Endogenous Group Formation in Contests: Unobservable Sharing Rules

By Kyung Hwan Baik*

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Abstract

We study contests in which players compete by expending irreversible effort to win a prize, the prize is awarded to one of the players, the winner shares the prize with other players in his group, if any, and each group's sharing rule is unobservable to the other groups and the singletons, if any, when the players expend their effort. The number of groups, their sizes, and the number of singletons are exogenous in the first model, whereas they are endogenous in the second model. We show that group formation occurs if the number of players is four or smaller, but does not occur otherwise. We examine the effect of endogenous group formation on total effort level and the profitability of endogenous group formation. In each of the two models, comparing the outcomes of the case of unobservable sharing rules with those of the case of observable sharing rules, we show that the two cases yield quite different outcomes.

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*Department of Economics, Sungkyunkwan University, Seoul 110-745, South Korea (e-mail: khbaik@skku.edu). I am grateful to Eric Bahel, Mike Baye, Francis Bloch, Hans Haller, Hanjoon Michael Jung, Amoz Kats, Amy Lee, Dongryul Lee, Wooyoung Lim, Hyeran Moon, Eric Rasmusen, Michael Rauh, Nathan Smith, Nic Tideman, two anonymous referees, and seminar participants at the Hong Kong University of Science and Technology, Virginia Polytechnic Institute and State University, Western Illinois University, and Indiana University for their helpful comments and suggestions. I am also grateful to Daehong Min and Chang-Geun Song for excellent research assistance. Earlier versions of this paper were presented at the 2012 Annual Conference of the Korean Econometric Society, Seoul, Korea, February 2012; and the 2013 Annual Meetings of the Allied Social Science Associations, San Diego, CA, January 2013. This work was supported by the National Research Foundation of Korea (NRF) Grant funded by the Korean Government (MEST) (No. 2012S1A5A2A01017808).

1. Introduction

Contests are common in which players, each as a member of a group or as a singleton (or, equivalently, a nonmember), compete with one another by expending irreversible effort or resources to win a prize.[1] Examples include various types of rent-seeking contests, patent competition among consortiums, competition among political parties or candidates for office, class action litigation, and sporting contests. In some contests, group formation may occur because players want to better cope with uncertainties including the possibility of failure in winning the prize. In others, group formation may occur because players try to gain strategic advantage through group formation.

In such contests, the players in each group jointly decide how to share the prize among themselves if they or one of them wins it – that is, they make a binding agreement on their sharing rule. One example for such an agreement comes from patent competition, where the firms in each R&D joint venture decide how to share the benefits of the innovation if they or one of them wins the patent. Another example comes from competition for presidential office, where the candidates in each political party may decide, before they campaign, how they will share the office benefits if one of them wins the election. Yet another example comes from college football, where the teams in each conference decide how to share or divide the prize among themselves if one of them wins a bowl game.

Often, each group's sharing rule is unobservable to the other groups and the singletons. For example, in patent competition, the firms in each R&D joint venture and the independent firms may not know the sharing rules of the other R&D joint ventures when they compete to win the patent. In competition for presidential office, the candidates in one political party and the independent candidates may not be sure, when they campaign, how the candidates in another party will share office benefits, policy influence, and the political posts among themselves in case of one of them winning the election. This situation of unobservable sharing rules may occur simply because the groups do not announce their sharing rules. Furthermore, it may occur, even though they announce publicly their sharing rules, because their announced sharing rules are unverifiable. Nitzan and Ueda (2011) say that, without restrictive assumptions that decisions made within a group are transparent and detection of changes is easy, a model of group contests with observable sharing rules is questionable.

In addition, in such contests, we may well expect that the number of groups, their sizes, and the number of singletons are endogenously determined.

Accordingly, it is those ideas, unobservable sharing rules and endogenous group formation, that motivate this paper. This paper studies contests in which risk-neutral players compete by expending irreversible effort to win a prize, the prize is awarded to one of the players, the winner shares the prize with other players in his group, if any, and each group's sharing rule is unobservable to the other groups and the singletons, if any, when the players expend their effort. It considers two models: the model with exogenous groups and the endogenous group formation model.

In the model with exogenous groups, the number of groups, their sizes, and the number of singletons are exogenous. We formally consider the following game. First, the players in each group jointly decide how to share the prize among themselves if one of them wins it – that is, they make a binding agreement on their sharing rule (or, equivalently, their winner's fractional share). Next, all the players in the contest choose their effort levels simultaneously and independently. When the players choose their effort levels, each group's sharing rule is unobservable to the other groups and the singletons. Finally, the winning player is determined, and the players in his group, if any, share the prize according to their sharing rule on which they agreed.

We show that each group's equilibrium effort level and the equilibrium total effort level are independent of the number of players in the contest and the sizes of the groups, as long as changes in these do not change the number of groups or the number of singletons. We show also that each player in a larger group expends less effort, and has a less equilibrium expected payoff, than each player in a smaller one. Finally, we show that the equilibrium expected payoff for each singleton is greater than that for each player in any actual group.

We compare the outcomes of the case of unobservable sharing rules – the game presented above – with those of the case of observable sharing rules, the game which is the same as the one presented above with the exception that each group's sharing rule is now observable to all the players in the game. By the situation of "observable sharing rules," we mean a situation in which the groups announce publicly their sharing rules and, furthermore, are committed to their sharing rules. We show that the two cases yield the same outcomes if there are one group and one singleton, whereas they yield different outcomes if there is more than one singleton or more than one group.

Next, we consider the endogenous group formation model. This extended model is the same as the model with exogenous groups with the exception that, in the extended model, the players decide first whether to form groups. Thus, in the extended model, the number of groups, their sizes, and the number of singletons are endogenously determined. We first obtain the equilibrium numbers of groups, the equilibrium group sizes, and the equilibrium numbers of singletons. Then, after obtaining the equilibrium total effort level and each player's equilibrium expected payoff, we examine the effect of endogenous group formation on total effort level and the profitability of endogenous group formation. We show the following. If the number of players in the contest is four or smaller, then group formation occurs. If the number of players is five or greater, then the individual contest occurs – that is, group formation does not occur. For any number of players, the equilibrium total effort level does not exceed the total effort level resulting from the individual contest – in other words, endogenous group formation does not increase prize dissipation, as compared with the individual contest. If the number of players is four or smaller, then endogenous group formation is beneficial to the players in the contest, as compared with the individual contest. Overall, we obtain quite different results from those in the corresponding model with observable sharing rules.

This paper is related to the literature on collective rent seeking, and to the literature on endogenous group formation in contests or conflicts. Bloch (2012), Kolmar (2013), and Flamand and Troumpounisy (2015) provide excellent surveys of these literatures. In the literatures, a standard assumption is that, when players choose their effort levels, each group's sharing rule, if any, is observable to all the players – in other words, public information is assumed regarding sharing rules. Nitzan (1991a, 1991b), Baik (1994), Baik and Shogren (1995), Hausken (1995), Lee (1995), Davis and Reilly (1999), Ueda (2002), Baik et al. (2006), and Ursprung (2012) are examples of collective rent seeking with observable sharing rules. Baik and Lee (2001) consider collective rent seeking with observable sharing rules in which the number of groups, their sizes, and the number of singletons are endogenously determined, and examine the profitability of endogenous group formation and the effect of endogenous group formation on rent dissipation. Garfinkel (2004) considers a three-stage model of distributional conflict with observable sharing rules in which individuals can form alliances in the first stage, and examine the effect of endogenous alliance formation on the severity of conflict. Bloch et al. (2006) extend the model of conflicts and contests introduced by Esteban and Ray (1999) to incorporate endogenous group formation. Sánchez-Pagés (2007) considers endogenous coalition formation in contests. Bloch (2012) provides a survey of the theoretical literature on alliance formation, and then considers different models of endogenous formation of alliances in conflicts. Recently, unlike the previous papers, Baik and Lee (2007) and Nitzan and Ueda (2011) consider collective rent seeking between groups in which each group's sharing rule is private information. Baik and Lee (2012) study collective rent seeking between two groups in which each group has the option of making its sharing rule observable or unobservable. They show that the case where both groups make their sharing rules observable does not occur in equilibrium. A striking difference between the current paper and the previous papers in the above-mentioned literatures is that, unlike the previous papers, the current paper studies contests with both unobservable sharing rules and endogenous group formation.

