delegate 1's effort level is increasing in v_1, but its limit is $v_2/4$, as v_1 approaches (plus) infinity. Delegate 2's effort level is decreasing in v_1, but its limit is 0, as v_1 approaches infinity. As v_1 increases, total effort level remains constant at $v_2/4$.

Part (*a*) of Proposition 2 says that, as her valuation for the prize increases, the higher-valuation player (player 1) makes her delegate more aggressive or stronger by offering him greater contingent compensation. Because of her higher valuation, she can do so with a lower value of α_1. Part (*c*) is interesting because a previous result in the literature on the theory of contests shows that, as v_1 increases from v_2, the equilibrium total effort level increases.[22] The explanation for part (*c*) follows. In equilibrium, the higher-valuation player (player 1) offers her delegate greater contingent compensation than her opponent – which leads to the 2*L* sequential-move subgame in the effort-expending stage – and the lower-valuation player (player 2) offers her delegate the "maximum" value of α_2, $\alpha_2^* = 1/2$, that she is willing to offer (see Proposition 1). Consequently, the equilibrium total effort level is equal to $\alpha_2^* v_2/2$.[23] Next, note that, because $\alpha_2^* = 1/2$, the equilibrium total effort level, $\alpha_2^* v_2/2$, is independent of v_1 – specifically, as v_1 increases from v_2, the delegates' equilibrium effort levels change in the opposite directions

by the same amount. Therefore, the equilibrium total effort level remains unchanged as v_1 increases from v_2.

Next, using Proposition 1, we examine the effects of *decreasing* player 2's valuation v_2 for the prize on the outcomes of the game. It may appear that, given that $v_1 \geq v_2$, the effects of decreasing v_2 are similar to those of increasing v_1. However, this appearance is partly incorrect. We obtain first that, as v_2 decreases from v_1, *ceteris paribus*, both $\alpha_1^* v_1$ and $\alpha_2^* v_2$ decrease, which is in contrast with part (*a*) of Proposition 2. This in turn leads to some other comparative statics results that are different from those in Proposition 2: As v_2 decreases from v_1, *ceteris paribus,* (i) both delegate 1 and delegate 2 expend less effort, (ii) total effort level decreases, and (iii) delegate 1's expected payoff increases if $v_2 > 4v_1/9$, and decreases if $v_2 < 4v_1/9$. The first result is interesting that both parties expend less effort as one of the players' valuations for the prize decreases. The intuitions behind these results are as follows. As v_2 decreases from v_1, α_1^* decreases and α_2^* is constantly equal to $1/2$, so that both $\alpha_1^* v_1$ and $\alpha_2^* v_2$ decrease. Given less contingent compensations, delegate 2 (the leader in the effort-expending stage) is less motivated, and thus expends less effort; delegate 1 who also is less motivated follows suit. The effect of decreasing v_2 on delegate 1's expected payoff is not unidirectional because, as v_2 decreases from v_1, delegate 1's probability of winning increases while α_1^* decreases at an increasing rate. An increase in delegate 1's probability of winning tends to increase his expected payoff while a decrease in his contingent compensation $\alpha_1^* v_1$ tends to decrease his expected payoff. At a "high" value of v_2, the former is less than offset by the latter, so that delegate 1's expected payoff increases as v_2 decreases. At a "low" value of v_2, the former is more than offset by the latter, so that delegate 1's expected payoff decreases as v_2 decreases.

7. Comparison to the simultaneous-move framework

In this section, assuming that $h(x_i) = x_i$, we compare the outcomes of the endogenous-timing framework – the full game analyzed so far – with those of the simultaneous-move

framework. The outcomes of the simultaneous-move framework are provided in Lemma B1 in Appendix B.

Figure 1 shows the players' reaction functions and the equilibrium contracts in the two frameworks when v_1 is greater, but not "significantly" greater, than v_2 (see footnote 24). Points *S* and *Q* represent the equilibrium contracts in the endogenous-timing framework and those in the simultaneous-move framework, respectively.