This paper is also related to the literature on the theory of endogenous coalition formation: See, for example, Bloch (1995), Yi (1998), Belleflamme (2000), Morasch (2000), and Yi and Shin (2000). These papers concentrate on examining the equilibrium structures of coalitions – specifically, associations, R&D joint ventures, or strategic alliances of firms – in oligopolies.

The paper proceeds as follows. Section 2 develops the model with exogenous groups, and sets up the game in which the number of groups, their sizes, and the number of singletons are exogenously given. In Section 3, we solve the game, and obtain the equilibrium sharing rules of the groups, the equilibrium effort levels of the players, and the equilibrium expected payoffs for the players. In Section 5, we consider an extended model in which the number of groups, their sizes, and the number of singletons are endogenously determined. In Sections 4 and 5, we compare the outcomes of the case of unobservable sharing rules with those of the case of observable sharing rules. Finally, Section 6 offers our conclusions, and discusses modifications and extensions of the models.

2. The model with exogenous groups

There are n risk-neutral players who compete by expending irreversible effort to win a rent (or, in general, a prize), where $n \geq 3$. They each are a member of one of N groups, 1 through N, or do not belong to any of these groups. Group i consists of m_i players, where $N \geq 1$ and $2 \leq m_i < n$. Without loss of generality, we assume that $m_1 \geq m_2 \geq \ ... \ \geq m_N$. The prize will be awarded to one of the players. Every player values the prize at V, which is positive and publicly known.

The players in group i, for $i = 1, \ldots, N$, write an agreement, before they choose their effort levels, on how to share or divide the prize if one of them wins the prize. The agreement takes the following form: If a player in group i wins the prize, then the winner takes $\sigma_i V$ and each of the other players takes $(1 - \sigma_i)V/(m_i - 1)$, where $\sigma_i \geq 1/m_i$. We call σ_i group i's winner's fractional share or, broadly, its sharing rule. If $1/m_i \leq \sigma_i < 1$, then the winner shares the prize with other players in his group, and thus he takes less than the prize. If $\sigma_i = 1$, then the winner takes all the prize and nothing is left for other players in his group. If $\sigma_i > 1$, then the winner takes "incentives" from other players in his group as well as the prize. Specific examples for such an agreement come from patent competition, competition for presidential office, and college football, which have been mentioned in Section 1. Consider, for example, patent competition. The firms in an R&D joint venture may write an agreement that, if one of them wins the patent, the winning firm (or the patent-holding firm) must license out its new innovation to the other firms in its R&D joint venture for a certain low fixed fee or a certain low royalty payment. Another specific example comes from competition among university professors and researchers for research grants. If a researcher in a university wins a grant, all of the researchers in the university may share the grant in the form of overhead or indirect costs collected by the university. Yet another example comes from competition among researchers for federal research grants. Many states in the United States have a matching grant program for researchers who secure federal grants.

Note that a group as defined in the preceding paragraph is an extremely loose form of association, and may be like a network of solidarity: Even the players in the same group compete with each other to win the prize. Section 6 will discuss a model in which the players in each group pool their effort to win the prize and the prize is awarded to one of the singletons or one of the groups. Note also that the players in each group create externalities for each other when they expend their effort.[2]

We may treat the singletons – that is, the players who do not belong to any of those N groups – as the members of group N+1 whose winner's fractional share is unity and publicly known. Let m_{N+1} denote the size of group N+1 or, equivalently, the number of singletons, where $m_{N+1} \geq 0$. We have then $n = \sum_{i=1}^{N+1} m_i$.

Let x_{ik} represent the effort level that player k in group i expends, for $i = 1, \ldots , N+1$, and let X_i represent the effort level that all the players in group i expend, so that $X_i = \sum_{k=1}^{m_i} x_{ik}$. Let X represent the effort level that all the players in the contest expend, so that $X = \sum_{i=1}^{N+1} X_i$. Each player's effort level is nonnegative, and measured in units commensurate with the prize. Let p_{ik}

denote the probability that player k in group i wins the prize. We assume that the contest success function for player k in group i is

$$p_{ik} = \begin{cases} x_{ik}/X & \text{for } X > 0 \\ 1/n & \text{for } X = 0. \end{cases} \tag{1}$$

Let π_{ik} denote the expected payoff for player k in group i. Then the payoff function for player k in group i is

$$\begin{aligned} \pi_{ik} &= (\sigma_i V - x_{ik})p_{ik} + \{(1-\sigma_i)V/(m_i - 1) - x_{ik}\}\sum_{j \neq k} p_{ij} + (-x_{ik})(1 - \sum_{j=1}^{m_i} p_{ij}) \\ &= \sigma_i V p_{ik} + \{(1-\sigma_i)V/(m_i - 1)\}\sum_{j \neq k} p_{ij} - x_{ik} \text{ for } i = 1, \ldots, N, \end{aligned}$$

and (2)

$$\pi_{ik} = Vp_{ik} - x_{ik} \qquad \text{for } i = N{+}1,$$

where $\sum_{j \neq k} p_{ij}$ is the probability that any one of the other players in group i wins the prize, and $(1 - \sum_{j=1}^{m_i} p_{ij})$ is the probability that no player in group i wins it.

We formally consider the following game. First, the players in group i, for $i = 1, \ldots, N$, jointly decide how to share the prize among themselves if one of them wins it. That is, they make a binding agreement on their sharing rule σ_i. Note that, because the players in the group are identical, their decision on σ_i is unanimous. Next, all the players in the contest choose their effort levels simultaneously and independently. When the players choose their effort levels, the players in each group know the sharing rule of their own group; however, each group's sharing rule is unobservable to the other groups and the singletons. Lastly, the winning player is determined, and the players in his group, if any, share the prize according to their sharing rule on which they agreed.

This game differs from a standard two-stage game in which the actions chosen in the first stage are *observed* by all the players before actions are chosen in the second stage. In the current game, the players in group i, for $i = 1, \ldots, N$, choose two *sequential* actions – specifically, their sharing rule and then effort levels – without observing those chosen by the players in the other groups or the effort levels chosen by the singletons. Thus they play a simultaneous-move game with the players in the other groups and the singletons (see Baik and Lee 2007).

Finally, we assume that all of the above is common knowledge among the players.

3. Equilibrium sharing rules, effort levels, and expected payoffs

The equilibrium sharing rules of groups 1 through N and the equilibrium effort levels of the players in the contest satisfy the following two requirements. First, each player's effort level is optimal given his group's sharing rule and given the effort levels of all the other players. In other words, each player's effort level is a best response both to his group's sharing rule and to the effort levels of the other players. Second, group i's sharing rule, for $i = 1, \ldots, N$, is optimal given the effort levels of the players in the other groups and the singletons and given the subsequent behavior, or rather effort levels, of its own players. The decision of the players in group i on their sharing rule is not *directly* affected by their beliefs about the other groups' sharing rules. This is because their expected payoffs do not depend *directly* on the other groups' sharing rules and because they play a simultaneous-move game with the players in the other groups and the singletons. Note, however, that their expected payoffs depend *indirectly* on the other groups' sharing rules.

To obtain the equilibrium sharing rules of the groups and the equilibrium effort levels of the players, working backward, we consider first the players' decisions on their effort levels, and then consider the players' decisions on their groups' sharing rules.

Consider the decision of player k in group i, for $i = 1, \ldots, N+1$, on his effort level. After observing only his group's sharing rule σ_i, he seeks to maximize his expected payoff π_{ik} in equation (2) over his effort level x_{ik}, taking the effort levels of all the other players as given. From the first-order condition for maximizing π_{ik}, we obtain

$$\sigma_i V(X - x_{ik}) \; - \; (1 - \sigma_i)V(X_i - x_{ik})/(m_i - 1) \; = \; X^2 \tag{3}$$

or, equivalently,

$$x_{ik} \;=\; \sqrt{\sigma_i V(X - x_{ik}) - (1-\sigma_i)V(X_i - x_{ik})/(m_i - 1)} \;-\; (X - x_{ik}).$$

It is straightforward to see that π_{ik} is strictly concave in x_{ik}, which implies that the second-order condition for maximizing π_{ik} is satisfied and the player's best response is unique.[3] Now, using equation (3), we see that the players in each group must choose the same effort level in equilibrium. Accordingly, let $x_{ik} = x_i$ for all k. Then equation (3) reduces to

$$\sigma_i VQ \;-\; Vx_i \;=\; Q^2,$$

where $Q = \sum_{i=1}^{N+1} m_i x_i$. This equation yields

$$x_i(\sigma_i, Q_{-i}) \;=\; \{(\sigma_i m_i - 1)V - 2m_i Q_{-i} + \sqrt{(\sigma_i m_i - 1)^2 V^2 + 4m_i VQ_{-i}}\}/2m_i^2, \tag{4}$$

where $Q_{-i} = m_1 x_1 + \cdots + m_{i-1}x_{i-1} + m_{i+1}x_{i+1} + \cdots + m_{N+1}x_{N+1}$.

Next, consider the decision of the players in group i, for $i = 1, \ldots, N$, on their group's sharing rule. Recall that $\sigma_{N+1} = 1$, which is known publicly. Taking Q_{-i} as given, the players in group i seek to maximize their expected payoffs (which are the same across the players) over their sharing rule σ_i, having perfect foresight about $x_i(\sigma_i, Q_{-i})$ for each value of σ_i. More precisely, they seek to maximize

$$\pi_i(\sigma_i, Q_{-i}) \;=\; Vx_i(\sigma_i, Q_{-i})/\{m_i x_i(\sigma_i, Q_{-i}) + Q_{-i}\} \;-\; x_i(\sigma_i, Q_{-i}) \tag{5}$$

with respect to σ_i, taking Q_{-i} as given. Note that we obtain equation (5) using equations (1), (2), and (4). Note also that $\pi_i(\sigma_i, Q_{-i}) > 0$ if and only if $V > Q$. From the first-order condition for maximizing $\pi_i(\sigma_i, Q_{-i})$ in equation (5), we obtain

$$\sigma_i(Q_{-i}) \;=\; \{1 + (m_i - 1)\sqrt{Q_{-i}/V}\}/m_i. \tag{6}$$

We are now ready to obtain the equilibrium sharing rules of the N groups and the equilibrium effort levels of the players. They satisfy N+1 equations from (4) and N equations from (6) simultaneously. Let $\sigma_i(\boldsymbol{m})$ represent the equilibrium sharing rule of group i, for $i = 1$, ... , N, and let $x_i(\boldsymbol{m})$ represent the equilibrium effort level of each player in group i, for $i = 1$, ... , N+1, where $\boldsymbol{m} = (m_1, \ldots, m_{N+1})$. Substituting equation (6) into equation (4), we have

$$x_i(Q_{-i}) = (\sqrt{VQ_{-i}} - Q_{-i})/m_i \tag{7}$$

or, equivalently,

$$Q^2 = VQ_{-i} \qquad \text{for } i = 1, \ldots, N. \tag{8}$$

Substituting $\sigma_{N+1} = 1$ into equation (4), we have

$$Q^2 = V(Q - x_{N+1}). \tag{9}$$

By solving N+1 simultaneous equations from (8) and (9), we obtain the players' equilibrium effort levels, $x_1(\boldsymbol{m})$ through $x_{N+1}(\boldsymbol{m})$ (see Appendix A). Next, substituting the players' equilibrium effort levels into equation (6), we obtain the groups' equilibrium sharing rules, $\sigma_1(\boldsymbol{m})$ through $\sigma_N(\boldsymbol{m})$. Lemma 1 reports the groups' equilibrium sharing rules, and Lemma 2 reports the effort levels of the players, those of the groups, and the total effort level in equilibrium.

Lemma 1. *Group i's equilibrium sharing rule is* $\sigma_i(\boldsymbol{m}) = 1 - (m_i - 1)/m_i(N + m_{N+1})$ *for* $i = 1$, ... , N.

It is immediate from Lemma 1 that group i's equilibrium winner's fractional share is greater than $1/m_i$, and is less than unity. Effort expended by a player generates a negative externality for other players in his group since it reduces the other players' winning probabilities. We may say that each group's equilibrium winner's fractional share, since it is less than unity, works as a device to amend this negative externality. Another result from Lemma 1 is that each group's equilibrium sharing rule is independent of the players' valuation V for the prize. This can be explained as follows. If V increases, for example, the players in each group do not decrease or increase their winner's fractional share because the higher valuation serves equally to motivate more the other players as well as the players in the group. We obtain from Lemma 1 that, as m_i increases without changing N or changing m_{N+1}, group i's equilibrium winner's fractional share decreases: $\partial\sigma_i(\boldsymbol{m})\big/\partial m_i < 0$. This makes intuitive sense. Competing with the players in the other groups and the singletons to win the prize, the players in group i optimally motivate themselves to win the prize by choosing the "right" sharing rule. If m_i increases, they can do so with a lower value of their winner's fractional share because their larger group size, too, serves to attain their goal. Note that an increase in m_i, without changing N or changing m_{N+1}, may occur if n increases or m_j decreases for some j, where $j = 1, \ldots, N$ with $i \neq j$. We also obtain from Lemma 1 that group i's equilibrium winner's fractional share increases as N increases without changing m_{N+1} or changing m_i; it increases as m_{N+1} increases without changing N or changing m_i; and it increases as both N and m_{N+1} increase without changing m_i. An increase in N or m_{N+1} increases the intensity of the competition, so that the players in group i need to increase their winner's fractional share. Finally, according to Lemma 1, two groups of the same size have the same equilibrium sharing rule, and the equilibrium winner's fractional share of a larger group is less than that of a smaller one; consequently, we have $\sigma_1(\boldsymbol{m}) \leq \ldots \leq \sigma_N(\boldsymbol{m})$.[4]

Let $Q_i(\boldsymbol{m})$ represent group i's equilibrium effort level, and $Q(\boldsymbol{m})$ the equilibrium total effort level: $Q_i(\boldsymbol{m}) = m_i x_i(\boldsymbol{m})$ and $Q(\boldsymbol{m}) = \sum_{i=1}^{N+1} Q_i(\boldsymbol{m}) = \sum_{i=1}^{N+1} m_i x_i(\boldsymbol{m})$.