Now, using Proposition 1 and Lemma B1 in Appendix B, we compare the outcomes of the endogenous-timing framework with those of the simultaneous-move framework. The superscripts * and ** in Proposition 3 indicate the outcomes of the endogenous-timing framework and those of the simultaneous-move framework, respectively.

Proposition 3. (*i*) *If* $v_1 = v_2$, *then we obtain:* $\alpha_i^* > \alpha_i^{**}$, $x_i^* > x_i^{**}$, $\pi_i^* > \pi_i^{**}$, *and* $G_i^* < G_i^{**}$, *for* $i = 1, 2$. (*ii*) *If* $v_1 > v_2$, *then we obtain:* $\alpha_2^* > \alpha_2^{**}$, $\pi_2^* > \pi_2^{**}$, *and* $G_2^* < G_2^{**}$. (*iii*) *Unless* v_1 *is significantly greater than* v_2, *then we obtain:* $\alpha_1^* > \alpha_1^{**}$, $\pi_1^* > \pi_1^{**}$, *and* $G_1^* < G_1^{**}$.[24]

Proposition 3 says that each player offers her delegate better contingent compensation in the endogenous-timing framework than in the simultaneous-move framework: $\alpha_i^* v_i > \alpha_i^{**} v_i$. This result is very interesting. The intuitions behind the result are as follows. In the endogenous-timing framework, the delegate with greater contingent compensation (than his counterpart) gains strategic advantage – as the second mover – against his counterpart (see Lemma 1). Such strategic advantage of the delegate is beneficial also to his employer. Hence, expecting that the delegates will endogenize the order of their moves in the remainder of the game, the higher-valuation player (in the first stage) has an incentive to offer – and actually offers – her delegate better contingent compensation than she does in the simultaneous-move framework. On the other hand, the lower-valuation player – who knows that she will fail to offer her delegate greater contingent compensation than her opponent – also does so for a different reason. In the endogenous-timing framework, the delegate with less contingent

compensation strategically softens the competition in the effort-expending stage by moving earlier (than his counterpart) and expending less effort, as compared with the simultaneous-move framework (see Lemma 1 and Section 3.2). Thus, expecting such behavior from her delegate, the lower-valuation player offers him better contingent compensation than she does in the simultaneous-move framework in order to let him fight harder.

Part (*i*) says that, if $v_1 = v_2$, each delegate's effort level and total effort level are greater in the endogenous-timing framework than in the simultaneous-move framework.[25] It follows from Lemma 1 and Section 3.2 that, given contracts such that $\alpha_1 v_1 \neq \alpha_2 v_2$, each delegate's effort level and total effort level are smaller in the case where the delegates decide endogenously when to expend their effort than in the case where they must choose their effort levels simultaneously. On the basis of this, one may conjecture that the opposite of part (*i*) – specifically, $x_i^* < x_i^{**}$ for $i = 1, 2$ – holds. But this conjecture is wrong. One should not overlook the fact that both delegates are offered better contingent compensation – so that they are motivated to exert more effort – in the endogenous-timing framework than in the simultaneous-move framework.

Another interesting result in Proposition 3 is that, unless v_1 is significantly greater than v_2, each delegate's expected payoff is greater in the endogenous-timing framework than in the simultaneous-move framework, whereas each player's expected payoff is less in the endogenous-timing framework than in the simultaneous-move framework. No doubt, the players' stiffer competition in the endogenous-timing framework, each trying to provide her own delegate with strategic advantage against his counterpart, makes the delegates better off but the players themselves worse off, as compared with the simultaneous-move framework. On the basis of this result, we argue that the players prefer the simultaneous-move framework (to the endogenous-timing framework) while the delegates prefer the endogenous-timing framework. We argue also that it benefits the delegates but hurts the players' expected payoffs to allow and facilitate the delegates to endogenize the order of their moves.