Lemma 2. *The equilibrium effort levels of the individual players, those of the groups, and the equilibrium total effort level are* $x_i(\boldsymbol{m}) = V(N + m_{N+1} - 1)/m_i(N + m_{N+1})^2$ *for* $i = 1, \ldots, N$; $x_{N+1}(\boldsymbol{m}) = V(N + m_{N+1} - 1)/(N + m_{N+1})^2$; $Q_i(\boldsymbol{m}) = V(N + m_{N+1} - 1)/(N + m_{N+1})^2$ *for* $i = 1, \ldots, N$; $Q_{N+1}(\boldsymbol{m}) = Vm_{N+1}(N + m_{N+1} - 1)/(N + m_{N+1})^2$; *and* $Q(\boldsymbol{m}) = V(N + m_{N+1} - 1)/(N + m_{N+1})$.

Note first that all the players in the contest are active in equilibrium – that is, the players' equilibrium effort levels, $x_1(\boldsymbol{m})$ through $x_{N+1}(\boldsymbol{m})$, are positive. Next, Lemma 2 says that group i's equilibrium effort level, $Q_i(\boldsymbol{m})$, for $i = 1, \ldots, N+1$, and the equilibrium total effort level, $Q(\boldsymbol{m})$, depend on the number N of groups and the number m_{N+1} of singletons; however, they are independent of the number n of players in the contest and the sizes of groups 1 through N, as long as changes in these do not change N or m_{N+1}. This result implies that, as n increases by increasing only the sizes of one or more existing actual groups, neither the equilibrium effort levels of groups 1 through $N+1$ nor the equilibrium total effort level changes.[5] This implication is in contrast with a standard result in the literature on the theory of contests – the one that the equilibrium total effort level increases as the number of players increases.

An interesting and important observation from Lemma 2 is that group i's equilibrium effort level, $Q_i(\boldsymbol{m})$, for $i = 1, \ldots, N$, and each singleton's equilibrium effort level, $x_{N+1}(\boldsymbol{m})$, are equal to the corresponding players' equilibrium effort levels obtained in a reduced contest in which only $(N + m_{N+1})$ players – one for each group, and m_{N+1} singletons – compete individually to win the prize. We can understand this observation by noting that, in equation (7), group i's effort level, $m_i x_i(Q_{-i})$, for $i = 1, \ldots, N$, does not depend on its own size. An observation similar to this is obtained in contests with group-specific public-good prizes (see, for example, Baik 2008).

Using Lemma 2, we obtain the following further results on the equilibrium effort levels. First, the equilibrium total effort level, $Q(\boldsymbol{m})$, is less than the players' valuation V for the prize. Second, we have $Q_1(\boldsymbol{m}) = \ldots = Q_N(\boldsymbol{m})$, and $Q_1(\boldsymbol{m}) = x_{N+1}(\boldsymbol{m})$ if there is at least one singleton, which we can readily understand by recalling the observation stated in the preceding paragraph. This result implies that the equilibrium effort levels of the groups are the same regardless of the groups' different sizes.[6] This happens because, given their different sizes, the groups choose their sharing rules which countervail the differences in group size – indeed, a smaller group chooses a larger winner's fractional share than a larger group in equilibrium. Third, each player in two groups of the same size expends the same effort, and each player in a larger group expends less effort than each player in a smaller one; consequently, we have $x_1(\boldsymbol{m}) \leq \ \ldots \ \leq x_N(\boldsymbol{m})$. This result is natural because the groups' equilibrium effort levels are the same, and $m_1 \geq m_2 \geq \ \ldots \ \geq m_N$. Furthermore, it makes intuitive sense because a larger group, possessing a competitive size advantage over a smaller one, allows its players to ease up by choosing a lower winner's fractional share.

We end this section by looking at the expected payoffs for the players in equilibrium, which are used in studying endogenous group formation in Section 5. Let $\pi_i(\boldsymbol{m})$ represent the equilibrium expected payoff for each player in group i, for $i = 1, \ldots, N$+1. Using equations (1) and (2), and Lemmas 1 and 2, we obtain Lemma 3.

Lemma 3. *Given the players' valuation V for the prize, we have*
$\pi_i(\boldsymbol{m}) = V/m_i(N + m_{N+1})^2$ *for* $i = 1, \ldots, N$, *and* $\pi_{N+1}(\boldsymbol{m}) = V/(N + m_{N+1})^2$.

Lemma 3 says that the equilibrium expected payoff for each player in group i, for $i = 1, \ldots, N$+1, does not depend on the size m_j of group j, for $j = 1, \ldots, N$ with $i \neq j$, as long as a change in m_j does not change m_i, N, or m_{N+1}. Lemma 3 implies that each player in two groups of the same size has the same equilibrium expected payoff, and each player in a larger group has a less equilibrium expected payoff than each player in a smaller one; consequently, we have $\pi_1(\boldsymbol{m}) \leq \ \ldots \ \leq \pi_N(\boldsymbol{m})$.[7] Lemma 3 implies also that $m_1\pi_1(\boldsymbol{m}) = \ \ldots \ = m_N\pi_N(\boldsymbol{m})$, and that $m_1\pi_1(\boldsymbol{m}) = \pi_{N+1}(\boldsymbol{m})$ if there is at least one singleton. We can readily understand this by recalling the observation, stated above, that $Q_i(\boldsymbol{m})$, for $i = 1, \ldots, N$, and $x_{N+1}(\boldsymbol{m})$ are equal to the corresponding players' equilibrium effort levels obtained in the reduced contest with only $(N + m_{N+1})$ players.

Another interesting result in Lemma 3 is that the equilibrium expected payoff for each singleton is greater than that for each player in any actual group. This makes intuitive sense because the singletons are more motivated than the players in any group – indeed, σ_{N+1} is greater than the equilibrium winner's fractional share of any group.

4. Comparison with the case of observable sharing rules

In this section, we compare the outcomes of the case of unobservable sharing rules with those of the case of observable sharing rules. The outcomes of the case of unobservable sharing rules are those of the game analyzed so far – in which each group's sharing rule is unobservable to the other groups and the singletons – and are provided in Lemmas 1 through 3 in Section 3. The outcomes of the case of observable sharing rules are provided in Lemmas 2 through 4 in Baik and Lee (2001). By the case of observable sharing rules, we mean the game which is the same as the one in Section 2 with the exception that each group's sharing rule is now observable to all the players in the game. More specifically, we refer to the following two-stage game as the case of observable sharing rules. In the first stage, the players in group i, for $i = 1, \ldots, N$, jointly make a binding agreement on their sharing rule σ_i, and then all the groups simultaneously announce (and commit to) their sharing rules. In the second stage, after knowing the sharing rules of the N groups, all the players in the contest choose their effort levels simultaneously and independently. The winning player is determined at the end of the second stage, and the players in his group, if any, share the prize according to their sharing rule announced in the first stage.

In the examples mentioned in the previous sections, the situation of observable sharing rules may occur. In patent competition, the firms in each R&D joint venture may announce publicly (and commit to) their sharing rule before they compete to win the patent: They may announce publicly their agreement that, if one of them wins the patent, the winning firm must license out its new innovation to the other firms in its R&D joint venture for a certain low fixed fee or a certain low royalty payment. In competition for presidential office, the candidates in each political party may announce publicly, before they campaign, how they will share office benefits, policy influence, and the political posts among themselves in case of one of them winning the election. In college football, the teams in each conference may announce publicly how they will share or divide the prize among themselves if one of them wins a bowl game.

Table 1 summarizes the outcomes of the case of unobservable sharing rules and those of the case of observable sharing rules. Comparing the outcomes of the two cases, we obtain

Proposition 1. The superscripts *ub* and *ob* in Proposition 1 indicate the outcomes of the case of unobservable sharing rules and those of the case of observable sharing rules, respectively.

Proposition 1. (*a*) *If there are one group and one singleton, then the two cases yield the same outcomes.* (*b*) *If there are one group and more than one singleton, then we obtain:* (*i*) $\sigma_i^{ub} < \sigma_i^{ob}$, (*ii*) $Q_i^{ub} < Q_i^{ob}$, (*iii*) $Q_{N+1}^{ub} > Q_{N+1}^{ob}$, (*iv*) $Q^{ub} < Q^{ob}$, (*v*) $\pi_i^{ub} < \pi_i^{ob}$, *and* (*vi*) $\pi_{N+1}^{ub} > \pi_{N+1}^{ob}$, *for* $i = 1, ... , N$. (*c*) *If there is more than one group, then we obtain:* (*i*) $\sigma_i^{ub} < \sigma_i^{ob}$, (*ii*) $Q_i^{ub} < Q_i^{ob}$ *unless n is large,* (*iii*) $Q_{N+1}^{ub} > Q_{N+1}^{ob}$, (*iv*) $Q^{ub} < Q^{ob}$, (*v*) $\pi_i^{ub} > \pi_i^{ob}$ *unless n is sufficiently large, and* (*vi*) $\pi_{N+1}^{ub} > \pi_{N+1}^{ob}$, *for* $i = 1, ... , N$.

The following complements part (*c*) of Proposition 1. If values of V, m_i, N, and m_{N+1} are given, then Q_i^{ub} and π_i^{ub} remain unchanged while Q_i^{ob} and π_i^{ob} change, as n increases. Thus $Q_i^{ub} > Q_i^{ob}$ may hold for some i, but not all i, if n is large; $\pi_i^{ub} < \pi_i^{ob}$ may hold for some i, but not all i, if n is sufficiently large. We find that $Q_i^{ub} > Q_i^{ob}$ may hold for some i, if $n \geq 7$, and that $\pi_i^{ub} < \pi_i^{ob}$ may hold for some i, if $n \geq 10$.

Part (*a*) happens because the players in the group choose the same sharing rule in the two cases. Facing just one singleton, the players in the group do not behave differently between the two cases.

On the other hand, parts (*b*) and (*c*) say that the unobservability of sharing rules makes differences, as compared to the case of observable sharing rules, if there is more than one singleton or more than one group. In other words, the players behave differently between the two cases, if the competition is fiercer. Most importantly, group *i*'s equilibrium winner's fractional share is less in the case of unobservable sharing rules than in the case of observable sharing rules, which provides the intuitions behind the rest of the comparative results.[8] This can be explained as follows. In the case of observable sharing rules, the players in a group can make a strategic commitment for the competition in the effort-expending stage by announcing, in the first stage, their sharing rule.[9] Naturally, to achieve their competitive advantage in the competition in the effort-expending stage, the players in the group have an incentive to strongly motivate themselves to win the prize, and actually do so by choosing a "large" winner's fractional share. Such strategic behavior of the group leads to the finding that $\sigma_i^{ub} < \sigma_i^{ob}$.

Next, the equilibrium total effort level is less in the case of unobservable sharing rules than in the case of observable sharing rules. Not surprisingly, this happens because, in the case of observable sharing rules, the players in a group choose a larger winner's fractional share – so that they are motivated to exert more effort – compared to the case of unobservable sharing rules. On the basis of this result, we may argue that, in rent-seeking contests, the unobservability of sharing rules reduces social costs due to rent-seeking activities, compared to the case of observable sharing rules. We may also argue that, in patent competition, the unobservability of sharing rules delays the expected date of invention; in competition for presidential office, it reduces campaign spendings, compared to the case of observable sharing rules.

Another interesting comparative result is that $\pi_i^{ub} < \pi_i^{ob}$ holds in part (*b*), while $\pi_i^{ub} > \pi_i^{ob}$ holds in part (*c*). This can be explained as follows. The observability of the sharing rule, which enables the players in the group to make a strategic commitment for the competition in the subsequent effort-expending stage, is beneficial to them as compared with the case of unobservable sharing rules, when there is just one group. However, it is harmful to the players in the groups, when there is more than one group, due to their stiffer competition.

Finally, parts (*b*) and (*c*) say that the equilibrium expected payoff for each singleton is greater in the case of unobservable sharing rules than in the case of observable sharing rules. This comes from the following facts. First, σ_{N+1} is equal to unity both in the case of unobservable sharing rules and in the case of observable sharing rules. Next, in the case of unobservable sharing rules, σ_{N+1} is greater than the equilibrium winner's fractional share of any group, so that each singleton possesses a competitive advantage over the players in the groups. However, in the case of observable sharing rules, σ_{N+1} may be less than the equilibrium winner's fractional share of some group (see footnote 8). Lastly, $\sigma_i^{ub} < \sigma_i^{ob}$ holds for all *i*, which implies that the singletons face less motivated opponents in the case of unobservable sharing rules than in the case of observable sharing rules.

5. An extension: Endogenous group formation

In the model with exogenous groups we have considered so far, the number of groups, their sizes, and the number of singletons are exogenously given. In this section, we consider an extended model in which the number of groups, their sizes, and the number of singletons are endogenously determined. This extended model is the same as the model with exogenous groups with the exception that, in the extended model, the players decide first whether to form groups. In the extended model, we assume that $n \geq 2$, $N \geq 0$, and $2 \leq m_i \leq n$. Recall from Section 2 that we may treat the singletons as the members of group N+1 whose winner's fractional share is unity and publicly known.

We formally consider the following game. At the start of the game, the players decide simultaneously and independently whether to form groups, knowing that the winner should share the prize with other players in his group, if any. Next, after knowing the number of groups and their sizes, the players in each group jointly decide how to share the prize among themselves if one of them wins it. That is, they make a binding agreement on their sharing rule σ_i. Next, all the players in the contest choose their effort levels simultaneously and independently. When the players choose their effort levels, the players in each group know the sharing rule of their own group; however, each group's sharing rule is unobservable to the other groups and the singletons. Lastly, the winning player is determined, and the players in his group, if any, share the prize according to their sharing rule on which they agreed.

Different procedures can be designed for the players' simultaneous decisions on forming groups at the start of the game. One possible procedure is that the players first talk one another about forming groups, and then they decide simultaneously and independently whether to join the potential groups. The following is another possible, but more formal, procedure. Each player throws his name tag into one of n jars. If more than one player throws their name tags into the same jar, they form a group. If only one player throws his name tag in one of the jars, he does not belong to any group; instead, he becomes a singleton. In order to facilitate group formation, one player may be allowed to throw his name tag in one of the jars before the other players do.

5.1. Equilibrium numbers of groups, group sizes, and numbers of singletons

To solve the game, we need to work backward. However, the players' decisions on their effort levels and on their groups' sharing rules, given the number of groups, the group sizes, and the number of singletons, have already been considered in Section 3. Thus we are ready to consider the players' decisions on forming groups at the start of the game.

When deciding on forming groups, the players have perfect foresight about their subsequent decisions on the sharing rules of their groups to be formed and on their effort levels. This means that, at the start of the game, the players can compute their expected payoffs reported in Lemma 3.