8. Conclusions

We have studied two-player contests with delegation in which each player first hires a delegate, then each delegate decides independently whether he will expend his effort in the first period or in the second period, and then the two delegates expend their effort to win the prize on behalf of their employers. First we have looked closely at the delegates' decisions on when to expend their effort, given contracts between the players and the delegates, and looked at the players' decisions on their contracts. Next, assuming that $h(x_i) = x_i$, we have found the players' contracts, the orders of the delegates' moves, their effort levels, their probabilities of winning, and the expected payoffs of the delegates and the players that are specified in the subgame-perfect equilibria. Finally, we have performed comparative statics of these outcomes with respect to each player's valuation for the prize, and compared these outcomes (of the endogenous-timing framework) with the outcomes of the simultaneous-move framework.

We have shown in Section 3 that the underdog, or the delegate with less contingent compensation, announces the first period while the favorite, or the delegate with greater contingent compensation, announces the second period. In Section 5, we have shown that the higher-valuation player offers her delegate greater contingent compensation than her opponent, the delegate hired by the higher-valuation player chooses his effort level after observing his counterpart's, the equilibrium expected payoff of the delegate hired by the higher-valuation player is greater than that of his counterpart, and economic rent for each delegate exists. In Section 6, we have shown that, as the valuation for the prize of the higher-valuation player increases, total effort level remains unchanged; as the valuation of the lower-valuation player decreases, total effort level decreases. In Section 7, we have shown that each player offers her delegate better contingent compensation in the endogenous-timing framework than in the simultaneous-move framework. We have shown also that, unless the valuation for the prize of the higher-valuation player is significantly greater than that of the lower-valuation player, then each delegate's expected payoff is greater in the endogenous-timing framework than in the simultaneous-move framework, whereas each player's expected payoff is less in the endogenous-

timing framework than in the simultaneous-move framework. On the basis of this result, we have argued that the players prefer the simultaneous-move framework while the delegates prefer the endogenous-timing framework, and that it benefits the delegates but hurts the players' expected payoffs to allow and facilitate the delegates to endogenize the order of their moves.

We have assumed that contracts between the players and the delegates are public information, so that the delegates decide when to expend their effort after knowing both contracts. It would be interesting to consider a model in which contracts between the players and the delegates are private information, so that each delegate decides when to expend his effort and chooses his effort level without knowing the contract for his counterpart. We have assumed that potential delegates have equal ability for the contest, and thus the same reservation wage. It would be interesting to consider a model in which delegates have different ability and different reservation wages. Another possible extension of this paper is a model in which each player decides first whether she will expend her own effort or hire a delegate. We leave these modifications or extensions for future research.

Appendix A: The shapes of the delegates' reaction functions

Delegate *i*'s reaction function, $x_i = r_i(x_j)$, is derived from the first-order condition for maximizing π_i with respect to x_i given x_j, where *j* is the other delegate. This first-order condition is

$$\partial \pi_i / \partial x_i \;=\; D_i(x_1, x_2) \;-\; 1 \;=\; 0, \tag{A1}$$

where $D_i(x_1, x_2) = \alpha_i v_i h'(x_i) h(x_j) / \{h(x_1) + h(x_2)\}^2$. Note that, under Assumption 1, we obtain $\partial D_i / \partial x_i < 0$, and thus the second-order condition for maximizing π_i is satisfied.

Differentiating along (A1), we obtain the derivative of delegate *i*'s reaction function:

$$dr_i(x_j)/dx_j \;=\; -\, h'(x_1) h'(x_2) \{h(x_i) - h(x_j)\} / h(x_j) K_i, \tag{A2}$$

where $K_i \equiv h''(x_i)\{h(x_1) + h(x_2)\} - 2(h'(x_i))^2$. The denominator in the right-hand side of (A2) is negative, and $h'(x_1)h'(x_2)$ in the numerator is positive, due to Assumption 1. Then, using (A2), we obtain Lemma A1.

Lemma A1. (*a*) *Delegate i's reaction function is increasing in* x_j *– in terms of the symbols,* $dr_i(x_j)/dx_j > 0$ *– when it lies in the region satisfying* $x_i > x_j$ *or, equivalently, when* $r_i(x_j) > x_j$. (*b*) *It is decreasing in* x_j *when it lies in the region satisfying* $x_i < x_j$ *or, equivalently, when* $r_i(x_j) < x_j$. (*c*) *It is stationary in* x_j *when* $r_i(x_j) = x_j$.