The players decide simultaneously and independently whether to form groups. Thus, in equilibrium, no player has an incentive to individually move out of a group or to individually move in a group. Expressed differently, an equilibrium is immune to any unilateral deviation. We do not consider coordinated deviations by coalitions of players. Instead, in Section 6, we discuss using the concept of coalition-proof Nash equilibrium, introduced by Bernheim et al. (1987), to obtain the equilibrium numbers of groups, the equilibrium sizes of the groups, and the equilibrium numbers of singletons.

Let N^* represent the equilibrium number of groups, and let m_i^* the equilibrium size of group i, for $i = 1, \dots, N^*$. Let $m_{N^*+1}^*$ represent the equilibrium number of singletons. Using Lemma B2 in Appendix B, we obtain the left part of Table 2 and Proposition 2.

Proposition 2. (a) *If the number of players in the contest is four or smaller, then group formation occurs*. (b) *If the number of players is five or greater, then group formation does not occur*. (c) *If the contest consists of two or three players, only the grand coalition occurs*.

Proposition 2 is in line with the results in the literature on endogenous group formation in contests or conflicts. See, for example, Bloch (2012) for related discussions.

As shown in Table 2, if the number of players in the contest is four, then the equilibrium number of groups is not unique. In equilibrium, all four players in the contest join a single group; or there are two groups, each of which consists of two players; or the players compete individually to win the prize. It follows from Table 2 that, in equilibrium, every player belongs to one of the groups or all the players compete individually to win the prize.

When there is a small number of players in the contest, the grand coalition occurs. This is because each player in the grand coalition claims the share V/n of the prize – without expending any effort – which is greater than his expected payoff resulting when he moves out of the group. However, as the number n of players increases, each player's share V/n of the prize in the grand coalition becomes smaller, whereas his expected payoff resulting when he moves out of the group remains unchanged. Consequently, for $n \geq 5$, the grand coalition is not formed; furthermore, no group is formed.[10]

Quite different results are obtained on the equilibrium numbers of groups and the equilibrium sizes of the groups in the case of observable sharing rules – that is, in the game which is the same as the one in this section with the exception that each group's sharing rule is now observable to all the players in the game. Baik and Lee (2001) show, for example, the following. The equilibrium number of groups is one for $n \leq 5$, but is not unique for $n \geq 6$. Only the grand coalition occurs for $n = 2$ or 3, the equilibrium group sizes are 3 and 4 for $n = 4$ or 5, and the grand coalition does not occur for $n \geq 5$. When just one group is formed in equilibrium, there may be players who do not belong to the group and, for $n \geq 6$, the equilibrium group size equals the smallest of integers greater than half the number of players. When more than one group is formed, every player belongs to one of the groups, and the difference in equilibrium group size between any two groups is at most one.

Comparing the results of the two cases, those of unobservable and observable sharing rules, we argue that the unobservability of sharing rules hinders the players from forming groups because it nullifies strategic advantage that the players could gain if they formed groups and announced (and committed to) their sharing rules. In the case of observable sharing rules, the players in a group may discourage the outsiders by showing the outsiders that they have boosted their incentives through their winner's fractional share greater than unity, whereas such strategic behavior is not available to a singleton. This implies that forming a group is advantageous, which leads to the conclusion that group formation is a rule in the case of observable sharing rules. On the other hand, in the case of unobservable sharing rules, the players in a group cannot affect the effort levels of the outsiders by the choice of their sharing rule. Given this inability, they focus rather on reducing the negative externality among themselves by choosing their winner's fractional share less than unity, which makes them less competitive in the effort-expending stage. By contrast, the benefit to be a singleton is relatively large, especially if many players participate in the contest. Accordingly, forming a group is not so attractive, and group formation is an exception in the case of unobservable sharing rules. On the basis of this comparative result, we may expect that, in patent competition, less R&D joint ventures are formed and more independent firms exist in the case of unobservable sharing rules than in the case of observable sharing rules. We may also expect that, in college football, less conferences are formed and more independent teams exist in the case of unobservable sharing rules than in the case of observable sharing rules.

In the literature on the theory of endogenous coalition formation, the equilibrium coalition structures usually involve multiple coalitions, and the grand coalition is typically not an equilibrium coalition structure (see, for example, Bloch 1995; Yi 1998; Yi and Shin 2000).[11] This is in contrast to Proposition 2. Examining the equilibrium structures of strategic alliances of firms, Morasch (2000) finds that all firms join a single alliance if the number of firms in the industry is four or smaller, but alliance structures involving two or more alliances are quite likely if the number of firms in the industry is five or greater. Clearly, the former is similar to Proposition 2, whereas the latter is in contrast to Proposition 2.

5.2. Equilibrium effort levels and expected payoffs

Let Q^* represent the equilibrium total effort level in the game, and let π^* represent each player's equilibrium expected payoff in the game. Using Lemmas 2 and 3, Table 1, the left part of Table 2, and Lemma B1, we obtain the right part of Table 2.

It is of interest that, in Table 2, the equilibrium total effort level, Q^*, increases while each player's equilibrium expected payoff, π^*, decreases, as the number of players, n, increases. Table 2 implies that, for any number of players in the contest, the equilibrium total effort level is less than the players' valuation V for the prize – in other words, complete dissipation or overdissipation of the contested prize never occurs in this contest. Furthermore, Table 2 together with Lemma B1 implies that, for any number of players, the equilibrium total effort level does not exceed the total effort level resulting from the individual contest – in other words, endogenous group formation does not increase prize dissipation, as compared with the individual contest.

Table 2 together with Lemma B1 implies that, if $n = 2$ or 3, then each player's equilibrium expected payoff is greater than each player's expected payoff resulting from the individual contest; if $n = 4$, then it is greater than or equal to each player's expected payoff resulting from the individual contest; if $n \geq 5$, then it is equal to each player's expected payoff resulting from the individual contest. The explanations for this are straightforward. When there is a small number of players in the contest, the grand coalition occurs, so that the players share the prize equally without expending any effort. However, when $n \geq 5$, no group is formed, so that the players compete individually to win the prize. On the basis of the above result, we may argue that, given $n \leq 4$, endogenous group formation is beneficial to the players in the contest, as compared with the individual contest.

As shown in Table 2, if $n = 4$, then the equilibrium number of groups (or the equilibrium) is not unique. However, we may select the equilibrium yielding the grand coalition as the distinguished one by the following two facts. First, each player has the highest expected payoff in that equilibrium. Second, as will be discussed later in Section 6, if we use the concept of coalition-proof Nash equilibrium, then only the grand coalition occurs in equilibrium.

In the case of observable sharing rules, quite different results are obtained on the equilibrium total effort level and each player's equilibrium expected payoff. Baik and Lee (2001) show, for example, the following. First, when just one group is formed in equilibrium, the equilibrium total effort level is smaller than with the individual contest; when two groups are formed in equilibrium, it is equal to the total effort level resulting from the individual contest; however, when more than two groups are formed in equilibrium, *it is greater than with the individual contest*. Second, when just one group is formed in equilibrium, endogenous group formation is beneficial both to the group members and to the singletons, as compared with the individual contest; however, when more than two groups are formed in equilibrium, *it is never profitable to any players in the contest*.

6. Conclusions

We have studied contests in which n risk-neutral players compete by expending irreversible effort to win a prize. The prize is awarded to one of the players. The probability that a player wins the prize depends on the effort levels of the n players. The winner shares the prize with other players in his group, if any, and each group's sharing rule is unobservable to the other groups and the singletons, if any, when the n players choose their effort levels.

We have considered two models: the model with exogenous groups and the endogenous group formation model. The number of groups, their sizes, and the number of singletons are exogenous in the former, whereas they are endogenous in the latter. We have shown the following in Section 5. If the contest consists of two or three players, then the grand coalition occurs in equilibrium. If $n = 4$, then the grand coalition occurs; or there are two groups, each of which consists of two players; or the players compete individually to win the prize. If $n \geq 5$, then the individual contest occurs – that is, group formation does not occur. In Section 5, we have also shown the following. For any number of players in the contest, the equilibrium total effort level is less than the players' valuation V for the prize and, further, does not exceed the total effort level resulting from the individual contest. If $n \leq 4$, then endogenous group formation is beneficial to the players in the contest, as compared with the individual contest. In Sections 4 and 5, we have compared the outcomes of the case of unobservable sharing rules with those of the case of observable sharing rules. We have shown in each of the two models that the two cases yield quite different outcomes.

In Section 5, to obtain the equilibrium numbers of groups, the equilibrium sizes of the groups, and the equilibrium numbers of singletons, we have examined only whether a player has an incentive to individually move out of a group or to individually move in a group. What happens if we use the concept of coalition-proof Nash equilibrium which takes into account also coordinated deviations by coalitions of players? In this case, we obtain the following sharper predictions. If $n \leq 4$, then only the grand coalition occurs in equilibrium. If $n \geq 5$, then no group is formed and the individual contest occurs in equilibrium.

In the two models that we have studied in this paper, we have assumed that the players have the common valuation V for the prize. Instead, we can assume that the players have different valuations. In Sections 2 and 3, an alternative model is tractable in which each player in group i, for $i = 1, \ldots, N$, values the prize at V_i. However, we obtain results similar to those in this paper. In Section 5, an alternative tractable model is one in which player 1 values the prize at V_1 and players 2 through n each value it at V_2, where $V_1 > V_2$. We obtain the following results which are similar to those in this paper. First, group formation may occur for $n \leq 5$. Second, group formation does not occur for $n \geq 6$. Third, the grand coalition may occur for $n \leq 3$, but does not occur for $n \geq 4$.

We have assumed in this paper that the prize is a private good. It would be interesting to consider models in which the prize has both private-good and group-specific public-good components. Group formation might be more attractive in the modified (endogenous group formation) model.

In the two models of this paper, we have assmued that the prize is awarded to one of the players in the contest, and even the players in the same group compete with each other to win the prize. It would be interesting to consider models in which the players in each group pool their effort to win the prize, the prize is awarded to one of the singletons or one of the groups, and the players in the winning group, if any, share the prize among themselves.[12]

In this paper, private information on sharing rules is exogenously assumed. That is, we have exogenously assumed that each group's sharing rule is unobservable to the other groups and the singletons, if any, when the *n* players choose their effort levels. However, we may well expect that each group has the option of making its sharing rule observable or unobservable. Thus it would be interesting to consider models that incorporate groups' decisions on whether they will make their sharing rules observable or unobservable. Another possible extension of this paper would be to consider models in which the contest organizer or the decision-maker who has authority to select the winner decides first whether she will require groups to release their sharing-rule information. Consider, for example, patent competition. The government may decide first whether it will require R&D joint ventures to release their sharing-rule information.

Finally, we have assumed in Section 5 that, at the start of the game, the players decide simultaneously and independently whether to form groups. It would be interesting to consider a model in which the players decide sequentially whether to form groups and, if a player decides to join an existing group, the existing group may block the player from joining it.

We leave these possible modifications and extensions for future research.

Footnotes

1. The literature on the theory of contests is enormous and growing. Important work in this literature includes Tullock (1980), Rosen (1986), Dixit (1987), Hillman and Riley (1989), Katz et al. (1990), Nitzan (1991b), Baye et al. (1993), Nitzan (1994), Skaperdas (1996), Clark and Riis (1998), Hurley and Shogren (1998), Moldovanu and Sela (2001), Hvide (2002), Che and Gale (2003), Szymanski (2003), Corchón (2007), Epstein and Nitzan (2007), Congleton et al. (2008), Konrad (2009), and Siegel (2010).

2. In this respect, this paper is related to the literature on contests with externalities (see, for example, Esteban and Ray 1999; Kolmar and Wagener 2013).

3. The second-order condition is satisfied for every maximization problem in this paper. For concise exposition, however, we do not state it explicitly in each case.

4. Baik and Lee (2001) obtain the same result in the case of observable sharing rules. Nitzan and Ueda (2011) study collective contests in which each group's sharing rule is endogenously determined and is unobservable to the other groups, and identify the cases where a larger group chooses a more egalitarian sharing rule than a smaller group.

5. An increase in m_i leads to a decrease in $x_i(\boldsymbol{m})$, so that $Q_i(\boldsymbol{m})$ remains unchanged. This reminds us of the "waterbed effect" in industrial organization, which is used to describe the offset of prices in two-sided markets (see, for example, Genakos and Valletti 2011).

6. This implies that $Q_h(\boldsymbol{m})/Q(\boldsymbol{m}) = Q_t(\boldsymbol{m})/Q(\boldsymbol{m})$ holds for any h and t with h, $t = 1, \ldots, N$. That is, in equilibrium, the probability that any one of the players in group h wins the prize is the same as the probability that any one of the players in group t wins it, regardless of the groups' sizes. Considering a noncooperative rent seeking contest with quadratic costs in which players form groups sequentially, Bloch et al. (2006) show that the equilibrium winning probability is the same across groups. By contrast, Esteban and Ray (2001) show, in a model with nonlinear lobbying costs, that the equilibrium winning probability of a larger group is higher than that of a

smaller one. Nitzan and Ueda (2011) show that the equilibrium winning probability of a larger group is higher than that of a smaller one, unless the prize is purely private.

7. Esteban and Ray (2001) show, in a model with nonlinear lobbying costs, that the equilibrium expected payoff to a player increases with group size when the collective good is purely public, and decreases with group size when the collective good is purely private.

8. The following are additional information regarding group *i*'s equilibrium winner's fractional share. Recall from Lemma 1 that $1/m_i < \sigma_i^{ub} < 1$ holds always. By contrast, Baik and Lee (2001) show in the case of observable sharing rules that $1/m_i < \sigma_i^{ob} < 1$ holds if $m_i > n/2$; $\sigma_i^{ob} = 1$ holds if $m_i = n/2$; and $\sigma_i^{ob} > 1$ holds if $m_i < n/2$.

9. Sharing rules may not have strategic effects if each group's sharing rule is unobservable to the other groups and the singletons. For related issues, see Katz (1991), Bagwell (1995), Gal-Or (1997), and Baik and Kim (2014).

10. Bloch et al. (2006) show that the grand coalition is a unique equilibrium coalition structure in the noncooperative rent seeking contest with quadratic costs in which players form groups sequentially.

11. However, if coalition formation among symmetric players creates negative externalities for singletons, the grand coalition is an equilibrium coalition structure under the open membership rule.

12. In this modified model, one may assume that the probability that group *i* wins the prize is X_i/X for $X > 0$, and it is $1/(N + m_{N+1})$ for $X = 0$. One may use the following sharing-rule specification which is used in Nitzan (1991a, 1991b), Baik and Shogren (1995), Hausken (1995), Lee (1995), Davis and Reilly (1999), Ueda (2002), Baik and Lee (2007), Nitzan and Ueda (2011), and Ursprung (2012): The fractional share λ_{jk} of player *k* in the winning group, say group *j*, is

$$\lambda_{jk} \;=\; \theta_j x_{jk}/X_j \;+\; (1 - \theta_j)/m_j,$$

where the parameter θ_j is chosen by the players in group *j* before they choose their effort levels. Note that the players in group *j* need to know how much effort each player expended when they share the prize. Based on the results in Baik et al. (2006), one may conjecture that the modified model with this sharing-rule specification yields the same (main) results as in the current paper.