Assume natually that $D_i(x_1, x_2)$ is greater than unity – or, equivalently, $\partial \pi_i / \partial x_i > 0$ – at points on the 45º line which are close to the origin. It is straightforward to show that $D_i(x_1, x_2)$ decreases as we move upward from the origin along the 45º line. Recall that, given x_j, $\partial D_i / \partial x_i < 0$, and that $D_i(x_1, x_2) = 1$ holds along delegate *i*'s reaction function. Using these, together with Lemma A1, we find the shapes of the delegates' reaction functions.

Appendix B: The simultaneous-move framework with the specific form of the function *h*

Consider the following two-stage game. In the first stage, each player hires a delegate, and the players simultaneously announce their contracts written independently with their delegates – that is, player 1 announces publicly the value of α_1, and player 2 announces the value of α_2. In the second stage, after knowing both contracts, the delegates choose their effort levels *simultaneously* and independently. At the end of the second stage, only the winning player pays compensation to her delegate according to her contract announced in the first stage.

To obtain a subgame-perfect equilibrium of the game, we work backward. At the Nash equilibrium of the second-stage subgame that ensues after the players announce α_1 and α_2 in the first stage, the effort levels of the delegates are $x_1^E = \alpha_1^2 v_1^2 \alpha_2 v_2/(\alpha_1 v_1 + \alpha_2 v_2)^2$ and $x_2^E = \alpha_1 v_1 \alpha_2^2 v_2^2/(\alpha_1 v_1 + \alpha_2 v_2)^2$. Next, using these (and others), we find the equilibrium contracts chosen by the players in the first stage, which satisfy the following reaction functions simultaneously:

$$\alpha_1 \;=\; B_1(\alpha_2) \;=\; (\sqrt{\alpha_2 v_1 v_2 + \alpha_2^2 v_2^2} - \alpha_2 v_2)/v_1$$

and

$$\alpha_2 \;=\; B_2(\alpha_1) \;=\; (\sqrt{\alpha_1 v_1 v_2 + \alpha_1^2 v_1^2} - \alpha_1 v_1)/v_2,$$

where $B_i(\alpha_j)$ represents player *i*'s best response to α_j, for $i, j = 1, 2$ with $i \neq j$.

Let α_i^{**} represent player *i*'s contract that is specified in the subgame-perfect equilibrium of the two-stage game. Let x_i^{**} represent the effort level of delegate *i* that is specified in the subgame-perfect equilibrium. Let π_i^{**} and G_i^{**} represent the expected payoff for delegate *i* and that for player *i*, respectively, in the subgame-perfect equilibrium. Finally, let $v_1 = \theta v_2$, where $\theta \geq 1$. Then we obtain Lemma B1.

Lemma B1. (*a*) $\alpha_1^{**} = 1/(2 + \theta k)$ *and* $\alpha_2^{**} = \theta k/(1 + 2\theta k)$, *where defining* $H \equiv \left(-4.5\theta^4 - 8\theta^3 + 1.5\sqrt{-96\theta^9 - 183\theta^8 - 96\theta^7}\right)^{1/3}$, *we have* $k = (4 + 6\theta)/3H + H/3\theta^2 - 2/3\theta$. (*b*) $x_1^{**} = \theta^3 k^3 v_2/(1 + \theta k)^2(1 + 2\theta k)$ *and* $x_2^{**} = \theta^2 k^2 v_2/(1 + \theta k)^2(1 + 2\theta k)$.

(*c*) $\pi_1^{**} = \theta^2 k v_2/(1+\theta k)(2+\theta k) - \theta^3 k^3 v_2/(1+\theta k)^2(1+2\theta k)$ *and*

$\pi_2^{**} = \theta k v_2/(1+\theta k)(1+2\theta k) - \theta^2 k^2 v_2/(1+\theta k)^2(1+2\theta k)$.

(*d*) $G_1^{**} = \theta^2 k v_2/(2+\theta k)$ *and* $G_2^{**} = v_2/(1+2\theta k)$.

Note that we use the computer program Mathematica to solve for k. Note also that k is a positive real for any value of θ, even though H is imaginary, where $\theta \geq 1$. If $\theta = 1$, then $k = 1$, so that $\alpha_1^{**} = \alpha_2^{**} = 1/3$, $x_1^{**} = x_2^{**} = v_2/12$, $\pi_1^{**} = \pi_2^{**} = v_2/12$, and $G_1^{**} = G_2^{**} = v_2/3$. Table 1 in Baik (2007) complements Lemma B1.

Footnotes

1. We define a contest as a situation in which players compete with one another to win a prize. The literature on the theory of contests deals with rent-seeking contests, litigation, tournaments, patent contests, sporting contests, election campaigns, all-pay auctions, environmental conflicts, arms races, etc. Important work in this literature includes Loury (1979), Tullock (1980), Rosen (1986), Dixit (1987), Appelbaum and Katz (1987), Hillman and Riley (1989), Ellingsen (1991), Baye et al. (1993), Nitzan (1994a), Clark and Riis (1998), Che and Gale (1998, 2003), Moldovanu and Sela (2001), Hvide (2002), Szymanski (2003), Corchón (2007), Epstein and Nitzan (2007), Congleton et al. (2008), and Konrad (2009).

2. Fershtman and Judd (1987) and Sklivas (1987) study strategic managerial delegation. Fershtman et al. (1991) and Katz (2006) study delegation with observable contracts, and Katz (1991) delegation with unobservable contracts.

3. Dixit (1987) studies two-player asymmetric contests without delegation or endogenous timing, and defines the favorite [the underdog] as the player who has a probability of winning greater [less] than $1/2$ at the Nash equilibrium of a simultaneous-move contest.

4. If both delegates announce the same period in the second stage, the game has exactly three stages. But if one delegate announces to expend his effort in one period and the other delegate in the other period, one may say that the game has four stages. For concise exposition, we do not break this third stage into two.

5. Asymmetric contests are studied by, for example, Baik (1994, 2004), Nti (1997), and Malueg and Yates (2005).

6. We can instead assume that player *i*'s contract specifies as follows: Compensation of $\alpha_i v_i$ is paid to delegate *i* if he wins the prize, and $\beta_i v_i$ if he loses it, where $\beta_i < \alpha_i < 1$ and $\beta_i \geq 0$. This contract specification is used in Baik (2007, 2008). Using this more general contract specification, however, we obtain exactly the same (main) results because player *i* chooses zero for β_i in equilibrium. A form of contract in which a delegate's compensation is contingent on the

outcome of the contest is widely used in the real world. A salient example is contracts between litigants and attorneys in various litigation around the world.

7. This assumption may indicate that we restrict attention to the pairs of the players' contracts at which neither of the delegates' participation constraints is binding. Baik (2007, 2008) studies contests in which delegate i has a reservation wage of R_i, where R_i is nonnegative.

8. We exclude or ignore zero effort for concise exposition because including or excluding zero effort does not affect our analysis.

9. Several forms of contest success functions have been used in the literature on the theory of contests. For example, logit-form contest success functions are used in Tullock (1980), Rosen (1986), Dixit (1987), Nitzan (1991), Baik and Shogren (1992), Baik (1994), Skaperdas (1996), Szidarovszky and Okuguchi (1997), Morgan (2003), Szymanski (2003), and Stein and Rapoport (2004). Probit-form contest success functions are used in Dixit (1987). Difference-form contest success functions are used in Hirshleifer (1989). Ratio-form contest success functions are used in Baik (2004). All-pay-auction contest success functions are used in Hillman and Riley (1989), Baye et al. (1993, 1996), Che and Gale (1998, 2003), Clark and Riis (1998), Moldovanu and Sela (2001), and Konrad and Leininger (2007).