Appendix A: Obtaining the players' equilibrium effort levels

From equations (8) and (9), we have the following system of N+1 simultaneous equations:

$$\begin{aligned} Q^2 &= V(m_2x_2 + m_3x_3 + \cdots + m_Nx_N + m_{N+1}x_{N+1}) \\ Q^2 &= V(m_1x_1 + m_3x_3 + \cdots + m_Nx_N + m_{N+1}x_{N+1}) \\ &\ \ \vdots \\ Q^2 &= V(m_1x_1 + m_2x_2 + \cdots + m_{N-1}x_{N-1} + m_{N+1}x_{N+1}) \end{aligned}$$

and $$Q^2 = V\{m_1x_1 + m_2x_2 + \cdots + m_Nx_N + (m_{N+1} - 1)x_{N+1}\}.$$

These equations can be rewritten as

$$\begin{aligned} Q^2 &= V(Q - m_1x_1) \\ Q^2 &= V(Q - m_2x_2) \\ &\ \ \vdots \\ Q^2 &= V(Q - m_Nx_N) \end{aligned} \tag{A1}$$

and $$m_{N+1}Q^2 = V(m_{N+1}Q - m_{N+1}x_{N+1}).$$

Adding these equations together, we have

$$(N + m_{N+1})Q^2 = V\{(N + m_{N+1})Q - Q\}.$$

This yields

$$Q = V(N + m_{N+1} - 1)/(N + m_{N+1}).$$

Substituting this expression for Q into the equations in (A1), we obtain the players' equilibrium effort levels, $x_1(\boldsymbol{m})$ through $x_{N+1}(\boldsymbol{m})$.

Appendix B: Proof of Proposition 2

Lemma B1, which can be easily proved, is useful in obtaining Lemma B2 below and the right part of Table 2.

Lemma B1. (a) *If a grand coalition is formed* (*i.e.*, $m_1 = n$), *then the equilibrium sharing rule of the group is* $1/n$, *each player in the group chooses zero effort levels, and each player's equilibrium expected payoff is* V/n. (b) *If the players compete individually to win the prize* (*i.e.,* $N = 0$), *then each player chooses an effort level of* $V(n-1)/n^2$, *the equilibrium total effort level is* $V(n-1)/n$, *and each player's equilibrium expected payoff is* V/n^2.

Using Lemma 3 and Lemma B1, we obtain Lemma B2, which is useful in obtaining Proposition 2.

Lemma B2. (a) *If the difference in group size between two actual groups is two or greater, then a player in the larger group has an incentive* (*to move out of the larger group and*) *to move in the smaller group. Otherwise, no player in one group has an incentive to move in the other group.* (b) *Suppose that* $N \geq 2$. *If* $N = 2$, $m_1 = m_2 = 2$, *and* $m_{N+1} = 0$, *then no player in the groups has an incentive to be a singleton. Otherwise, there is at least one player in the groups who has an incentive to be a singleton.* (c) *Suppose that* $N = 1$ *and* $m_{N+1} \geq 1$. *If* $m_1 = 2$ *and* $m_{N+1} = 1$, *then no player in the group has an incentive to be a singleton. Otherwise, a player in the group has an incentive to be a singleton.* (d) *Suppose that a grand coalition is formed. If* $m_1 = n = 2$, 3, *or* 4, *then no player in the group has an incentive to be a singleton. Otherwise, a player in the group has an incentive to be a singleton.* (e) *Suppose that* $N \geq 1$ *and* $m_{N+1} \geq 1$. *If* $N = 1$, $m_N = 2$, *and* $m_{N+1} = 1$, *then the singleton has an incentive to move in group* N. *Otherwise, no singleton has an incentive to move in group* N. (f) *Suppose that* $N = 0$. *If* $m_{N+1} = n = 2$ *or* 3, *then a singleton has an incentive to unilaterally deviate* (*by throwing his *name tag into another player's jar*). *Otherwise, no singleton has an incentive to unilaterally deviate*.

The proof of Lemma B2 is straightforward and therefore omitted. In proving part (*a*), for example, we compare the expected payoff for each player in one group – computed before a player moves out of the group – with the expected payoff for each player in the other group resulting when a player moves out of the first group and moves in the second group. Note that, in Lemma B2, a player has an incentive to move out of a group and/or to move in a group if such a movement increases his expected payoff.

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TABLE 1

The Outcomes of the Two Cases

	Unobservable Sharing Rules	Observable Sharing Rules
Group *i*'s Sharing Rule	$1-\frac{m_i-1}{m_i(N+m_{N+1})}$	$1+\frac{n-2m_i}{m_i\{n(N-1)+2m_{N+1}\}}$
Effort Level of Each Player in Group *i*	$\frac{V(N+m_{N+1}-1)}{m_i(N+m_{N+1})^2}$	$\frac{V(n-m_i)\{n(N-1)+2m_{N+1}-1\}}{m_i\{n(N-1)+2m_{N+1}\}^2}$
Effort Level of Each Singleton	$\frac{V(N+m_{N+1}-1)}{(N+m_{N+1})^2}$	$\frac{V\{n(N-1)+2m_{N+1}-1\}}{\{n(N-1)+2m_{N+1}\}^2}$
Group *i*'s Effort Level	$\frac{V(N+m_{N+1}-1)}{(N+m_{N+1})^2}$	$\frac{V(n-m_i)\{n(N-1)+2m_{N+1}-1\}}{\{n(N-1)+2m_{N+1}\}^2}$
Singletons' Total Effort Level	$\frac{Vm_{N+1}(N+m_{N+1}-1)}{(N+m_{N+1})^2}$	$\frac{Vm_{N+1}\{n(N-1)+2m_{N+1}-1\}}{\{n(N-1)+2m_{N+1}\}^2}$
Total Effort Level	$V(1-\frac{1}{N+m_{N+1}})$	$V[1-\frac{1}{n(N-1)+2m_{N+1}}]$
Expected Payoff for Each Player in Group *i*	$\frac{V}{m_i(N+m_{N+1})^2}$	$\frac{V(n-m_i)}{m_i\{n(N-1)+2m_{N+1}\}^2}$
Expected Payoff for Each Singleton	$\frac{V}{(N+m_{N+1})^2}$	$\frac{V}{\{n(N-1)+2m_{N+1}\}^2}$

TABLE 2

The Equilibrium Numbers of Groups, Group Sizes, Numbers of Singletons, Effort Levels, and Expected Payoffs

n	N^*	m_1^*	m_2^*	$m_{N^*+1}^*$	Q^*	π^*
2	1	2		0	0	$V/2$
3	1	3		0	0	$V/3$
4	1	4		0	0	$V/4$
4	2	2	2	0	$V/2$	$V/8$
4	0			4	$3V/4$	$V/16$
5	0			5	$4V/5$	$V/25$
6	0			6	$5V/6$	$V/36$
$\vdots$	$\vdots$			$\vdots$	$\vdots$	$\vdots$
s	0			s	$V(s-1)/s$	V/s^2
$\vdots$	$\vdots$			$\vdots$	$\vdots$	$\vdots$