10. For concise exposition, we omit the detailed analysis of the second and third stages including the proofs of the characterization of the delegates' equilibrium effort levels. They are available from the authors upon request. See also Baik and Shogren (1992), Leininger (1993), Baik (1994), and Morgan (2003).

11. We use 1L as a shorthand for "with delegate 1 as the leader," and 2L as a shorthand for "with delegate 2 as the leader."

12. See Szidarovszky and Okuguchi (1997) for a proof of the existence and uniqueness of a pure-strategy Nash equilibrium of the simultaneous-move subgame.

13. Baik and Shogren (1992) also obtain this result. Other papers on endogenous timing, mentioned in the introduction, obtain similar results. For example, Fu (2006) shows that the uninformed player chooses the first period, while the informed player chooses the second period.

Konrad and Leininger (2007) show that the strongest player – the player who has the lowest cost in expending a given effort level – typically chooses the second period, whereas all the other players are indifferent with respect to their choice of timing.

14. If player i chooses a value of α_i such that $\alpha_1 v_1 = \alpha_2 v_2$, then we have $x_k^{kL} = x_k^{tL} = x_k^N$, for k, $t = 1, 2$ with $k \neq t$, so that player i's expected payoffs are $G_i^{2L} = G_i^{1L} = G_i^N$. For concise exposition, we will henceforth combine this case with the other two cases to have the case of $\alpha_1 v_1 \leq \alpha_2 v_2$ and that of $\alpha_1 v_1 \geq \alpha_2 v_2$.

15. In this case, the contest success function for delegate i is $p_i(x_1, x_2) = x_i/(x_1 + x_2)$. This contest success function is extensively used in the literature on the theory of contests. Examples include Tullock (1980), Appelbaum and Katz (1987), Hillman and Riley (1989), Hirshleifer (1989), Nitzan (1991), Leininger (1993), Hurley and Shogren (1998), Morgan (2003), Baik (2004), and Stein and Rapoport (2004).

16. For concise exposition, we do not provide the full derivations of the technical results presented in Sections 5, 6, and 7 and Appendix B. They are available from the authors upon request.

17. With the specific form of the function h, we have $G_i^{iL} = (1 - \alpha_i) v_i p_i(x_1^{iL}, x_2^{iL})$ and $G_j^{iL} = (1 - \alpha_j) v_j p_j(x_1^{iL}, x_2^{iL})$, for $i, j = 1, 2$ with $i \neq j$, where $p_i(x_1^{iL}, x_2^{iL}) = \alpha_i v_i / 2\alpha_j v_j$.

18. Stated differently, when player j offers a "low" value of α_j to her delegate, player i offers a relatively "high" value of α_i to her delegate and makes delegate i the favorite, in which case delegate i chooses to be the follower in the effort-expending stage. However, when player j offers a "significantly high" value of α_j to her delegate, player i steps back and offers a relatively "low" value of α_i to her delegate and makes delegate i the underdog, in which case delegate i chooses to be the leader in the effort-expending stage.

19. Proposition 1 may hold for a more general form of the function h. First, consider the case where $v_1 = v_2$. In this case, one may well expect that $\alpha_1^* = \alpha_2^*$ and $\alpha_1^* v_1 = \alpha_2^* v_2$ hold, which implies that part (*b*), $x_1^* = x_2^*$, $p_1(x_1^*, x_2^*) = 1/2$, $\pi_1^* = \pi_2^*$, and $G_1^* = G_2^*$ hold. Next, consider the case where $v_1 > v_2$. Using payoff function (2) and maximization problem (4), we have

$$\partial G_1^{2L}/\partial \alpha_1 \;=\; v_1\{ - p_1(x_1^{2L}, x_2^{2L}) + (1 - \alpha_1)\partial p_1/\partial \alpha_1\}$$

and

$$\partial G_2^{2L}/\partial \alpha_2 \;=\; v_2\{ - p_2(x_1^{2L}, x_2^{2L}) + (1 - \alpha_2)\partial p_2/\partial \alpha_2\}.$$

Since $p_1(x_1^{2L}, x_2^{2L}) = p_2(x_1^{2L}, x_2^{2L})$ and $\alpha_1 < \alpha_2$ hold at values of α_1 and α_2 such that $\alpha_1 v_1 = \alpha_2 v_2$, we obtain that $\partial G_1^{2L}/\partial \alpha_1 > \partial G_2^{2L}/\partial \alpha_2$ holds in the neighborhood of these values unless $\partial p_2/\partial \alpha_2$ is significantly greater than $\partial p_1/\partial \alpha_1$. This, together with the first-order conditions $\partial G_1^{2L}/\partial \alpha_1 = 0$ and $\partial G_2^{2L}/\partial \alpha_2 = 0$, implies that $\alpha_1^* v_1 > \alpha_2^* v_2$ holds. This in turn implies that part (*b*), $x_1^* > x_2^*$, and $p_1(x_1^*, x_2^*) > 1/2$ hold. Similarly, we may obtain that $\alpha_1^* < \alpha_2^*$ holds, which, together with $p_1(x_1^*, x_2^*) > 1/2$, implies that $G_1^* > G_2^*$ holds.

20. Santore and Viard (2001), Schoonbeek (2002), and Baik (2007, 2008) also obtain this result in similar or different contexts.

21. We believe that most of the results in Propositions 1 and 2 may hold for the function $h(x_i) = x_i^r$, where $0 < r < 1$. In this case, however, it is not possible to obtain the equilibrium contracts, α_1^* and α_2^*, of the players – and other outcomes of the game – because it is computationally intractable to derive closed-form solutions for the delegates' equilibrium effort levels, x_i^{iL} and x_j^{iL}, in the *iL* sequential-move subgame, for $i, j = 1, 2$ with $i \neq j$ (see Section 4).

22. See, for example, Baik (1994, 2004). He shows that, if $h(x_i) = x_i$, then the equilibrium total effort level increases as the valuation of the higher-valuation player increases. Baik (1994, 2004) considers *simultaneous-move* asymmetric contests *without delegation*, whereas we consider asymmetric contests with delegation in which the delegates decide endogenously when to expend their effort. Note that Proposition 2 implies that the equilibrium total effort level of the delegates remains unchanged as the equilibrium contingent compensation $\alpha_1^* v_1$ of delegate 1 increases from that of delegate 2.

23. Given a pair, α_1 and α_2, of the players' contracts chosen and announced in the first stage, the delegates' effort levels which are specified in the subgame-perfect equilibrium of the 2*L* sequential-move subgame are $x_1^{2L} = (2\alpha_1 v_1 \alpha_2 v_2 - \alpha_2^2 v_2^2)/4\alpha_1 v_1$ and $x_2^{2L} = \alpha_2^2 v_2^2/4\alpha_1 v_1$, so that total effort level is $x_1^{2L} + x_2^{2L} = \alpha_2 v_2/2$. This means that, if the equilibrium contracts, α_1^* and α_2^*,

lead to the $2L$ sequential-move subgame, then the equilibrium total effort level is equal to $x_1^* + x_2^* = \alpha_2^* v_2/2$.

24. If $v_1 = 2v_2$, then we obtain that $\alpha_1^* > \alpha_1^{**}$, $\pi_1^* > \pi_1^{**}$, and $G_1^* < G_1^{**}$. If $v_1 = 3v_2$, then we obtain that $G_1^* > G_1^{**}$. If $v_1 = 5v_2$, then we obtain that $\alpha_1^* > \alpha_1^{**}$ and $\pi_1^* > \pi_1^{**}$. If $v_1 = 6v_2$, then we obtain that $\alpha_1^* < \alpha_1^{**}$ and $\pi_1^* > \pi_1^{**}$.

25. If $v_1 = 2v_2$, then we obtain that $x_1^* > x_1^{**}$ and $x_2^* < x_2^{**}$. If $v_1 = 4v_2$, then we obtain that $x_i^* < x_i^{**}$ for $i = 1, 2$.

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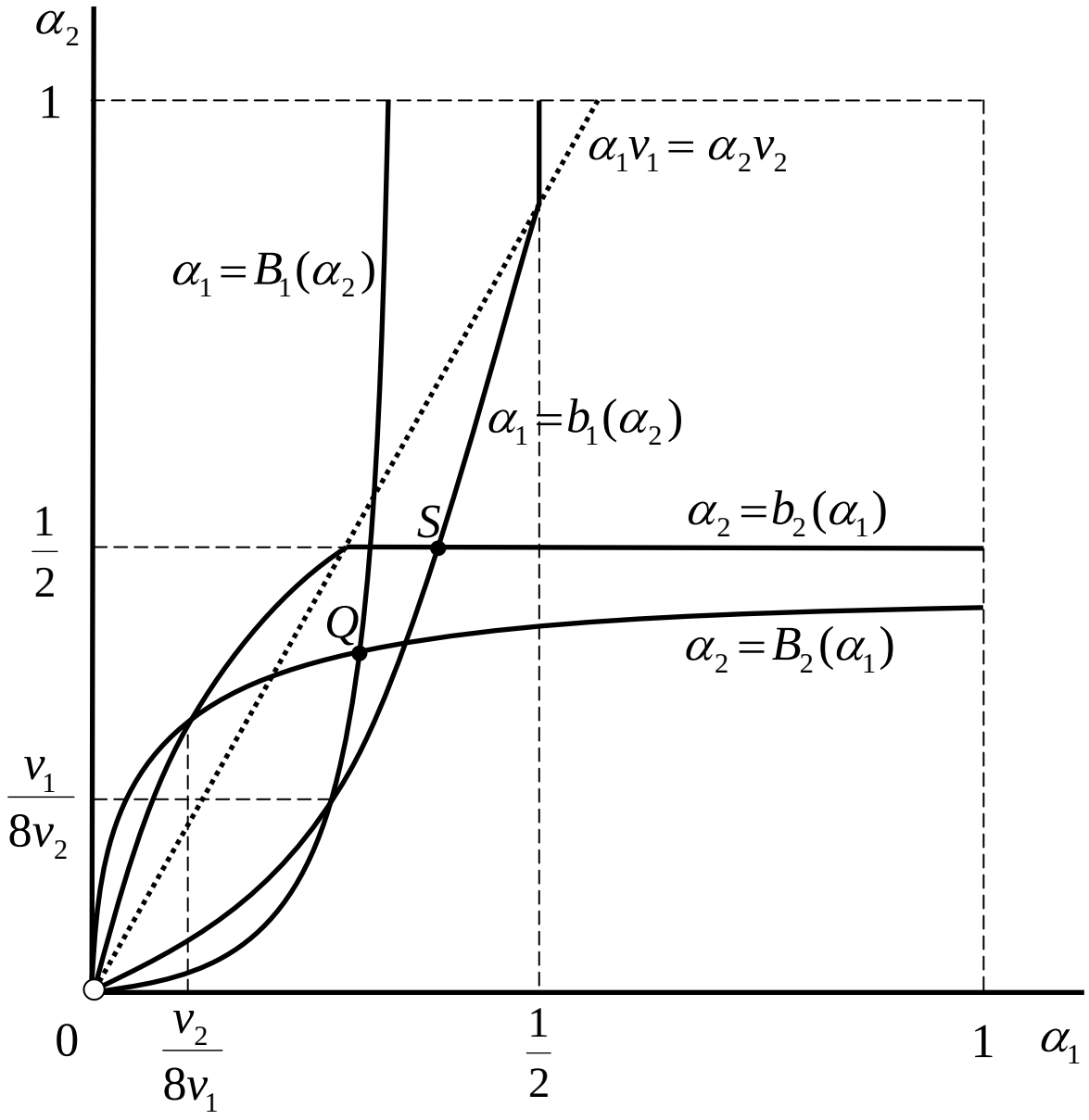

Figure 1. The Players' Reaction Functions and the Equilibrium Contracts in the Two Frameworks